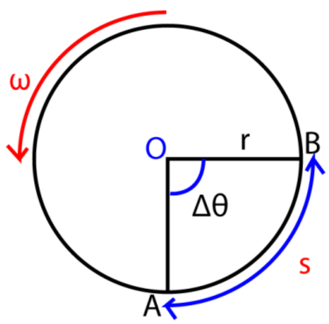
SENIOR FIVE TERM 1

TOPIC 1/6: CIRCULAR MOTION

Competency: The learner examines the motion of bodies on curved paths and ensures safety precautions for people using automobiles on such paths.

Circular motion

This is the motion of an object moving in a circular path with a uniform speed around a fixed point O. consider a body moving from A to B in a small time Δt such that the radius r, sweeps through a small angle Δθ in radians



Distance, AB = s = rΔθ

But speed, v = $\frac{distance}{time} = r\frac{\Delta\theta}{\Delta t}$

As Δt → 0, $\frac{\Delta\theta}{\Delta t} \rightarrow \frac{d\theta}{dt}$

v = $r\frac{d\theta}{dt}$

Since, $\frac{d\theta}{dt}$ = ω

v = rω

Terminology in circular motion

Angular velocity, ω

This is the rate of the angle for an object moving in a circular path about in a circular path about the center. S.I units are $rads^{-1}$.

Period, T
Time taken to make one complete revolution

Time = $\frac{Distance}{speed}$
$= \frac{2\pi r}{v}$
$= \frac{2\pi r}{r\omega}$
$= \frac{2\pi}{\omega}$

Frequency, f
This is the number of revolutions made in one second. The S.I units are hertz (Hz)
F = $\frac{1}{T}$
$= \frac{\omega}{2\pi}$

Centripetal acceleration (a)
This is the rate of change of velocity for a body moving in a circular path and it is directed towards the center of that circular path.
a = $\frac{v^2}{r} = r\omega^2$

Derivation of a = $\frac{v^2}{r}$ or $r\omega^2$

Consider an object moving with a constant speed, v, round a circle of radius, r.

In figure (i) below; at A, its velocity v_A is in direction of the tangent AC, a short time dt later at B, its velocity v_B is in the direction of tangent BD

Since their directions are different, the velocity v_B is different from the velocity v_A although their magnitude are both equal to v.

Thus a velocity change or acceleration has occurred from A to B

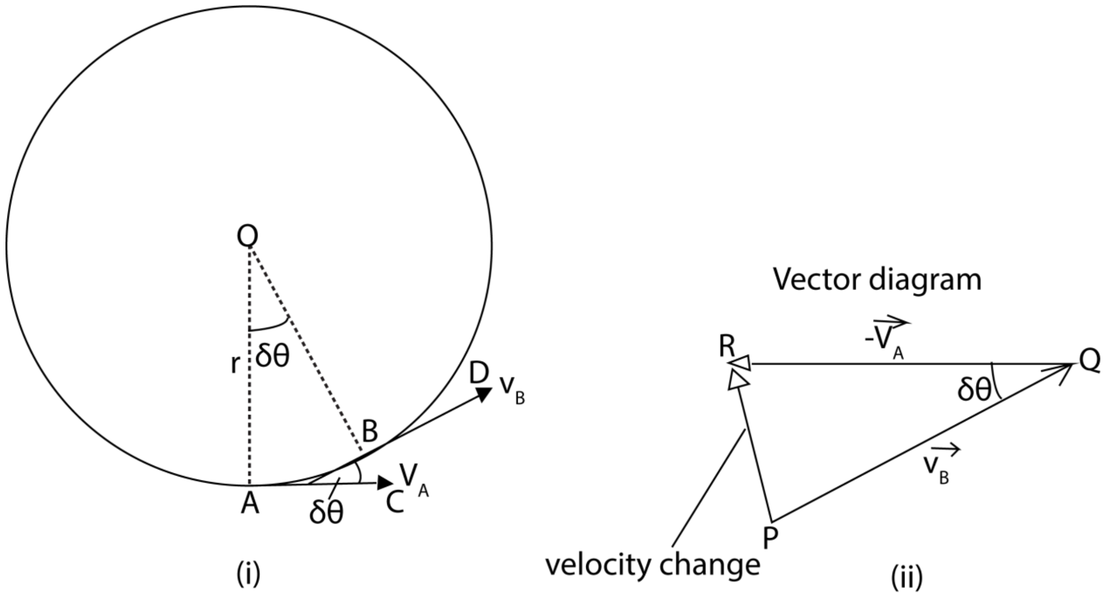


Fig. 2: Acceleration in circle

The velocity change from A to B = $v_B - v_A$ or $v_B + (-v_A)$.

In figure 2(ii) above, PQ represents v_B in magnitude (v) and direction BD; QR represents $-v_A$ in magnitude (v) and direction (CA).

Velocity change = $v_B + (-v_A)$ = PR

When δt is small, the angle AOB or δθ is small;
Also angle PQR equal to δθ is small

PR or acceleration then points toward O, the center of the circle.

a = $\frac{velocity\ change}{time} = \frac{PR}{\delta t} = \frac{v\delta\theta}{\delta t}$
but $\frac{\delta\theta}{\delta t}$ =ω and v = rω

a = rω x ω = $r\omega^2$

thus an object moving in a circle of radius r with a constant speed v has a constant acceleration towards the center equal to $\frac{v^2}{r} = r\omega^2$

Centripetal force
This is the force which keeps the body moving in a circular path and it is directed towards the center of the circular path.

F = ma = $m\frac{v^2}{r} = mr\omega^2$

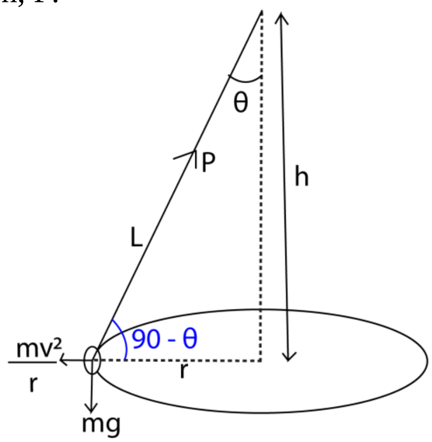
Centrifugal force

This is the force acting on a body moving in a circular path.

Examples of circular motion

1. A conical circular pendulum

Consider a mass , m, attached to a string of length L moving around a horizontal circle of radius, R, at a constant speed, v, and the string make an angle θ with the vertical and have a tension, P.



Resolving vertically

Pcosθ = mg ……………………………………………….. (i)

Resolving horizontally

Psinθ = m$\frac{v^2}{r}$ ……………………………………………… (ii)

Eqn (i) and Eqn (ii)

tan θ =$\frac{v^2}{rg}$

From the diagram

sin θ = $\frac{r}{L}$, r = Lsinθ

cos θ = $\frac{h}{L}$, r = Lcos θ

Pcos θ = mg

⇨ P . $\frac{h}{L}$ = mg

P = $\frac{mgL}{h}$ …………………………….. (iii)

Psinθ = m$\frac{v^2}{r}$

P . $\frac{r}{L}$ = m$\frac{v^2}{r}$

P = $\frac{mv^2L}{r^2}$

But v = rω

$P = \frac{m(r\omega)^2 L}{r^2} = mL\omega^2$ …………………….. (iv)

Eqn (iii) and (iv)

$mL\omega^2 = \frac{mgL}{h}$

$\omega^2 = \frac{g}{h}$

From $T = \frac{2\pi}{\omega}$

$\omega = \frac{2\omega}{T}$

$\frac{g}{h} = \left[\frac{2\pi}{T}\right]^2 = \frac{4\pi^2}{T^2}$

$T^2 = \frac{4\pi^2 h}{g}$

$T = \sqrt{\frac{2\pi^2 h}{g}} = 2\pi\sqrt{\frac{h}{g}}$

Example 1

A mass of 0.2kg is whirled in a horizontal circle of radius 0.5m by a string inclined at 300 to the vertical. Find

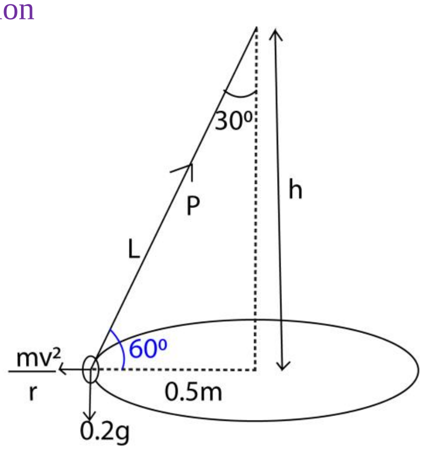
(i) The tension in the string

(ii) The speed of the mass in the horizontal plane

(iii) The length of the string

(iv) The angular speed

Solution



(i) Resolving vertically

$P\cos\theta = mg$

$P = \frac{mg}{\cos\theta} = \frac{0.2 \; x \; 9.81}{\cos 30} = 2.2655N$

Tension P = 2.2655N

(ii) Resolving vertically

$P\sin\theta = m\frac{v^2}{r}$

$v^2 = \frac{rP\sin\theta}{m}$

$v = \sqrt{\frac{rP\sin\theta}{m}} = \sqrt{\frac{(0.5 \; x \; 2.2655 \; x \; \sin 30^0}{0.2}} = 1.6828ms^{-1}$

(iii) From $\sin\theta = \frac{r}{L}$

$$L = \frac{0.5}{\sin 30} = 1m$$

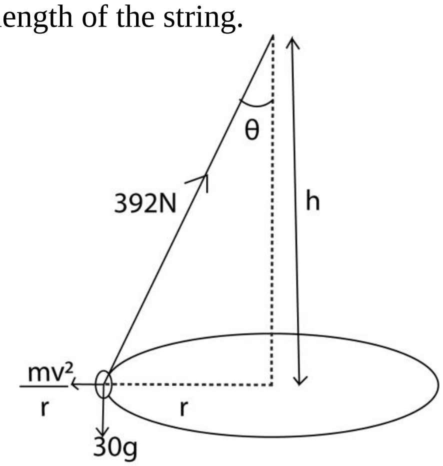
(iv) From $\omega = \frac{v}{r} = \frac{1.6828}{0.5} = 3.265 rads^{-1}$

Example 2

A 30kg body is swirled in a horizontal circle as a conical pendulu by means of inelastic string that has a breaking strength of 392N. when the speed of the body is $8ms^{-1}$, the string broke. Calculate

(i) The angle the string made at that instant.

(ii) The length of the string.



(i) From $P\cos\theta = mg$

$$\cos\theta = \frac{30 \times 9.81}{392}$$

$$\theta = 41.34$$

(ii) From $P\sin\theta = m\frac{v^2}{r}$

$$r = \frac{30 \times 8^2}{392 \sin 41.34^0} = 7.42$$

But $\sin\theta = \frac{r}{L}$

$$L = \frac{7.42}{\sin 41.34} = 11.23m$$

Example 3

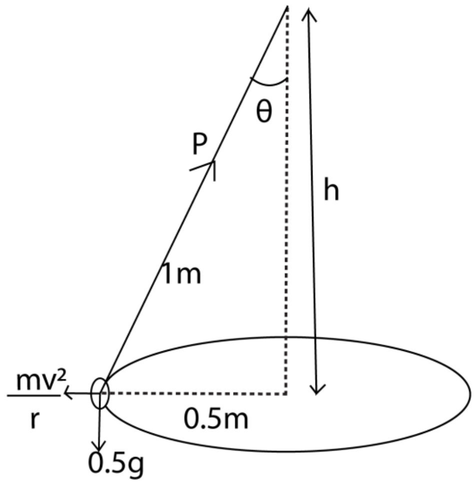
A steel ball of mass 0.5kg is suspeneded from a light inelastic string of length 1m, the ball is whirled in horizontal cirle of radis 0.5m. find

(i) Centripetal force and tension in the string

(ii) The angular speed of the ball

(iii) The angle between the string and the radius of the circle is thetension in string is 10N

Solution



(i) From sinθ = $\frac{r}{L}$

$\theta = \sin^{-1}\frac{0.5}{1} = 30^0$

From Pcos θ = mg

Tension, P = $\frac{mg}{\cos\theta} = \frac{0.5\ 9.81}{\cos 30}$= 5.664N

But, Psinθ = F

F = 5.664 sin 30= 2.832N

(ii) tanθ = $\frac{v^2}{rg}$

v = $\sqrt{rgtan\ \theta} = \sqrt{[0.5\ x\ 9.81\ x \tan 30^0]}$ = 1.683

But, ω =$\frac{v}{r} = \frac{1.683}{0.5} = 3.366 rads^{-1}$

(iii) From Pcos θ = mg

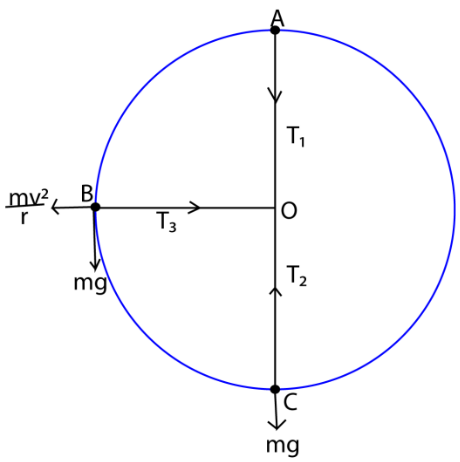
cos θ = $\frac{0.5x\ 9.81}{10}$

θ = 60.60

the required angle = 90 – 60.6 = 29.4^0

Motion in verticle circle

Consider a body of mass, m, attached to a string of length, r, and whirled in a vertical circle at constant velecity, v.



At A, $m\frac{v^2}{r} = T_1 + mg$

$T_1 = m\frac{v^2}{r} - mg$

At B, $T_3 = m\frac{v^2}{r}$

At C, $T_4 = m\frac{v^2}{r} + mg$

From the above expressions, tension in the string is minimum aat the top of the circle and maximum at the bottom of the circle. So the string is most likely to break when the body is at the bottom of the circle.

Example 4

A mass of 0.4kg is rotated by a string at a constant speed, v, in a vertical circle of radius 1m. If the minimum tension in a string is 3N. calculate

(i) The velocity
(ii) The maximum tension
(iii) Tension when the string is just horizontal

Solution

Minimum tension, $T_1 = m\frac{v^2}{r} - mg$

$3 = \frac{0.4v^2}{1} - 0.4 \ x \ 9.81$

$v = 4.16ms^{-1}$

Maximum tension $T_3 = m\frac{v^2}{r} + mg$

$= \frac{0.4 \ x \ 4.16^2}{1} + 0.4 \ x \ 9.81$

$= 10.85N$

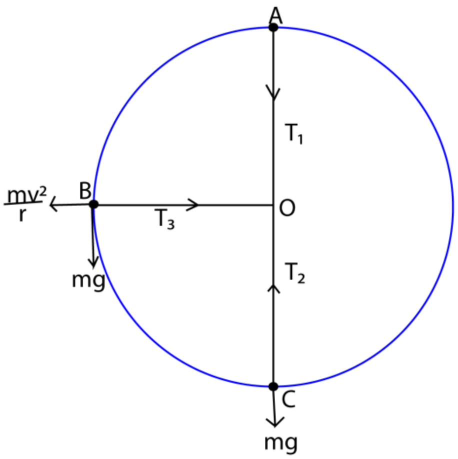
Tension when the string is horizontal $T = m\frac{v^2}{r} = \frac{0.4 \ x \ 4.16^2}{1} = 6.92N$

Example 5

A particle of mass 5kg describes a complete vertical circle at the end of a light inextensible string of length 2m. given that the speed of the particle is $5ms^{-1}$ at the highest point. Find

(i) Speed at the lowest point
(ii) Tension in the string when it is horizontal
(iii) Magnitude of centripetal acceleration when the string is horizontal

Solution



(i) Mechanical energy at A = $m\frac{v^2}{r}$ + mg

r = 2m, m = 5kg, v = $5ms^{-1}$

Mechanical energy at A = $\frac{1}{2}x\ 5\ x\ 5^2 + 5\ x\ 9.81\ x\ 4$ = 258.7J

Mechanical energy at C = $m\frac{v^2}{r}$ + mg = $\frac{1}{2}x\ 5\ x\ v^2 + 5\ x\ 9.81x\ 0$ = $2.5v^2$ J

But from the principle of conservation of energy mechanical energy

Mechanism energy at A = mechanical energy a t C

$258.7 = 2.5v^2$

$v = 10.2ms^{-1}$.

(ii) Mechanical advantage at B, = $m\frac{v^2}{r} + mgh$ = 5 x $\frac{v^2}{2}$ + 5 x 9.81 x 2 = 258.7

v = 8ms

tension at B = $m\frac{v^2}{r}$ = $5\ x\ \frac{8^2}{2}$ =160N

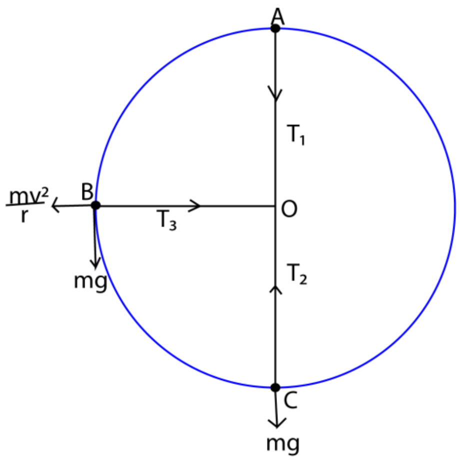
(iii) From a = $\frac{v^2}{r} = \frac{8^2}{2}$ =$32ms^{-1}$

Example 6

A particle of mass m describes a complete vertical inextensible string of length, r, given that the speed at the lowest point is twice the speed at highest point. Show

(i) The speed of the particle at the lowest point is = v= $4\sqrt{\frac{gr}{3}}$.

(ii) The tension in the string when the particle is at the highest point, T = $\frac{mg}{3}$



(i) Mechanical energy at A = mechanical energy at B = m$\frac{v^2}{r}$ + mg

Let the speed at A = u

The speed at C = 2u

⇨ m$\frac{u^2}{2} + mg.2r = m\frac{(2u)^2}{2} + mg\ x\ 0$

$u^2 + 4gr = 4vu^2$

$3u^2 = 4gr$

u = $2\sqrt{\frac{gr}{3}}$.

At the lowest point velocity = 2u = $4\sqrt{\frac{gr}{3}}$.

(ii) Tension at the highest point = m$\frac{v^2}{r}$ – mg = $\frac{4mgr}{3r} - mg$ = $\frac{4mg-3mg}{3}$ = $\frac{mg}{3}$

Revision Exercise 1

1. An object of mass 0.kg on the end of a string is whirled around in a horizontal circle of radius 2m, with a constant speed of 10ms^{-1}. Find its angular velocity and the tension in the string [Ans. ω = 5rads^{-1}, T = 25.5N)
2. A small ball of mass 0.1kg is suspended by an inextensible string of length 0.5m and is caused to rotate in a horizontal circle of radius 0.4m. find
   (i) The tension in the string [Ans. 1.3N]
   (ii) The period of rotation [ans. 1.2s]
3. A pendulum bob of mass 0.2kg is attached to one end of an inelastic string of length 1.2m. the bob moves in a horizontal circle with the string inclined at 30^0 to the vertical. Calculate
   (i) Tension in the string [Ans. 2.27N]

(ii) The period of motion [Ans. 2.02s]

4. The period of oscillation of a conical pendulum is 2.0s. if the string makes an angle 60^0 to the vertical at the point of suspension, calculate the
   (i) vertical height of the point of suspension above the circle [h = 0.994m]
   (ii) length of the string [L= 1.99m]
   (iii) velocity of mass attached to the string. [v = 5.41ms^{-1}]

Motion of Bicycle Rider Round Circular Track

When a person on a bicycle rides round a circular racing track, the frictional force F at the ground provides the inward force· towards the centre or centripetal force, Fig. 1below

This produces a moment about his centre of gravity G which is counterbalanced, when he leans inwards, by the moment of the normal reaction R.

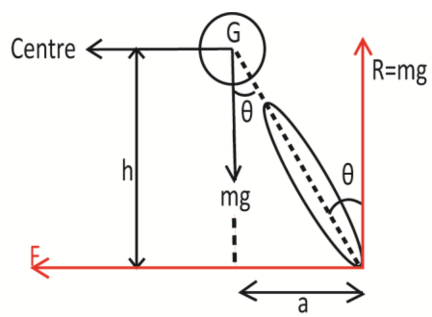


Fig.1. Rider on circular track

Thus provided no skidding occurs,

F . h = R . a= mg . a, since R = mg for no vertical motion .

tanθ = $\frac{a}{h} = \frac{F}{mg}$

But F = $\frac{mv^2}{r}$

∴ tanθ = $\frac{v^2}{rg}$

When F is greater than the limiting friction, skidding occurs. In this case F > µmg, or mg tan θ > µmg. Thus tan θ > µ is the condition for skidding.

Motion of Car (or Train) Round Circular Track

Suppose a car (or train) is moving with a velocity v round a horizontal circular track of radius r, and let R_1, R_2 be the respective normal reactions at the wheels A, B, and F 1, F 2 the corresponding frictional forces, Fig. 2.

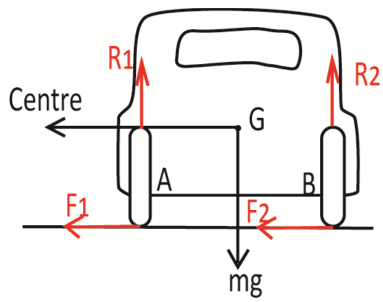


Fig. 2 Car on circuar track

Then, for circular motion we have

Horizontally, $(F_1 + F_2) = \frac{mv^2}{r}$ ....(i)

and vertically, $(R_1 + R_2) = mg$ .... (ii)

Taking moments about G

$(F_1 + F_2)h + a(R_1 - R_2)$..(iii)

where 2a is the distance between the wheels, assuming G is mid-way between the wheels, and h is the height of G above the ground.

From these three equations (i), (ii), (iii)

$$R_2 = \frac{1}{2}m\left(g + \frac{v^2h}{ra}\right)$$

and vertically $R_1 = \frac{1}{2}m\left(g - \frac{v^2h}{ra}\right)$

R2 never vanishes since it always has a positive value. But if $v^2 = \frac{arg}{h}$, $R_1 = 0$, and the car is about to overturn outwards. R_1 will be positive if $v^2 < \frac{arg}{h}$

Motion of Car (or Train) Round Banked Track

Suppose a car (or train) is moving round a banked track in a circular path of horizontal radius r, Fig. 3. the only forces at the wheels

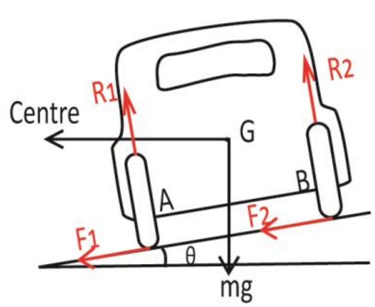


Fig. 3. Car on banked track

If A, B are the normal reactions R_1, R_2 respectively, that is, there is no side-slip or strain at the wheels, the force towards the centre of the track is $(R_1 + R_2)\sin\theta$, where θ is the angle of inclination of the plane to the horizontal.

$$\therefore (R_1 + R_2)\sin\theta = \frac{mv^2}{r} \quad \text{..(i)}$$

For vertical equilibrium, $(R_1 + R_2)\cos\theta = mg$ ....(ii)

Dividing (i) by (ii), $\tan\theta = \frac{v^2}{rg}$ .... (iii)

Thus for a given velocity v and radius r, the angle of inclination of the track for no side-slip must be $\tan^{-1}\left(\frac{v^2}{rg}\right)$. As the speed v increases, the angle θ increases, from (iii). A racing-track is made saucer-shaped because at higher speeds the cars can move towards a part of the track which is steeper and sufficient to prevent side-slip. The outer rail of a curved railway track is· raised about the inner rail so that the force towards the centre is largely provided by the component of the reaction at the wheels. It is desirable to bank a road at comers for the same reason as a racing track is banked.

Thrust at Ground

Suppose now that the car (or train) is moving at such a speed that-the frictional forces at A, B are F_1, F_2 respectively, each acting towards the centre of the track.

Resolving horizontally, $(R_1 + R_2)\sin\theta + (F_1 + F_2)\cos\theta = \frac{mv^2}{r}$ ....(i)

Resolving vertically, $(R_1 + R_2)\cos\theta + (F_1 + F_2)\sin\theta = mg$ ....(ii)

Simplifying. We find $(F_1 + F_2) = m\left(\frac{v^2}{r}cos\theta - gsin\theta\right)$ .... (iii)

If $\frac{v^2}{r}cos\theta > gsin\theta$, then $(F_1 + F_2)$ is positive; and in this case both the thrusts on the wheels at the ground are towards the centre of the track.

If $\frac{v^2}{r}cos\theta < gsin\theta$, then $(F_1 + F_2)$ is negative. In this case the forces F_1 and F_2 act outwards away from the centre of the track.

For stability, we have, by moments about G,

$(F_1 + F_2)h + R_1a - R_2a = 0$

$\therefore (F_1 + F_2)\frac{h}{a} = R_1 - R_2$

From (iii)

$$\therefore \frac{mh}{a}\left(\frac{v^2}{r}cos\theta - gsin\theta\right) = R_1 - R_2 \quad \text{...(iv)}$$

The reactions R_1, R_2 can be calculated by finding $(R_1 + R_2)$ from equations (i), (ii), and combining the result with equation (iv). This is left as an exercise to the student.

Topical revision questions

1. (i) Explain the significance of the banked tracks. (02 marks)

   It enables vehicles to navigate turns at higher speeds without skidding because, on banked track, a component of normal reaction supplements friction to provide the centripetal force.

   (ii) Explain why a cyclist bends inward while going round a curved path. (03marks)

   A cyclist bends inwards while going round a curved path to counterbalance centripetal force provided by friction between the tyres and the road such that skidding does not occurs.

   (iii) Derive an expression for the speed of a bicycle rider around a circular path. (03 marks)

   Assumption

   v = speed of the bicycle

   r = radius of the circular path

   m= mass of the bicycle and the rider

   g = acceleration due to gravity

   θ = banking angle of the track (if applicable)

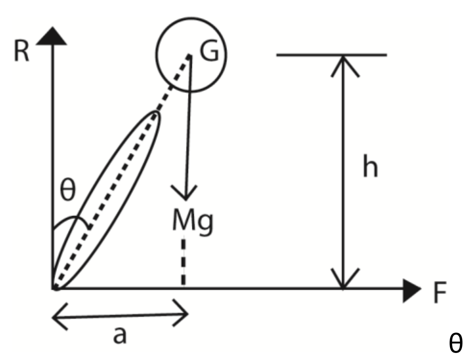
   F_c = centripetal force= $\frac{mv^2}{r}$

   F_r = friction = μR=μmg

   For a circular motion centripetal force = friction

   $$\Rightarrow \frac{mv^2}{r} = \mu mg$$
   $$v = \sqrt{\mu mgr}$$

   For banked track



Ncosθ = mg (vertical component)

N = $\frac{mg}{cos\theta}$ …………………………………………………………….(i)

Nsinθ =$\frac{mv^2}{r}$(horizontal component) ……………………(ii)

Substituting (i) into (ii)

$$\frac{mv^2}{r} = \frac{mg}{cos\theta}sin\theta$$

$$v = \sqrt{rgtan\theta}$$

(iv) A body of mass 1.5kg moves once round a circular path to cover 44.0cm in 5s. Calculate the centripetal force acting on the body. (04marks)

Radius of circular path, r

Circumference = 2πr = 44cm

r = $\frac{44}{2\pi}$ =7cm

Linear velocity, v = $\frac{0.44}{5s}$ = 0.088ms^{-1}

Centripetal force, $F = \frac{mv^2}{r} = \frac{1.5 \, x \, 0.088^2}{7 \, x \, 10^{-2}}$ = 0.166N

2. A stone tied to a string is whirled in a horizontal circle. Explain the motion of the stone when the string breaks. (05marks)

- **Before the String Breaks**:
  - The stone is kept in a circular path by the tension in the string, which provides the necessary centripetal force directed towards the center of the circle.
  - The stone continuously changes direction as it moves along the circular path, but the tension in the string ensures it stays in the circle.
- **At the Moment the String Breaks**
  - The string likely to break at the lowest point in the circle because tha tis where tension in the string is highest.
  - The centripetal force disappears immediately because the tension in the string is no longer present.
  - Without this force, there is nothing to keep the stone moving in a circular path.
- **After the String Breaks**:
  - According to Newton's First Law of Motion (inertia), an object in motion will continue in motion with the same speed and in the same direction unless acted upon by an external force.
  - Therefore, the stone will move in a straight line tangent to the circle at the point where the string breaks. This path is called a tangential path.
  - The stone's velocity at the moment the string breaks determines the direction of this straight-line motion.

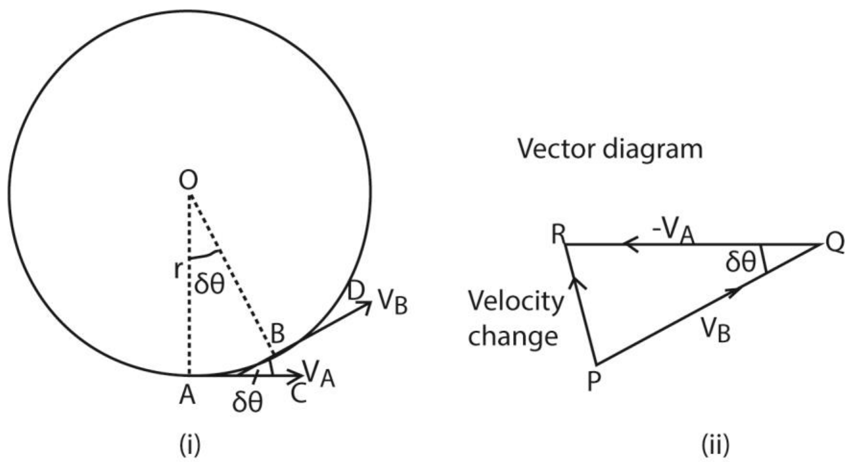
3. (a) (i) Define angular velocity. (01mark)

Angular velocity is the rate of change of angular displacement.

(ii) Explain why a body moving with constant speed along a circular path has an acceleration. (03 marks)

Acceleration is a rate of change of velocity with respect to time. In the **circular motion**, the **speed** of the **body** is **constant** but velocity changes continuously due to changes of its direction. Hence it is an **accelerated** motion

(iii) Derive an expression for the acceleration of a body moving in a circular path of radius r with a constant speed V. (04marks)



Acceleration in circle

The velocity change from A to B = $V_B - V_A$ or $V_B + (-V_A)$.

In figure 2(ii) above, PQ represents V_B in magnitude (V) and direction BD; QR represents $-V_A$ in magnitude (V) and direction (CA).

Velocity change = $v_B + (-v_A)$ = PR

When δt is small, the angle AOB or δθ is small;
Also angle PQR equal to δθ is small

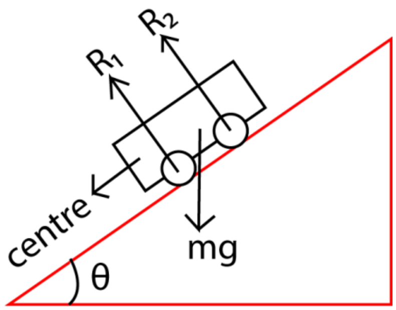
PR or acceleration then points toward O, the centres of the circle.
PR = $-V_A \sin \delta\theta \approx -V_A\delta\theta = V\delta\theta$

a = $\frac{velocity\ change}{time} = \frac{PR}{\delta t} = \frac{V\delta\theta}{\delta t}$
but $\frac{\delta\theta}{\delta t}$ =ω and V = rω

a = rω x ω = $r\omega^2$ but ω = $\frac{V}{r}$
a = $\frac{v^2}{r}$

(iv) Derive the condition for a car to move round a banked circular track without slipping. (04 marks)

Consider car negotiating a bend inclined at an angle θ to the horizontal. It is assumed that there is no tendency to slip at the wheels, therefore no frictional forces.



Resolving horizontally

$R_1\sin\theta + R_2\sin\theta = m\frac{v^2}{r}$ .... (i)

Resolving vertically

$R_1\cos\theta + R_2\cos\theta = mg$ .... (ii)

Eqn. (i) ÷ Eqn. (ii)

$$\frac{R_1\sin\theta + R_2 sin\theta}{R_1\cos\theta + R_2 cos\theta} = {m\frac{v^2}{r}}\Big/{mg}$$

$$\tan\theta = \frac{v^2}{rg}$$

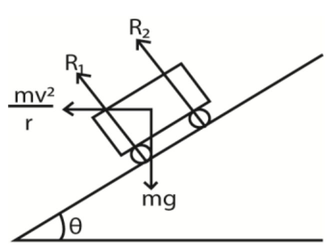
$$v^2 = rgtan\,\theta$$

$$v = \sqrt{rgtan\,\theta}$$

Hence car will not skip when $v \leq \sqrt{rgtan\,\theta}$

(v) A car of mass 1000kg moves round a banked track at constant speed of 108kmh^{-1}. Assuming the total reaction at the wheels is normal to the track, and the radius of curvature of the truck is 100m, calculate the:

(i) angle of inclination of the track to the horizontal (04marks)



Frθm tanθ = $\frac{v^2}{rg}$

v = 108kmh^{-1} = $\frac{108\ x1000}{60\ x\ 60} = 30ms^{-1}$

tan θ = $\frac{30^2}{100\ x\ 9.81}$

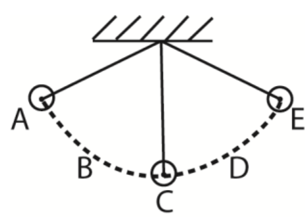
θ = 42.5^0

(ii) reaction at the wheels (02marks)

From (R_1 + R_2) sinθ = $\frac{mv^2}{r}$

=> R_1 + R_2 = $\frac{mv^2}{r sin\theta}$ = $\frac{1000\ x\ 30^2}{100\ \sin 42.5^0}$ = 1.33 x 10^4N

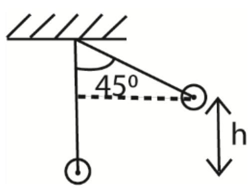
4. (a) Explain the energy changes which occur when a pendulum is set into motion. (03marks)



Potential energy at A → Kinetic + potential energy at B → kinetic at C → Kinetic + potential energy at D→ potential energy at E

(b) A simple pendulum of length 1m has a bob of mass 100g. It is displaced from mean position A to position B so that the string makes an angle of 45^0 with the vertical. Calculate the

(i) maximum potential energy of the bob. (03marks)



h = 1 – cos 45 = 0.293

P.E = mgh = 0.1 x 9.81 x 0.293 = 0.287J

(ii) velocity of the bob when the string makes an angle of 30^0 2ith the vertical. [Neglect air resistance]

Height of the ball when the string makes 300 = 1 – cos 30 = 0.134

Potential energy at this point = mgh = 0.1 x 9.81 x 0.134 = 0.122J

Kinetic energy = change in potential energy = 0.287 – 0.122 = 0.165J

Let the velocity be v

⇨ $\frac{1}{2}\ x\ 0.1\ x\ v^2 = 0.165$

$v = 1.8ms^{-1}$

(c) A body moving in a circular path of radius 0.5m makes 40 revolutions per second. Find the centripetal force if the mass is 1kg (03marks)

F = mrω^2but ω = 2πf

= 1 x 0.5 x (2π x 40)^2 = 1.36 x 10^{4}N

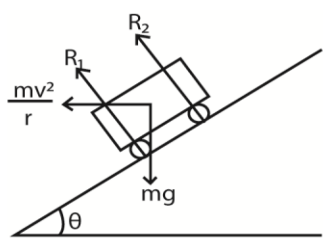
(d) Explain why a mass attached to a string rotating at constant speed in a horizontal circle will fly off at a tangent of the string breaks. (02marks)

If the spring breaks, the mass will not have any centripetal force and will continue in a straight line along the tangent according Newton's first law of motion.

5. (a) (i) What is meant by banking a track? (01mark)

Banking of roads is defined as the phenomenon in which the outer edges are raised for the curved roads above the inner edge to provide the necessary centripetal force to the vehicles so that they take a safe turn.

(ii) Derive an expression for the angle of banking θ for a car of mass, m, moving at speed, v, round a banked tract of radius, r. (04marks)



Resolving horizontally: (R_1 + R_2)sinθ = $\frac{mv^2}{r}$ .... (i)

Resolving vertically: (R_1 + R_2)cosθ = mg ....(ii)

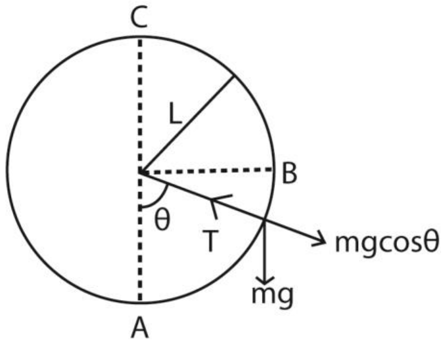
Eqn. (i) and Eqn. (ii);

tan θ = $\frac{v^2}{rg}$

θ = tan$^{-1}\left(\frac{v^2}{rg}\right)$

(c) A bob of mass, m is tied to an inelastic thread of length, L, and whirled with constant speed in a vertical circle

(i) With the aid of a sketch diagram, explain the variation of tension in the string along the circle. (05 marks)



At A, θ = 0

=> T = $\frac{mv^2}{L}$ + mg

At B, θ = 90^0

=> T = $\frac{mv^2}{L}$

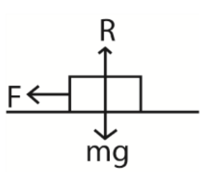
At C, θ = 1800

=> T = $\frac{mv^2}{L}$ - mg

(ii) If the string breaks at one point along the circle, state the most likely position and explain the subsequent motion of the bob.

The string breaks at the lowest point, A of the circle because tension the string is highest. Motion is tangential to the circle and when the string breaks, the bob follows a parabolic path.

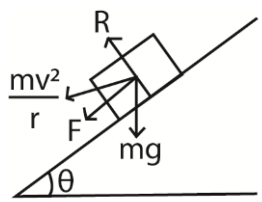
(d) Explain why maximum speed of a car on a banked road is higher than on an unbanked road. (04marks)



Along a circular path on a horizontal road, the frictional force provides the centripetal force

i.e. centripetal force = $\frac{mv_1^2}{r}$ = μR

On a banked road, the centripetal force is provided by the component of normal reaction, R, and the component of friction



Centripetal force = $\frac{mv_2^2}{r}$ = Fcosθ + Rsinθ

But F = μR

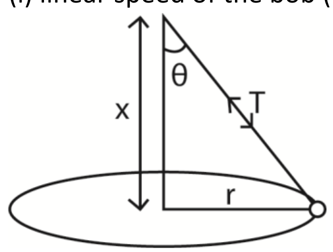
Centripetal force = $\frac{mv_2^2}{r}$ = μRcosθ + Rsinθ

For $0^0 < \theta < 90^0$, μcosθ+ sinθ > μ

Therefore, $v_2 > v_1$

(e) A small bob of mass 0.2kg is suspended by an inextensible string of length 0.8m. The bob is then rotated in a horizontal circle of radius 0.4m. Find

(i) linear speed of the bob (03marks)



$x^2 = 0.8^2 - 0.4^2$

x = 0.692m

Tsinθ = $\frac{mv^2}{r}$ ………………………….. (i)

Tcosθ = mg

Eqn. (i) and Eqn (ii)

tanθ = $\frac{v^2}{rg}$

v^2 = rgtanθ

v = $\sqrt{0.4 \times 9.81 \times \frac{0.4}{0.692}}$ = 1.51ms^{-1}

(ii) tension in the string. (02marks)

T = $\frac{mg}{\cos\theta} = 0.2 \times 9.81 \times \frac{0.8}{0.692}$ = 2.29N

6. (a) (i) Define angular velocity. (01marks)

Angular velocity is the rate of change of angle of rotation of an object moving in a circular path about the centre.

(ii) Derive an expression for the force, F, on a particle of mass, m, moving with angular velocity, ω, in a circle of radius, r. (03marks)

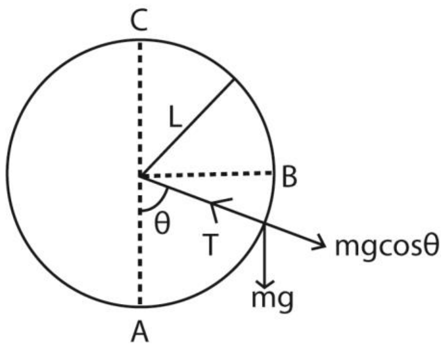
F = ma = $\frac{mv^2}{r}$ since a = $\frac{v^2}{r}$

But v = ωr

∴ F = $\frac{m(\omega r)^2}{r}$ = $m\omega^2 r$

(b) A stone of mass 0.5kg is attached to a string of length 0.5m which will break if the tension in it exceeds 20N. The stone is whirled in a vertical circle, the axis of rotation being at a vertical height of 1.0m above ground. The angular speed is gradually increased until the string breaks.

(i) In what position is the string most likely to break? (02marks)



At A, θ = 0

=> T = $\frac{mv^2}{L}$ + mg

At B, θ = 90^0

=> T = $\frac{mv^2}{L}$

At C, θ = 1800

=> T = $\frac{mv^2}{L}$ - mg

The string breaks at the lowest point, A of the circle because tension the string is highest.

(ii) At what angular speed will the string break? (03marks)

String breaks T = $m\omega^2 r$ + mg

20 = 0.5 x 0.5 x ω2 + 0.5 x 9.81

Ω = 7.77rads^{-1}

v = ωr=7.77 x 0.5 = 3.9ms^{-1}

(i) Find the position where the stone hits the ground when the string breaks. (03marks)

Vertical distance to be covered = 0.5m

s = ut + $\frac{1}{2}at^2$

but initial component of vertical velocity = 0

=> 0.5 = $\frac{1}{2}\ x\ 9.81\ x\ t^2$

t = 0.3s

Horizontal distance = 3.9 x 0.3 = 1.17m

(c) Explain briefly the action of a centrifuge. (03marks)

Consider a body falling through a viscous fluid. For small speed, the liquid opposes motion with a resisting force, f, proportional to the velocity, v.

f = kv where k is a constant of proportionality.

At terminal velocity, $v_t = \frac{mg}{k}$

If a container is whirled at high speed, the particles will move approximately in circles with speed, v, and acceleration= $\frac{v^2}{r}$.

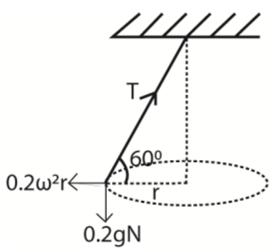
When they reach their terminal velocity, v_t, relative to the fluid, the resisting force of the fluid on the particle, f = kv_t must be equal to the mass of the particle multiplied by its acceleration such that the terminal velocity, vt of the particles relative to the fluid is given by $v_t = \frac{mv^2 \div r}{k}$

The terminal speed or sedimentation rate is increased by a factor of $\frac{v^2}{rg}$ which may be in thousands.

Centrifuges are used to separate cream from milk, silt from river water, blood cells from plasma, etc.

7. A pendulum bob of mass 0.2kg is attached to one end of an inelastic string of length 1.2m. The bob moves in a horizontal circle with the string inclined at 30^0 to the vertical. Calculate

(i) the tension in the string(02marks)



Resolving vertically

$T\sin 60^0 = 0.2g = 0.2 \times 9.81$

T = 2.27V

(ii) the period of motion (04marks)

$T\cos 60 = 0.2\omega^2 r$; $\omega = \frac{2\pi}{T}$; r = 1.2cos 60 = 0.6m

$2.27\cos 60 = 0.2 \times \frac{4\pi^2}{T^2} \times 0.6$

Period, T = 2.04s

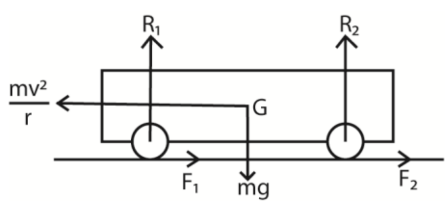
8. (a) Define the following terms:
   (i) Angular velocity (01mark)
   Angular velocity is the rate of change of the angle swept out by the radius joining a body to the centre of circular path.
   (ii) Centripetal acceleration (01mark)
   Centripetal acceleration is the rate of change of velocity for a body describing a circular path and is always directed towards the centre of the path.

(b) (i) Explain why a racing car can travel faster on a banked road than on flat track of the same curvature. (04marks)

(ii) Derive an expression for the speed with which a car can negotiate a bend on a banked track without skidding. (03marks)

- **On a flat track**

For Suppose the car is moving with velocity, v, around a horizontal circular track of radius, r. if m is the mass of the car and R_1 and R_2 are normal reactions at the inner and outer wheels respectively and F_1 and F_2 are the corresponding frictional forces, then for circular motion:



$F_1 + F_2 = \frac{mv^2}{r}$ ………………………………………… (i)

For vertical equilibrium

$R_1 + R_2 = mg$ …………………………………………(ii)

Taking moments about G

Clockwise moments = anticlockwise moments

$(F_1 + F_2)h + R_1\frac{a}{2} = R_2\frac{a}{2}$ (a = distance between the wheels, h = the height of the centre of gravity from the ground)

$(F_1 + F_2)h = \frac{a}{2}(R_2 - R_1)$ = …………………… (iii)

Substituting Eqn. (i) into Eqn. (iii)

$\frac{mv^2h}{r} = \frac{a}{2}(R_2 - R_1)$ ……………………………………(iv)

From (ii)

$R_1 = mg - R_2$

From equation (iv)

$\frac{mv^2h}{r} = \frac{a}{2}(R_2 - (mg - R_2)) = \frac{a}{2}(2R_2 - mg)$

$2R_2 = \frac{2mv^2h}{ar} + mg = m\left(\frac{2v^2h}{ar} + g\right)$

$R_2 = \frac{m}{2}\left(\frac{2v^2h}{ar} + g\right)$

Also $R_2 = mg - R_1$

From equation (iv)

$\frac{mv^2h}{r} = \frac{a}{2}((mg - R_1) - R_1) = \frac{a}{2}(mg - 2R_1)$

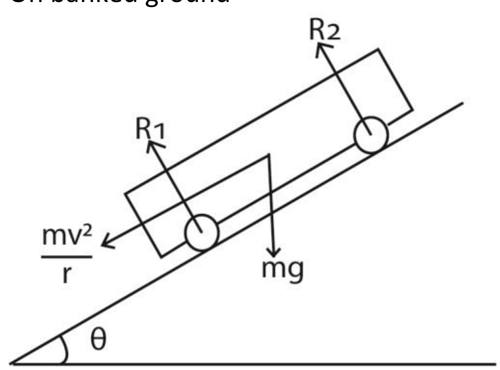
$2R_1 = mg - \frac{2mv^2h}{ar}$

$R_1 = \frac{m}{2}\left(g - \frac{2v^2h}{ar}\right)$

When the car is about to overturn, $g = \frac{2v^2h}{ar}$, $R_1 = 0$, $v^2 = \frac{gar}{2h}$

The maximum velocity of a car to negotiate a bend of radius r on a flat track, $v = \sqrt{\frac{gar}{2h}}$

- On banked ground



Resolving horizontally

$(R_1 + R_2)\sin\theta = \frac{mv^2}{r}$ …………………… (i)

Resolving vertically;

$(R_1 + R_2)\cos\theta = mg$ ………………… (ii)

Divide equation (i) by Eqn. (ii)

$\tan\theta = \frac{v^2}{rg}$

$v = \sqrt{rg\tan\theta}$

Thank you
Dr. Bbosa Science