SENIOR SIX TERM 1

TOPIC 5/6: STATIONARY WAVES

Competency: The learner investigates the behaviour of stationary waves and their applications in different situations.

Stationary/standing waves

A **stationary wave, or standing wave**, is a wave pattern that appears to stay in one place, formed by the interference of two waves with the same frequency and amplitude traveling in opposite directions, like a wave reflecting back on itself.

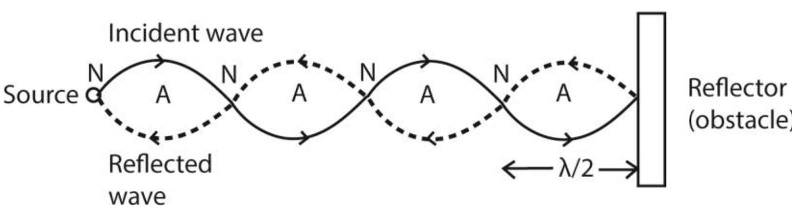
Formation of stationary wave.

Stationary wave is formed when two progressive waves having the same speed and frequency and approximately equal amplitude but travelling in opposite direction are superposed.

Conditions for formation of stationary waves

- Superposition of two waves travelling in opposite direction
- The waves should have the same frequency and speed
- The waves should have nearly equal amplitude

Progressive waves which agree with above condition can be obtained when a wave travels along a given path and strikes an obstacle such that it reflected perpendicularly and returns along the same path. The incident and reflected waves superpose to form a stationary wave in which the wave profile does not move along the medium.



The superposition of the two waves results into points where the particles are permanently at rest. These points are known as nodes. These points midway between successive nodes will have their particles vibrating with maximum amplitude. The points are known as antinodes.

The distance between any two successive nodes or any two successive antinodes is equal to half the wavelength of the stationary waves.

i.e. distance between successive node = $\frac{\lambda}{2}$

the distance between successive antinodes =$\frac{\lambda}{2}$

the distance between a node and neighboring antinode = $\frac{\lambda}{4}$.

A stationary wave unlike progressive waves does not transfer energy since the wave profile does not move along the medium.

Also, at points between successive nodes, all vibrations are in phase.

Characteristics of a stationary wave

- The amplitude of the wave varies from place to place along the wave profile.
- Wave energy is not transmitted along the wave profile.
- It has nodes and antinodes
- At points between successive nodes, the vibrations are in phase
- At nodes, particles are permanently at rest while at the antinodes the particles have maximum displacement.
- The medium is split up into segments. The particles in the segments are out of phase with particles in the neighboring segments by 180^0.
- In a given segment, the particles attain maximum velocity and acceleration at the same instant.
- Compression and rarefactions do not travel forward as progressive waves. They appear and disappear alternatively in the same place.
- During each vibration, all particles pass simultaneously through their mean position twice with maximum velocity which is different for different particles.

Differences between stationary and progressive waves

Stationary waves	**Progressive waves**
Each point along the wave has different amplitude of vibration from the neighboring points	The vibration of particles are of the same amplitude.
At points between successive nodes the vibrations are in phase i.e. the phase of vibration does not vary.	The phase of vibration of points near each other are all different.
They do not transfer energy	They transfer energy from one point to another.
The wave profile does not move along the medium	The wave profile moves along the medium with the wave speed.
The medium does not move	The medium moves

NB. The velocity of stationary wave is zero

The stationary wave equation

Stationary waves are formed by superposition of two progressive waves in opposite direction which have equal amplitude and frequency. The equations of such progressive waves will be

y_1 = Asin(ωt - $\frac{2\pi}{\lambda}$)

y_2 = Asin(ωt + $\frac{2\pi}{\lambda}$)

the equation of the stationary waves would then be the sum of these two equations

Y = y_1 + y_2 = Asin(ωt - $\frac{2\pi}{\lambda}$) + Asin(ωt + $\frac{2\pi}{\lambda}$)

= 2Acos$\frac{2\pi x}{\lambda}$sin(ωt)

But ω = $\frac{2\pi}{T}$

∴ 2Acos$\frac{2\pi x}{\lambda}$sin($\frac{2\pi t}{T}$)

Amplitude of a wave, A = 2acos$\frac{2\pi x}{\lambda}$

Since x varies from point to point, the amplitude also varies from point to point along the wave.

For antinodes, the amplitude should be maximum (regardless of the sign)

Amplitude is maximum when 2Acos$\frac{2\pi x}{\lambda}$ = ±1, this occurs when $\frac{2\pi x}{\lambda}$ = 0, π, 2π, 3π …….

Then amplitude maximum = 2A

The separation between successive antinodes can be obtained

The 1st antinode when $\frac{2\pi x}{\lambda}$ = 0, x = 0

The next antinode when $\frac{2\pi x}{\lambda}$ = π, x = $\frac{\lambda}{2}$

∴ the distance between two successive antinode is $\frac{\lambda}{2}$

For the node the amplitude is zero since the particles are permanently at rest.

From amplitude, A = 2acos$\frac{2\pi x}{\lambda}$; A = 0 when cos$\frac{2\pi x}{\lambda}$ = 0

This occurs when $\frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ …

At the first node, occurs when $\frac{2\pi x}{\lambda} = \frac{\pi}{2}$ or when x = $\frac{\lambda}{4}$

The 2nd node occurs when $\frac{2\pi x}{\lambda} = \frac{3\pi}{2}$ or when x = $\frac{3\lambda}{4}$

Distance between successive nodes = $\frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$

Example 1

Show that the distance a successive node and antinode is $\frac{\lambda}{4}$

Solution

For the 1st antinode, $\frac{2\pi x}{\lambda}$ = 0; x = o

At the first node, occurs when $\frac{2\pi x}{\lambda} = \frac{\pi}{2}$ or when x = $\frac{\lambda}{4}$

Distance between successive node and antinode = $\frac{\lambda}{4} - 0 = \frac{\lambda}{4}$

Example 2

A plane progressive wave is given by the equation

y = a $\sin\left(100\pi t - \frac{10\pi x}{9}\right)$ where x and y are in millimeters and t in second.

(i) Write the equation for progressive waves which would give rise to a stationary wave if superposed on the above

y_1 = a $\sin\left(100\pi t + \frac{10\pi x}{9}\right)$

i.e. it is a wave in opposite direction with equal frequency, speed and amplitude.

(ii) Find the equation of stationary wave and hence determine its amplitude of vibration.

$Y = y + y_1 = a\sin\left(100\pi t - \frac{10\pi x}{9}\right) + a\sin\left(100\pi t + \frac{10\pi x}{9}\right)$

$= 2a\cos\frac{10\pi x}{9}\sin 100\pi t$

Amplitude, A = $2a\cos\frac{10\pi x}{9}$ mm

(iii) Determine the frequency and velocity of stationary wave

$\omega = 100\pi$

$2\pi f = 100\pi$

$f = 50$Hz

Velocity v =0 because the wave is stationary.

MUSICAL NOTES

Musical notes are regular vibrations of sound waves.

Properties of musical notes

Musical notes have three properties namely:

Pitch, loudness and quality/timbre

Pitch

A pitch is the sharpness or mildness of the soft musical note. A pitch is directly proportional to the frequency in that high frequency gives high pitch (sharper note).

A high pitched note has high frequency and short wavelength and vice versa.

Loudness and intensity

This is the amount of sound energy entering the ear per second. The greater the amount of energy, the louder the sound.

The intensity or loudness of sound depends on the amplitude of the vibration in that a loud note has a large amplitude while a quiet note has a small amplitude.

A mother and child may both shout a note of the same frequency but the note from a mother is louder because the amplitude of the sound wave made by the mother is larger so a greater mass of air is set into vibration.

In general, the greater the mass of air which can be set into vibration the louder will be the sound. The prongs of a vibrating tuning fork, make a small mass of air vibrate. So the sound is soft. However if the end of the fork is placed on a table the sound is much louder because a large mass of air in contact with surface of the table is set into vibration.

A violin string directly produces soft sound because very little air is set into vibration as the violin has small surface area.

When the same violin string is attached to a sounding box a much louder sound is produced because the large surface area of the sounding box results in large mass of air to vibrate as the box vibrates,

The sound from a telephone earpiece is heard distinctly only when the ear is placed close to the earpiece because the vibrating circular metal plate inside has a small area so only a small mass of air is set into vibration.

A loud sound may be heard from a small transistor set or a television set because the loud speaker has a vibrating cone with relatively large surface area so a large mass of air in contact is set into vibration.

Intensity of sound

The intensity of sound is the rate of flow of energy per unit area perpendicular to the direction of sound. Intensity is proportional to;

i) The square of the amplitude,
ii) The square of the frequency
iii) The density of the medium

Loudness of sound

Loudness depends on;-

i) The varying pressure exerted on the eardrum by the sound,
ii) ii) The sensitivity of the ear to the different frequencies,
iii) The sound intensity.

Loudness of sound is the sensation of a note in the mind of an individual.

Quality or tone (timbre) of a musical note

The same note on different instruments sounds different, the notes are said to differ in quality or timbre. The difference arises because no instruments except a tuning fork and single generator can emit a pure note i.e. of one frequency but they produced notes consisting of main or fundamental frequency mixed with overtones.

Overtones have frequencies that are exact multiples of the fundamental. The number and strength of overtones determines the quality of a note.

A violin has more and stronger higher overtones than a piano.

Beats

Beats are periodic rise and fall in sound intensity when two sound waves of nearly equal frequency but same amplitude are sounded together.

Rise in sound is due to constructive interference while fall in sound is due to destructive interference.

Conditions for formation of beats

Beats are formed from notes of

- nearly equal frequency
- same amplitude sounded together

Beat frequency

This Is the number of beats formed in one second.

Beat period

This Is the time taken to fish one beat after completion of the previous one

Derivation of the expression for beat frequency

Consider two notes of frequencies f_1 and f_2 sounded together. Suppose the wave of frequency f_1 makes one cycle more than that of f_2 in time T_0.

In time T, the number of waves of f_1 is f_1T
The number of waves of f_2 is f_2T
Therefore $f_1T - f_2T = 1$

$$f_1 - f_2 = \frac{1}{T}$$

$$\text{but } \frac{1}{T} = f$$

$\therefore f_1 - f_2 = f$, the beat frequency

Example 3

Two whistles are sounded simultaneously. The wavelengths of the sounds emitted are 5.5m and 6.0m respectively. Find the beat frequency if the speed of sound is $330ms^{-1}$.

For 1st sound $f_1 = \frac{V}{\lambda_1} = \frac{330}{5.5} = 60Hz$

For 2nd t sound $f_2 = \frac{V}{\lambda_2} = \frac{330}{6} = 55Hz$

Beat frequency $f_b = f_1 - f_2 = 60 - 55 = 5Hz$

Uses of beats

(i) Tuning a musical instrument to desired note.
(ii) Determination of frequency of a musical note.

WAVES IN STRINGS AND PIPES

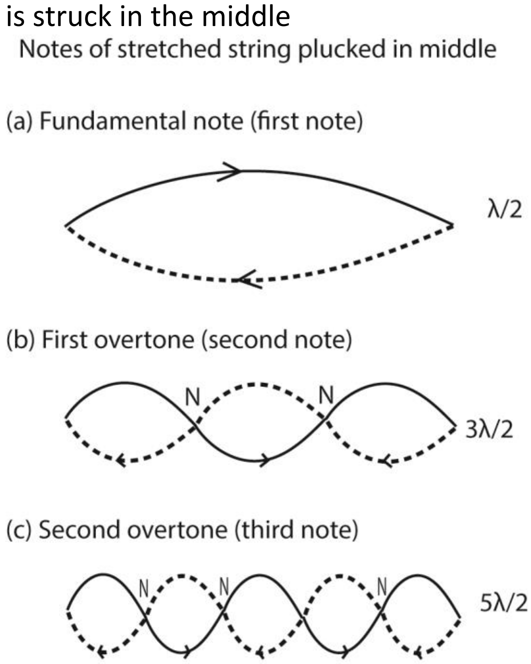
Vibrating string

Factors on which the frequency of a wave in a vibrating string depend

- Length of the string.
- Tension in the string.
- Mass per unit length of the string.

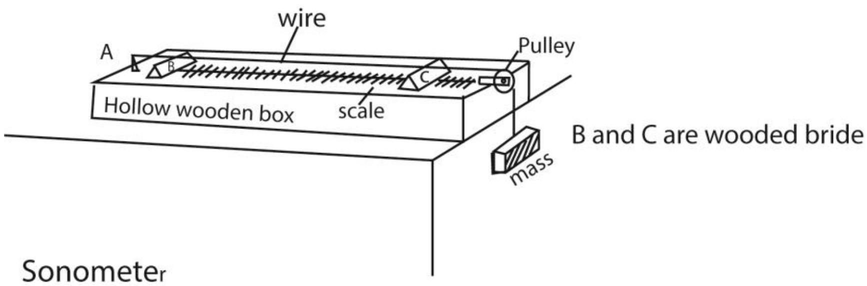
A stationary wave can be set up in a string which has both ends fixed.

The figure below shows the number of waves that can be formed when a string of fixed length is struck in the middle



An experiment to investigate the variation of frequency with length for vibrating wires

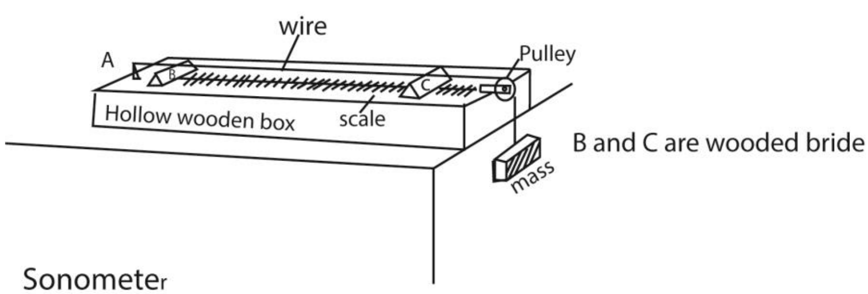
A sonometer below is used



- The wooden bridges B and C vary the effective length of the wire, L.
- Constant tension in the wire is maintained by the fixed mass
- A paper rider is placed on the wire in the middle of BC and a sounding fork placed near it.
- The position of the bridge C is varied until sound is heard.
- The distance between the bridges L and the frequency, **f**, of the tuning fork is noted.
- The procedure is repeated for various tuning forks and values of L, **f** and $\frac{1}{L}$ are tabulated
- A plot of **f** against $\frac{1}{L}$ gives a straight line showing that the frequency of vibration of the wire is inversely proportional to length.

An experiment to show the fundamental frequency varies with the tension in a given wire.

A sonometer below is used.



- The length L between the bridges is fixed.
- A suitable mass, m, is attached to the free end of the wire.
- The wire is plucked in the middle and a tuning fork of known frequency, f, is sounded
- The mass m corresponding to the frequency, f, are recorded in the table including values of f^2.
- The procedure is repeated for different values of f.
- A graph of f^2 against m is plotted.
- A straight line graph is obtained through the origin, implying that $f^2 \propto m$

  But m = $\frac{T}{g}$

  $\therefore$ f = $\sqrt{\frac{T}{g}}$ thus increase in tension, T, increases the frequency, f, of the wire.

Example 4

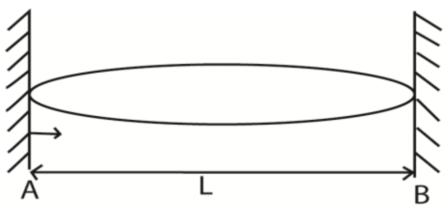
(a) What are overtones?

Overtones are notes of higher frequencies produced by an instrument after fundamental (or main) notes.

(b) Explain why a music tone played on one instrument sound differently from the same note played on another instrument.

- When an instrument is played, all the allowed vibration take place producing different frequencies.
- The quality of the musical note is determined by the number and strength (intensity) of the overtones.
- Thus when the note played on an instrument has fewer overtones, it sounds different from the same note played on different instrument with more overtones.

(c) A stretched string of length L, is fixed at both ends and then set to vibrate in its allowed modes. Derive an expression for frequency of the second overtone in terms of fundamental frequency.

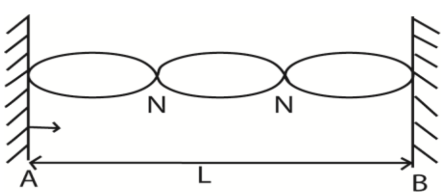


When the string is vibrating with fundamental frequency, the number of waves between A and B is $\frac{1}{2}$

L = $\frac{1}{2}\lambda$ i.e. λ =2L

Let the velocity of the wave be v, then frequency of fundamental note, $f_0 = \frac{v}{\lambda} = \frac{v}{2L}$

When vibrating with 2^{nd} overtone, the number of waves made by the length of the string between A and B is $\frac{3}{2}\lambda$.



L = 2λ i.e. λ =$\frac{2}{3}$L

The $f_2 = \frac{v}{\lambda} = \frac{v}{\frac{2}{3}L}$

$= \frac{3v}{2L} = 3(\frac{v}{2L})$

$\therefore f_2 = 3f_0$

(d) A wire of length 0.60m and mass 9 x 10^{-4}kg is under tension of 135N. The wire is plucked that it vibrates in its third harmonic. Calculate the frequency of the third harmonic.

$$\mu = \frac{m}{l} = \frac{9\,x\,10^{-4}}{0.6} = 1.5\,x\,10^{-3}kgm^{-1}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{135}{1.5\,x\,10^{-3}}} = 300\text{ms}^{-1}$$

The frequency of the third harmonic

$$f_2 = 3f_0 = \frac{2v}{2l} = \frac{2\,x\,300}{2\,x\,0.6} = 750\text{Hz}$$

Example 5

A uniform wire of length 1.00m and mass 2.0 x 10^{-2}kg is stretched between two fixed points. The tension in the wire is 200N. The wire is plucked in the middle and released. Calculate the

(i) Speed of the transverse waves. (03marks)

L=1.0m, m = 2.0 x 10^{-2}kg. T= 200M

$$V = \sqrt{\frac{T}{m}} = \sqrt{\frac{200}{2\,x\,10^{-2}}} = 100\text{ms}^{-1}.$$

(ii) Frequency of the fundamental note (03marks)

λ = 2L and V = fλ

$$f = \frac{V}{\lambda} = \frac{100}{2\,x\,1} = 50Hz$$

Example 6

A string of length 0.5m and mass 50g is stretched between two fixed points. If the tension in the string is 100N, find the frequency of second harmonic. (04marks)

From $f = \frac{v}{\lambda}$ ………………………… (i)

But $v = \sqrt{\frac{T}{\mu}}$; where μ is the mass per unit length.

$$\mu = \frac{5\,x\,10^{-3}}{0.5} = 0.01\text{kgm}^{-1}$$

$$v = \sqrt{\frac{100}{0.01}} = 100\text{ms}^{-1}$$

For second harmonic, first overtone, λ = L = 0.5m

Substituting in eqn. (i) $f = \frac{100}{0.5} = 200\text{Hz}$

Resonance

Resonance occurs when a body or system is set into vibrations with its own natural frequency by a nearby body or system vibrating with the same frequency.

The vibrations combine to produce a larger vibration with a larger amplitude.

Thus, resonance occurs when a body is set into vibration with its own natural frequency by another nearby body which vibrates with the same frequency.

Resonance tubes/ vibrating air column /Vibration of air in pipes

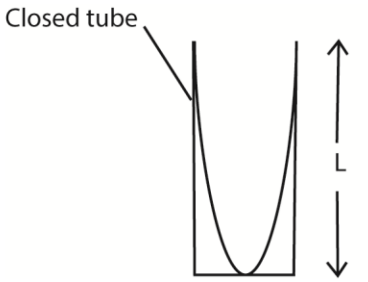
In vibrating air column, stationary waves are set up.

There are two types of pipe for air vibration:
(i) Closed pipe
(ii) Open pipe

Resonance in a closed pipe

a) First position of resonance

This occurs when a node is at the closed end and antinodes at the open end



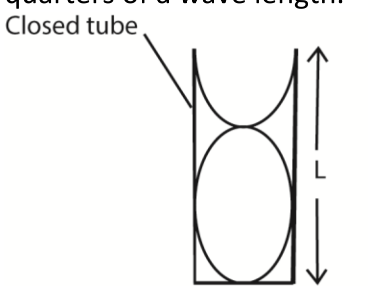
$L = \frac{\lambda}{4}$

$\lambda = 4L$

The fundamental note $f_0 = \frac{V}{\lambda}$ is given out, $f_o = \frac{V}{4L}$

(b) Second resonance

This occurs when a second node is formed such that the length L of vibrating air equals three quarters of a wave length.



$L = \frac{3\lambda}{4}$

$\lambda = \frac{4L}{3}$

The note given out is the first overtone f_1

$f_1 = \frac{V}{\lambda}\frac{3V}{4L} = 3f_0$

where $f_0 = \frac{V}{4L}$

The third resonance is obtained when air column L equals $\frac{5\lambda}{4}$

The note given out is the second overtone f_2

$L = \frac{5\lambda}{4}$ i.e. $\lambda = \frac{4L}{3}$

$f_2 = \frac{V}{\lambda} = \frac{5L}{4L} = 5f_0$

Where $f_0 = \frac{V}{4L}$

Where f_0 is the fundamental note.

For closed pipes, the possible harmonics are, f_0, $3f_0$, $5f_0$, $7f_o$.....

NB: A closed pipe produces only odd notes.

Example 7

A pipe closed at one end has a length of 10cm. If the velocity of sot in air of the pipe is 340 m/s, calculate

(a) the frequency of fundamental note

$L = \frac{\lambda}{4}$ L = 10cm = 0.1m

$\lambda = 4L$

$f_0 = \frac{V}{4L}$

$= \frac{340}{0.1 \text{ x } 4} = 850\text{Hz}$

(b) the frequency of first overtone L = 10cm = 0.1m

for Ist overtone

$L = \frac{3\lambda}{4}$ L = 10cm = 0.1m

$\lambda = \frac{4L}{3} = \frac{0.1 \text{ x } 4}{3} = \frac{0.4\text{m}}{3}$

$F_1 = \frac{V}{\lambda} = \frac{340}{\frac{0.4}{3}}$

$= 2550\text{Hz}$

Alternative method

$F_0 = 850$

For 1 overtone

= 3 x 850

= 2550 Hz

Note:

The frequency of fundamental may alter during the day because this temperature changes of air during the day result in change of velocity of sound.

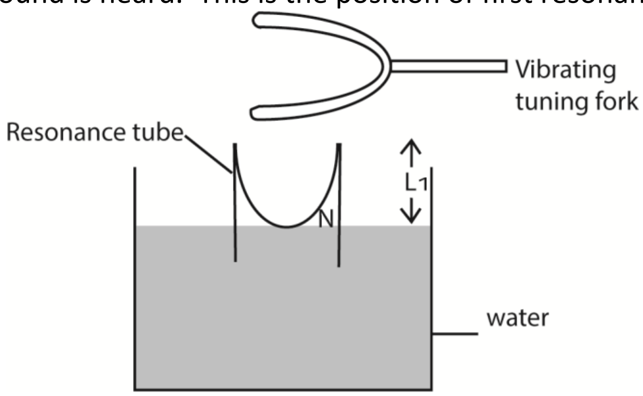
Vibrating turning fork

When the vibrating tuning fork is held over the mouth of tube T, the air inside the tube is set into vibration. The wave sent downwards is reflected from the surface of' water and a stationary wave is set.

Experiment 1: to determine the speed of sound in air by resonance method- air obtained by raising the tube from water

a) A tuning fork of known frequency, f, is set into vibration and placed above near the open tube as in the diagram.

b) The tube is raised from water until the length of air column, L_1 is reached when a loud sound is heard. This is the position of first resonance.



$L_1 = \frac{\lambda}{4} + C$ .... (i)

where C is the end correction and λ is the wave length.

c) While is sounding fork is kept in place; the tube is raised further to length L_2 of the air column until a second loud sound is heard. This is the position of the second resonance.

The tube is raised to length L_2 and the air column is varied until a loud sound is heard. This is the position of the second resonance.

$L_1 = \frac{3\lambda}{4} + C$ .... (ii)

where C is the end correction and λ is the wave length.

Subtracting resonance: 2nd position – 1st position

$$L_2 - L_1 = \left(\frac{3\lambda}{4} + c\right) - \left(\frac{\lambda}{4} + c\right)$$

$$L_2 - L_1 = \left(\frac{3\lambda}{4} - \frac{\lambda}{4} + c - c\right.$$

$$= \frac{3\lambda}{4} - \frac{\lambda}{4}$$

$$= \frac{2\lambda}{4}$$

$$= \frac{\lambda}{2}$$

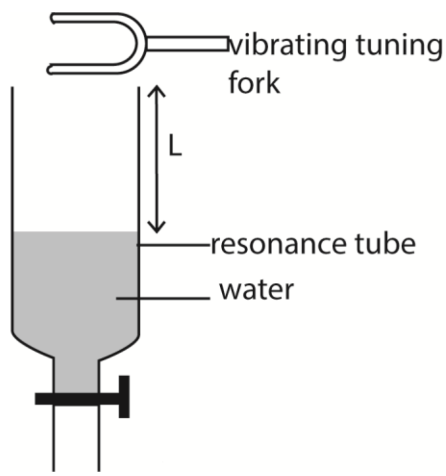
$$\boldsymbol{\lambda = 2(L_2 - L_1)}$$

But $V = \lambda f$

$$\therefore V = \mathbf{2(L_2 - L_1)f}$$

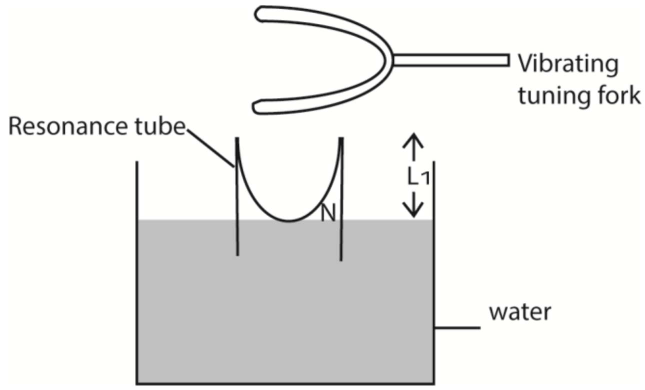
where v is the velocity of sound in air.

Experiment 2: to determine the speed of sound in air by resonance method- air obtained by draining the tube.



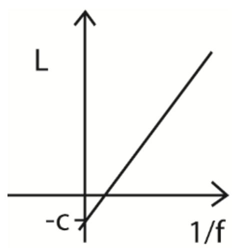
- A glass tube which can be drained from bottom is filled with water.
- A sounding tuning fork of frequency f is brought to the mouth of the tube.
- The water is slowly drained until a loud sound is heard. The tap is closed and length L_1 is measured.
- The tuning fork is again sounded at the mouth of the tube and the water drained further until a loud sound is heard and distance L_2 is noted
- Velocity, v, of sound in air = $2f(L_2-L_1)$

Experiment 3: to determine the speed of sound in air using tuning forks of different frequencies and resonate tube.



A resonance tube is place to stand in a tall jar full of water.
Starting with a short length air column, a vibrating tuning fork is held over the mouth of resonance tube. The tube is raised until a point where a loud sound is heard. The length L of the air column and frequency f of tuning fork are recorded.

The experiment is repeated with other five tuning forks of different frequencies
Values of L, f and $\frac{1}{f}$ are tabulated.
A graph of L against $\frac{1}{f}$ is plotted and slope S is determined



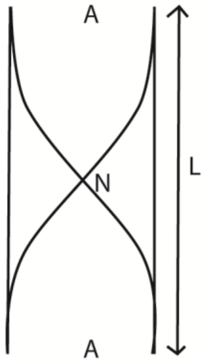
The velocity, v, of sound = 4S.

The end correction c is an intercept on L axis.

Note that; this experiment shows that frequency is inversely proportional to length of the air column.

Vibration in open pipes

An open pipe is one which is open at both ends as shown below, Here the antinodes are open at both ends.
(a)



For fundamental note f_0 the air column 'L' equals half the wave length.

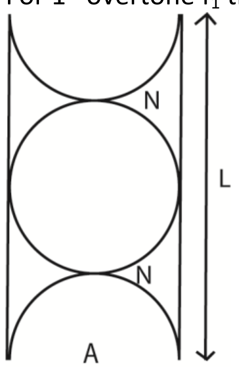
$L = \frac{\lambda}{2}$

$\therefore \lambda = 2L$

$f_0 = \frac{V}{\lambda} = \frac{V}{2L}$

For 1st overtone f_1

For 1st overtone f_1 the air column 'L' equals the wave length.

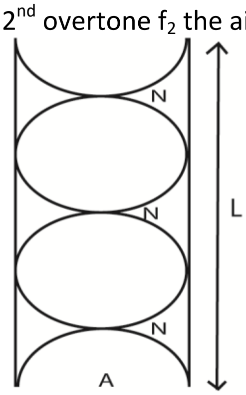


$f_1 = \frac{V}{\lambda} = \frac{V}{L} = \frac{2V}{2L} = 2f_0$

where $f_0 = \frac{V}{2L}$

For 2nd overtone f_2.

For 2nd overtone f_2 the air column 'L' equals 3/2 the wave length.



$L = \frac{3\lambda}{2}$

$\therefore \lambda = \frac{2L}{3}$

$v = f\lambda$

$f_2 = \frac{V}{\lambda} = \frac{v}{\frac{2L}{3}} = \frac{3V}{2L} = 3f_0$

The harmonics present in the open pipe are f_0, $2f_{tf}$, $3f_0$, $4f_0$...
NB: open pipe produces both odd and even notes.

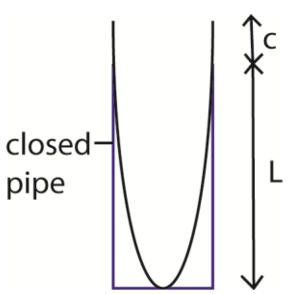
End correction, c

The end correction of an air pipe is a small distance that must be added to actual length of the pipe to account for increase in the effective length of the pipe as a result of the vibrating air column extending beyond the open end of the pipe.

The effective length of the pipe is therefore longer than actual length of the pipe because the vibrating air column extends beyond the open end(s) of the pipe.

The end correction is proportional to the radius of the pipe and the wavelength of sound wave. Experiment show that end correction, c= 0.6r where r is the radius of the pipe.

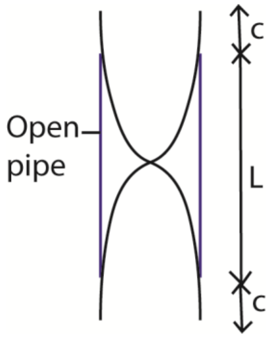
End correction in open pipe



$L + c = \frac{\lambda}{4}$; $\lambda = 4(L + c)$

The fundamental frequency in closed pipe, $f_0 = \frac{v}{4(L+c)}$

End correction in open pipe



L +2c = $\frac{\lambda}{2}$; λ =2(L+ 2c)

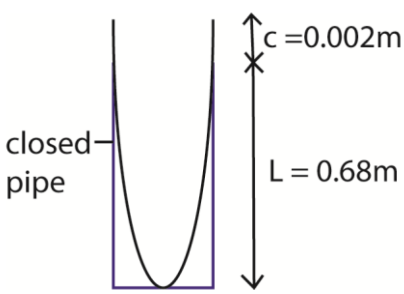
The fundamental frequency in closed pipe, $f_0 = \frac{v}{2(L+2c)}$

Example 8

A closed pipe of length 0.68m is blown at its open end so that it produced the first harmonic.

(a) Sketch the wave pattern of the pipe
(b) Determine the frequency of the note produced.
(c) What is the radius of the pipe?
(End correction of the pipe = 0.002m and velocity of sound is 340m/s)

(a)



(d) $f_0 = \frac{v}{4(L+c)}$

$f_0 = \frac{340}{4(0.68+0.002)}$ = 124.6Hz

(e) From c = 0.6r

r = $\frac{0.002}{0.6}$ = 3.3×10^{-3}m

Example 9

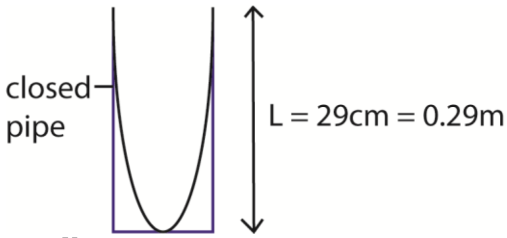
A circular pipe of length 29cm is closed at one end. The air in the pipe resonated when a turning fork of frequency 860Hz is sounded. Determine the

(i) Mode of vibration of the pipe
(ii) End correction (the speed of sound in air is 340m/s)

Solution

Let f_n = 860Hz = nf_0

Ignoring end correction



$f_0 = \frac{V}{4L} = \frac{340}{4 \times 0.29} = 293.1$

but $f_n = nf0$

$n = \frac{860}{293.1} = 2.943 \approx 3$

∴ the mode of vibration is 3rd harmonic of 2nd overtone.

(ii) $L + c = \frac{3\lambda}{4}$

From $v = f\lambda$

$340 = 860 \times \frac{4}{3}(0.29 + c)$

$c = 0.00651m$

Advantages of open pipes over closed pipes

Open pipes produce notes of better quality than closed pipes because they produce both odd and even harmonics while closed pipes produce only odd harmonics.

The Doppler Effect

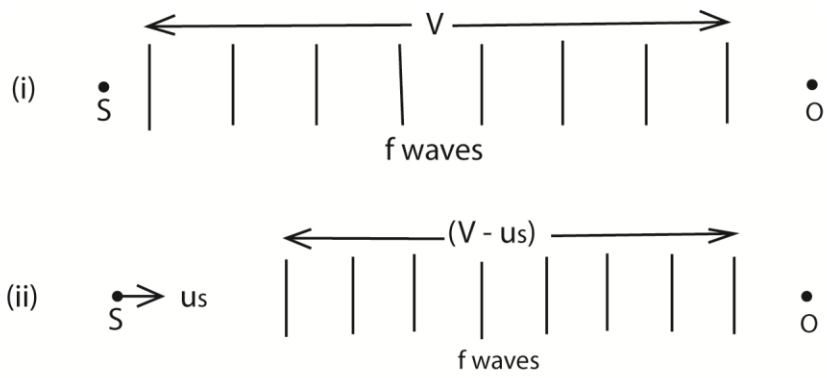
It is the apparent change in frequency of a wave motion due to relative motion between the source and observer.

Calculation of Apparent Frequency based Doppler Effect

Suppose V is the velocity of sound in air, u_s is the velocity of the source of sound S, u_0 is the velocity of an observer 0, and f is the true frequency of the source.

(a) Source moving towards stationary observer.

(i) If the source S were stationary, the f waves sent out in one second towards the observer O would occupy a distance V, and the wavelength would be V/f



Source moving towards stationary observer.

(ii) If S moves with a velocity u_s towards 0, however, the f waves sent out occupy a distance $(V - u_s)$, because S has moved a distance u_s towards 0 in 1s, fig. (ii).

Thus the wavelength λ' of the waves reaching O is now $\frac{(V - us)}{f}$

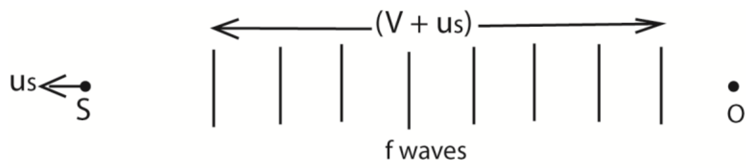
But velocity of sound = V

∴ Apparent frequency, $f' = \frac{V}{\lambda'} = \frac{V}{(V - u_s)/f} = \frac{V}{(V - u_s)} f$

Since $(V - u_s)$ is less than V, f' is greater than f; the apparent frequency thus appears to increase when a source is moving towards an observer.

(b) Source moving away from stationary observer.

In this case the f waves sent out towards 0 in 1s occupy a distance $(V + u_s)$,



Source moving away stationary observer.

Thus the wavelength λ' of the waves reaching O is now $\frac{(V+us)}{f}$

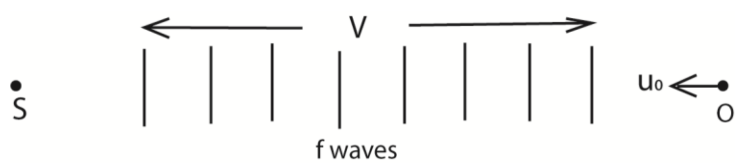
But velocity of sound = V

∴ Apparent frequency, f' = $\frac{V}{\lambda'} = \frac{V}{(V+u_s)/f} = \frac{V}{(V+u_s)}f$

Since (V + u_s) is greater than V, f' is less than f; the apparent frequency thus appears to reduce when a source is moving towards an observer.

(c) Source stationary, and observer moving towards it.

Since the source is stationary, the f waves sent out by S towards the moving observer O occupies a distance V,



Source stationary while observer moving toward source.

The wavelength of the waves reaching O is hence V/f, and thus unlike the cases already considered, the wavelength is unaltered.

The velocity of the sound waves relative to O is not V, however, as O is moving relative to the source.

The velocity of the sound waves relative to O is given by (V + u_o) in this case, and

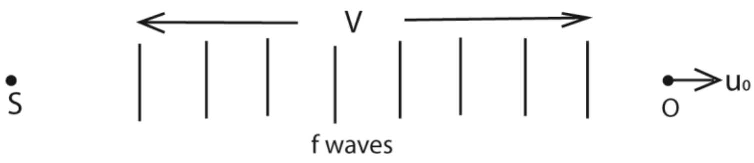
Hence the apparent frequency f' is given by

∴ Apparent frequency, f' = $\frac{V+u_0}{wavelength} = \frac{V+u_0}{V/f} = \left(\frac{V+u_0}{V}\right)f$

Since (V + u_0) is greater than V, f' is greater than f; the apparent frequency thus appears to increase when an observer is moving towards the source.

(d) Source stationary, and observer moving away from it.

Since the source is stationary, the f waves sent out by S towards the moving observer O occupies a distance V,



Source stationary while observer moving away from source.

The wavelength of the waves reaching O is hence V/f, and thus unlike the cases already considered, the wavelength is unaltered.
The velocity of the sound waves relative to O is not V, however, as O is moving relative to the source.
The velocity of the sound waves relative to O is given by ($V - u_o$) in this case, and
Hence the apparent frequency f' is given by

$$\therefore \text{ Apparent frequency, } f' = \frac{V - u_0}{wavelength} = \frac{V - u_0}{V/f} = \left(\frac{V - u_0}{V}\right) f$$

Since ($V - u_0$) is less than V, f' is less than f; the apparent frequency thus appears to reduce when an observer is moving away from the source.

Source and Observer Both Moving

If the source and the observer are both moving, the apparent frequency f' can be found from the formula $f' = \frac{V'}{\lambda'}$

where V' is the velocity of the sound waves relative to the observer, and λ' is the wavelength of the waves reaching the observer. This formula can also be used to find the apparent frequency in any of the cases considered before.

(i) Suppose that the observer has a velocity, u_o, the source a velocity u_s, and that both are moving in the same direction. Then

$V' = V - u_o$

And

$\lambda' = (V - u_s)/f$

$$f' = \frac{V'}{\lambda'} = \frac{V - uo}{(V - u_s)/f} = \left(\frac{V - u_O}{V - u_s}\right) f$$

Suppose that the observer has a velocity, u_o, the source a velocity u_s, and If the observer is moving towards the source, then

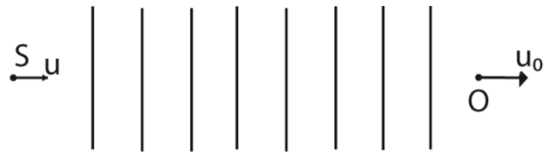
$V' = V + u_o$

And

λ' = (V - u_s)/f

f' $= \frac{V'}{\lambda'} = \frac{V + uo}{(V - u_s)/f} = \left(\frac{V+u_O}{V-u_s}\right) f$

(ii) A source of sound moving with velocity, u_s. approaches an observer moving with velocity u_0 in the same direction. Derive the expression for frequency of sound heard by observer. (05marks)



Let c be the velocity of sound from a source of frequency, f.

Apparent wave length, λ' $= \frac{c-u}{f}$

Apparent velocity c'= c-u_0

∴ Apparent frequency $= \frac{c'}{\lambda'} = \frac{(c-u_0)}{\frac{(c-u)}{f}} = \left(\frac{(c-u_0)}{(c-u)}\right) f$

Example 10

One species of bats locates obstacles by emitting high frequency sound waves and detecting the reflected waves. A bat flying at a steady speed of 5ms^{-1} emits sound of frequency 78.0 kHz and is reflected back to it.

(i) Derive the equation for the frequency of the sound waves reaching the bat after reflection (05marks)

Suppose the velocity of sound wave is c and that of the bat is v_0 and the frequency is f.

The velocity v' of reflected sound relative to the bat is given by v' = v_0 + c.

The apparent wave length λ' $= \frac{c - v_0}{f}$

But the apparent frequency, f' $= \frac{v'}{\lambda'} = \frac{c + v_0}{\frac{c - v_0}{f}}$

Hence f' $= \frac{c + v_0}{c - v_0} f$

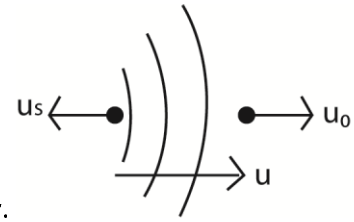
(ii) Calculate the frequency of sound received by the bat given the speed of sound in air is 340ms^{-1}. (02 marks)

From, f' $= \frac{c + v_0}{c - v_0} f$

f' $= \frac{340+5}{340-5}\ x\ 78\ x\ 10^3$ = 90.3kHz

Example 11

A motor cyclist and police car are approaching each other. The motor cyclist is moving at 10ms-1 and the police car at 20ms-1. If the police siren is sounded at 480Hz. Calculate the frequency of the note heard by the cyclist after the police car passes by.



Apparent wavelength reaching the observer, $\lambda' = \frac{v + u_s}{f}$

Apparent velocity of sound received, $v' = v - u_0$

Apparent frequency of sound received, $f' = \frac{v'}{\lambda'} = \frac{v - u_0}{v + u_s} f = \frac{340-10}{340+20} x\ 480 = 440Hz$

Example 12

(a) A source of sound moving with velocity, u. approaches an observer moving with velocity u_0 in the same direction. Derive the expression for frequency of sound heard by observer. (05marks)

Let c be the velocity of sound from a source of frequency, f.

Apparent wave length, $\lambda' = \frac{c-u}{f}$

Apparent velocity $c' = c - u_0$

∴ Apparent frequency = $\frac{c'}{\lambda'} = \frac{(c - u_0)}{\frac{(c-u)}{f}} = \left(\frac{(c-u_0)}{(c-u)}\right) f$

(b) Two whistles are sounded simultaneously. The wavelengths of the sounds emitted are 5.5m and 6.0m respectively. Find the beat frequency if the speed of sound is $330ms^{-1}$.

For 1st sound $f_1 = \frac{V}{\lambda_1} = \frac{330}{5.5} = 60Hz$

For 2nd t sound $f_2 = \frac{V}{\lambda_2} = \frac{330}{6} = 55Hz$

Beat frequency $f_b = f_1 - f_2 = 60 - 55 = 5Hz$

Thank you
Dr. Bbosa Science