SENIOR FIVE TERM 2

TOPIC 1/7: FLUID MECHANICS

Competency: The learner investigates the effect of molecular forces in fluids, fluid pressure and their applications in fluid systems.

Surface tension

Common observation explained by surface tension

1. A drop of water, on closing a tap remained dinging on the tap, as if the water was held in a bag.
2. A thin needle can be made to float on the surface as though it is denser than water
3. Mercury gathers in small spherical drops when poured on a smooth surface.
4. When a capillary tube is dipped in water, water is seen rising up in a tube
5. Insects can walk on the surface of water
6. A. Rain water forms beads on the surface of a waxy surface, such as a leaf. Water adheres weakly to wax and strongly to itself, so water clusters into drops. Surface tension gives them their near-spherical shape, because a sphere has the smallest possible surface area to volume ratio.

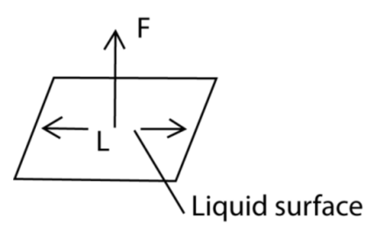
All the above observation show that a liquid surface behaves as if it was or it is in a state of tension. The phenomenon is called surface tension.

Surface Tension or Co-efficient of surface tension, γ

Is the force acting at right angle at one side of immaginary line of length 1m drawn in the surface of a liquid.

Or

Surface tension is an energy necessary to create a unit area of a surface under constant temperature, volume and chemical potential.



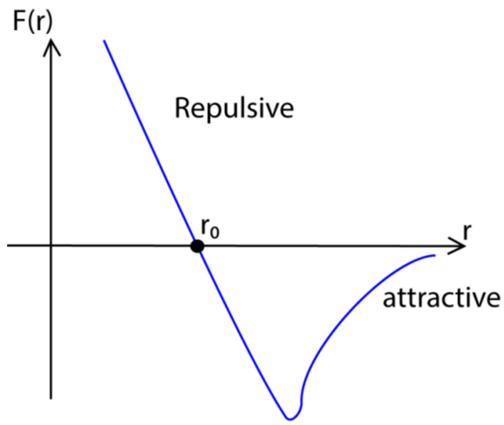
γ (gamma) = $\frac{F}{L}$

[γ] = $\frac{MLT^{-2}}{L}$ = MT^{-2}

Units = Nm^{-1}

Molecular theory of surface tension

The force F(r) between two molecules of a liquid varies with their separation r as shown below

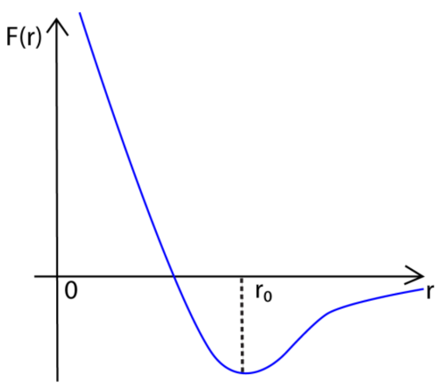


At the average equilibrium separation, r_o, F(r) = 0

For r> r_0 = the force is attractive to bring the distance between molecules to equilibrium separation, r_0.

For R < r_0 =the force is repulsive to restore the distance between molecules to equilibrium separation, r_0.

The corresponding potential energy variation with molecular separation, r, is shown below

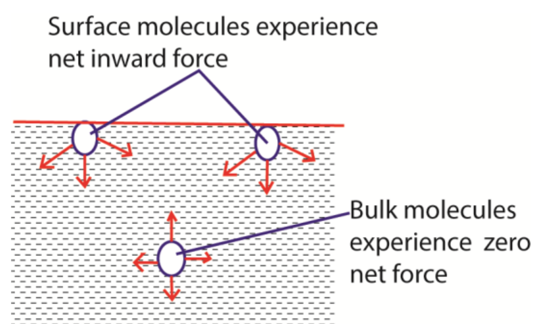


(i) The molecules within the body of the liquid (bulk) molecules is attracted equally by neighbors in all direction, hence, the force on the bulk molecules is zero, so the intermolecular separation for bulk molecules is r_0.

(ii) For a surface molecules, there is a net inward force because there are no molecules above the surface to attract them equally.

(iii) To the surface, work must be done against the inward attractive force, hence, a molecule in the surface of a liquid has a greater potential energy than a molecule in the bulk. The potential energy stored in molecules at the surface is called free surface energy.

(iv) Molecules at the surface their separation r > r_0. The attractive forces experienced by surface molecules due to their neighbours put them in a state of tension causing the surface to contract and behave as a stretched skin. This tension is called surface tension



Surface energy and shape of a drop of a liquid

System arrange themselves to chieve the least potntial energy.

In liquids, the least potential energy is achieved by having the fewest number molecules at surface or by contraction of liquid surface to the smallest possible area.

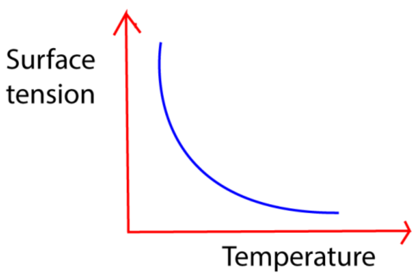
For this reason, free liquid drops are spherical because the sperical shape for any volume of a liquid gives the least surface area.

A large drop however, due to its large weight flattens out in order to mimimize the gravitational potential energy which tends to exceed the surface energy.

Factors affecting surface tension

(i) **High Temperature** lowers surface tension of molecules because molecules are far apart and moving faster decreasing time to form temporary bonds.

A graph of surface tension against temperature

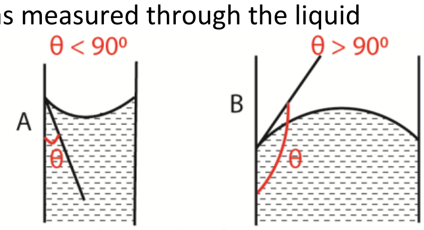


(ii) **Impurities:** these lower the surface tension forces because they displace molecules from their equilibrium positions and breaking bonds between them.

(iii) **Detergents:** reduce surface tension because they displace molecules from their equilibrium positions and break bond between them.

Angle of contact

This is the angle made between the solid surface and the tangent to the liquid surface at the point of intersection with the solid surface as measured through the liquid



θ = angle of contact

A liquid makes an acute angle of contact with solid surface when the adhesive force between the liquid and the solid are greater than the cohesive forces between the liquid molecules themselves.

A liquid makes an obtuse angle of contact with solid surface when the adhesive force between the liquid and the solid are less than the cohesive forces between the liquid molecules themselves.

A liquid that makes an acute angle with the solid is said to wet the solid surface for example water wets glass. While a liquid that makes obtuse angle with the solid does not wet it, e.g. mercury does not not wet glass but forms droplets on it.

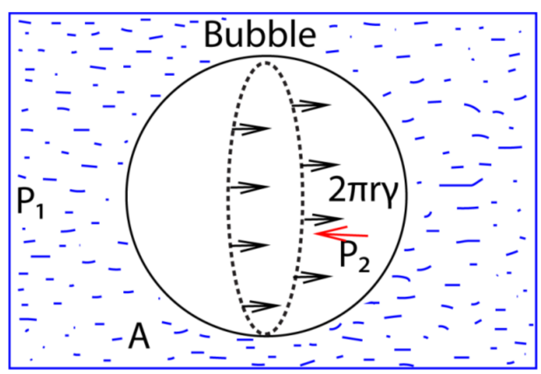
Water and mercury on glass

Addition of detergents to water reduces the angle of contact and that is why it helps in washing.

Pressure difference in a bubble or curved surface

(a) **Liquid curved surface**

Consider a bubble formed inside a liquid as shown below



If we consider equilibrium of one half A,

The force of tension on A plus the force on A due to external pressure P_1 = the force on A due to the internal pressure

P_2 inside the bubble
Force = area x pressure
(since Pressure is force per unit area.)

The force on A due to pressure $P_1 = \pi r^2 \times P_1$ (where πr^2 is the area of circular face A

and

the force on A due to pressure $P_2 = \pi r^2 \times P_2$

The surface tension force acts around the circumference of the bubble which has a length $2\pi r$, thus the force = $2\pi r\gamma$

At equilibrium:

$$2\pi r\gamma + \pi r^2 \times P_1 = \pi r^2 \times P_2$$

$$\therefore 2\gamma = r(P_2 - P_1)$$

$$\text{Or } (P_2 - P_1) = \frac{2\gamma}{r}$$

If $p = (P_2 - P_1)$, the excess pressure in the bubble over outside pressure

Then, $p = \frac{2\gamma}{r}$ ……………………………… (i)

The same formula for excess pressure holds for any curved liquid surface or meniscus, where r is its radius of curvature and γ is its surface tension, provided the angle of contact is zero.

If the angle of contact is θ, the formula is modified by replacing γ by γcosθ.

Thus in general, excess pressure,

$P = \frac{2\gamma\cos\theta}{r}$ …………………………….. (ii)

(b) Excess pressure in a soap bubble

A soap bubble has two liquid surfaces in contact with air, one inside the bubble and the other outside the bubble.

The force on one half, A, of the bubble due to surfaces is thus = $\gamma \times 2\pi r \times 2$
$= 4\pi r\gamma$

For equilibrium of A, it follows that;

$$4\pi r\gamma + \pi r^2 \times P_1 = \pi r^2 \times P_2$$

Where P_2 and P_1 are pressure inside and outside the bubble respectively

Simplifying

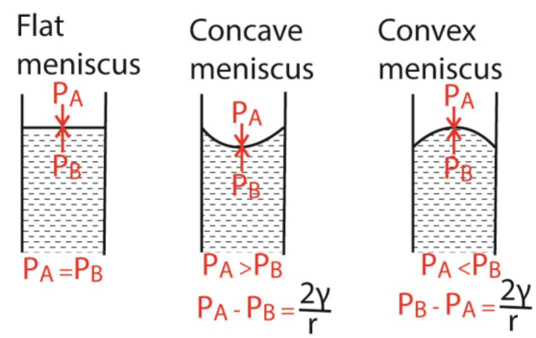
$$P_2 - P_1 = \frac{4\gamma}{r}$$

Therefore, excess pressure,

$p = \frac{4\gamma}{r}$ ……………………………………… (3)

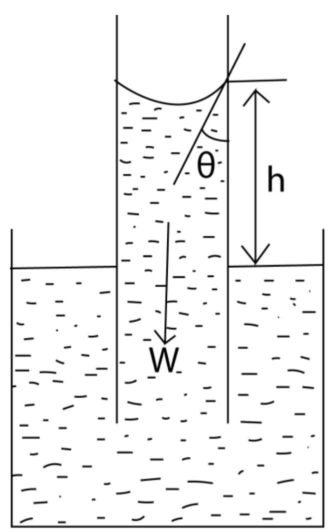
Note that,

(i) the pressure on the concave side of the liquid is always greater than that on a convex side, e.g.



(ii) When the atmospheric pressure exceeds pressure inside the bubble, it collapses.

Capillary rise



The liquid rises until the vertical

component of the upward forces due to surface tension is equal to the weight of the liquid column.

$F\gamma\cos\theta = W$

$\gamma = \frac{F}{L}$

$F = \gamma L$

$L = 2\pi r$

But W = mg and m = Vρ (where ρ is the density of the liquid in kg/m^3)

$W = v\rho g = 2\pi r^2 h\rho g$

$F\gamma\cos\theta = 2\pi r^2 h\rho g$

$\gamma.2\pi r\cos\theta = 2\pi r^2 h\rho g$

$$h = \frac{2\gamma\cos\theta}{r\rho g}$$

γ – coefficient of surface tension

θ – angle of contact

r – radius of capillary tube

ρ – density of the liquid

Example 1

A capillary tube is immersed in a liquid of density $13600kgm^{-3}$ and the angle of contact is 140^0. Find the capillary rise if the surface tension is $0.52Nm^{-1}$ and a diameter of the tube is 0.32mm

$$h = \frac{2\gamma\cos\theta}{r\rho g}$$

$$h = \frac{2 \ x \ 0.52 \ x \ 2 \ x \cos 140}{0.32 \ x \ 10^{-3} \ x \ 9.81 \ x \ 13600}$$

$$= -0.0096m$$

the negative implies there was a capillary depression of 0.0096m

Example 2

(a) Define what is meant by surface tension in terms of surface energy.

(b) (i)Calculate the work done against surface tension in blowing a soap bubble of diameter 15mm if the surface tension of the soap bubble is $3 \ x \ 10^{-2}N/m$

(ii) A soap bubble of radius r_1 is attached to another bubble of radius r_2. If r_1 is less than r_2, show that the radius of curvature of the common interface is $\frac{r_1 r_2}{r_2 - r_1}$

(c) (i) Sketch a graph of potential energy against separation of two molecules in the liquid of a substance.

(ii) Explain the main features of the graph in(c)(i)

Solution

(a) Surface tension is an energy necessary to create a unit area of a surface under constant emperature, volume and chemical potential.

(b) (i) Work done

= surface tension x increase in surface area

Surface Area of a sphere = $4\pi r^2$

The soap buble has two surface in contact with air, and thus its surface area = $2 \ x \ 4\pi r^2$

∴increase in surface of the soap bubble

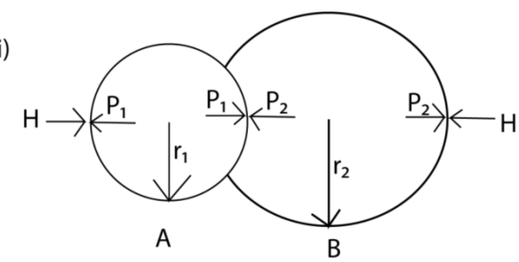
$= 2\ [4\pi(\frac{15}{2}\ x\ 10^{-3})]^2$

Hence work done

$= 3\ x\ 10^{-2}\ x 2[4\pi(\frac{15}{2}\ x\ 10^{-3})]^2$

$= 4.24 \ x \ 10^{-7}J$

(b)(ii)



For Bubble of radius r_1:

$P_1 - H = \frac{4\gamma}{r_1}$ .... (i)

For bubble of radius r^2:

$P_2 - H = \frac{4\gamma}{r_2}$ .... (ii)

Since $r_1 < r_2$; $P_1 > P_2$

From equations (i) and (ii)

$P_1 - P_2 = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$ .... (iii)

But

$P_1 - P_2 = \frac{4\gamma}{r}$ .... (iv)

From equation (iii) and (iv)

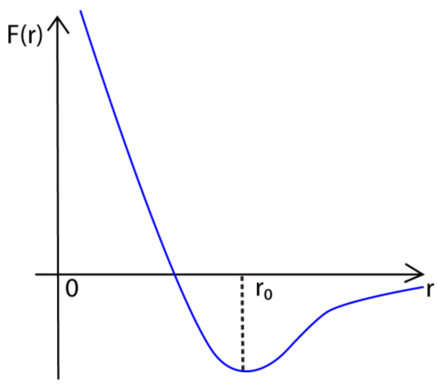
$$\frac{4\gamma}{r} = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$$

$$\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$$

$\frac{1}{r} = \frac{r_2 - r_1}{r_2 r_1}$

$r = \frac{r_2 r_1}{r_2 - r_1}$

(c)(i)



(c)(ii) At $r = r_0$, the resultant force is zero and the corresponding potential energy is minimum. So r_0 is the equilibrium separation

For $r < r_0$, the net force is repulsive, whereas $r > r_0$, the net force is attractive in order to restore the e separation to the equilibrium separation of r_0.

Example 3

(a) Define surface tension and derive its dimensions

(b) Explain using the molecular theory, the occurrence of surface tension

(c) Describe an experiment to measure surface tension of a liquid by capillary tube method.

(d) (i) Show that the excessive pressure in a soap bubble is given by $P = \frac{4\gamma}{r}$

(ii) Calculate the total pressure within the bubble of air of radius 0.1mm in water if the bubble is formed 10cm below the water surface. And the surface tension of water is 7.27×10^{-2}N/m.(atmospheric pressure = 1.01×10^5Pa)

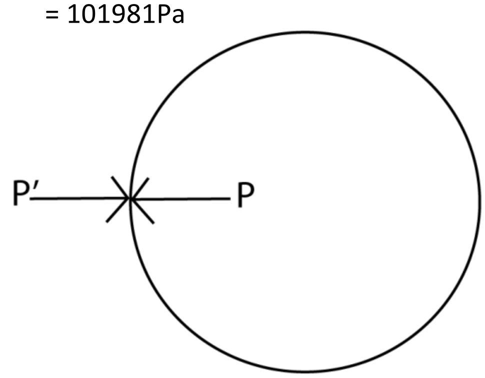
Solution

Let the pressure at 10cm below the water surface be P'

$P' = H + h\rho g$

$= 1.01 \times 10^5\text{Pa} + \frac{10}{100} \times 1000 \times 9.81$

$= 101981\text{Pa}$



Excess pressure = P – P', where P is the pressure inside the bubble

$P = \frac{2\gamma}{r} + P'$

$= \frac{2 \times 7.27 \times 10^{-2}}{0.1 \times 10^{-2}} + 101{,}981$

$= 1.03 \times 10^5\text{Pa}$

Revision exercise 1

1. (a) (i) Define the term **surface tension** and **angle constant**. (02marks)

- Surface tension is the force per metre length acting in the surface at right angles to one side of the line drawn in the surface.
- Angle of contact is the angle between the solid surface and the tangent plane to the liquid surface measured through the liquid.

(ii) Account for the temperature dependency of surface tension. (03marks)

Increase in temperatures increases the kinetic energy of liquid molecules and thus reduces the cohesive force and/or van der Waal forces among the molecules of the liquid thereby lowering the surface tension.

(b) When a capillary tube is held in a vertical position with one end just dipping in a liquid of surface tension, **γ**, and density, **ρ**, the liquid rises to a height **h**. Derive an expression for **h** in terms of **γ**, **ρ** and radius, **r** of the tube. Assume the angle of contact is zero. (04 marks)

The liquid rises until the vertical component of the upward forces due to surface tension is equal to the weight of the liquid column.

$F\gamma\cos\theta = W$ but $\theta = 0$

⇨ $F\gamma = W$

$\gamma = \frac{F}{L}$

$F = \gamma L$

$L = 2\pi r$

But W = mg and m = Vρ (where ρ is the density of the liquid in kg/m^3)

$W = v\rho g = 2\pi r^2 h\rho g$

$F\gamma = 2\pi r^2 h\rho g$

$\gamma.2\pi r = 2\pi r^2 h\rho g$

$h = \frac{2\gamma}{r\rho g}$

γ – coefficient of surface tension

θ – angle of contact

r – radius of capillary tube

ρ – density of the liquid

2. (i) A capillary tube is held in a vertical position with one end dipping in a liquid of surface tension γ and density ρ. If the liquid rises to a height, h, derive an expression for h in terms of γ, ρ and radius r of the tube assuming the angle of contact is zero. (04marks)

The liquid rises until the vertical component of the upward forces due to surface tension is equal to the weight of the liquid column.

$F\gamma\cos\theta = W$ but $\theta = 0$

⇨ $F\gamma = W$

$\gamma = \frac{F}{L}$

$F = \gamma L$

$L = 2\pi r$

But W = mg and m = Vρ (where ρ is the density of the liquid in kg/m^3)

$W = v\rho g = 2\pi r^2 h\rho g$

$F\gamma = 2\pi r^2 h\rho g$

$\gamma.2\pi r = 2\pi r^2 h\rho g$

$h = \frac{2\gamma}{r\rho g}$

γ – coefficient of surface tension

θ – angle of contact

r – radius of capillary tube

ρ – density of the liquid

(ii) A mercury drop of radius 2mm falls vertically and on hitting the ground, it splits into two drops each of radius 0.5mm. Calculate the change in surface energy given that the surface tension of mercury is $0.52Nm^{-1}$. (05marks)

Solution

Surface area of a drop = $4\pi r^2$

Surface area of a big drop = $4\pi(0.002)^2$

$= 5.03 \times 10^{-5} m^2$

Surface area of two small drops

$= 2 \times 4\pi(0.0005)^2$

$= 6.28 \times 10^{-6} m^2$

Change in area = $5.03 \times 10^{-5} - 6.28 \times 10^{-6} m^2$

$= 4.402 \times 10^{-5} m^2$

Change in surface energy

= change in area x coefficient of surface tension

$= 4.402 \times 10^{-5} \times 0.52$

$= 2.289 \times 10^{-5} J$

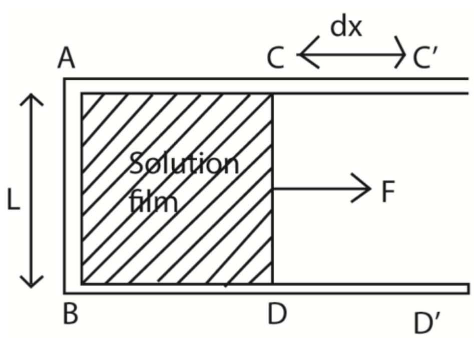
(iii) State the effect of temperature on surface tension of a liquid. (01mark)

Increase in temperature lowers surface tension because increase in temperature increases kinetic energy of molecules and increases movement of molecules reduces the cohesive forces between them.

3. (a) (i) Distinguish between surface tension and surface energy (01mark)

- Surface tension is a force per unit length acting at right angle to one side of an imaginary line drawn in the liquid surface
- Surface energy is the work done in increasing area of the surface by 1m^2 under isothermal conditions.

(ii) Show the surface energy and surface tension are numerically equal. (03marks)



- F a wire frame ABCD is put in a solution of surface tension γ and a film of the solution forms on ABCD; and if a force F is used to extend the film to ABC'D' ;
- Then surface tension, $\gamma = \frac{F}{2L}$
- Surface energy, $\sigma = \frac{Fdx}{2Ldx} = \frac{2\gamma Ldx}{2Ldx} = \gamma$
- ∴ Surface energy, σ = surface tension, γ

(iii) Explain why water dripping out of a tap does so in spherical shapes. (03marks)

For any given volume, a sphere is a shape that offer minimum surface area and therefore the most stable

(b) Two soap bubbles of radius 2.0cm and 4.0cm respectively coalesce under isothermal conditions. If the surface tension of the soap solution is 2.5x 10^{-2}Nm^{-1}, calculate the excess pressure inside the resulting soap bubble. (04marks)

$$2(\pi r_1^2) + 2(\pi r_2^2) = 2(\pi R^2)$$

$R = \sqrt{r_1^2 + r_2^2)} = \sqrt{2^2 + 4^2}$ = 4.47 cm

$P_1 - P_0 = \frac{4\gamma}{R} = \frac{4 \ x \ 2.5 \ x \ 10^{-2}}{4.47 \ x \ 10^{-2}}$ 2.24Pa

4. (a) (i) Define surface tension (01mark)

Is the force acting at right angle at one side of immaginary line of length 1m drawn in the surface of a liquid.

(ii) Explain the origin of surface tension. (03marks)

- Liquid molecules attract each other.
- The molecules within the body of the liquid (bulk) molecules are attracted equally by neighbors in all direction, hence, the force on a bulk molecule is zero,
- For a surface molecule, there is a net inward force because there are no molecules above the surface to attract them equally.
- This net inward pull puts tension on surface molecules causing the surface to contract and behave as a stretched skin. The tension is called free surface energy or surface tension.

5. (a) Define the terms surface tension and surface energy. (01mark)

It is the work done per unit area in increasing surface area of a liquid under isothermal conditions.

(b) (i) Calculate the work done against surface tension in blowing a soap bubble of diameter 15mm, if the surface tension of soap solution is 3.0 x 10^{-2}Nm. (03marks)

New surface area created = 2 x $4\pi r^2$

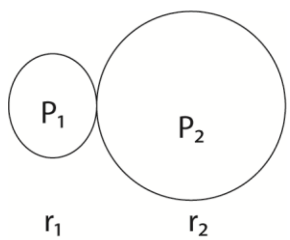
Energy required = γA

= $3.0 \times 10^{-2} \times 2 \times 4\pi \times (7.5 \times 10^{-3})^2$

= 4.24×10^{-5}J

(ii) A soap bubble of radius r_1 is attached to another bubble of radius r_2. If r_1 is less than r_2. Show that the radius of curvature of the common interface is $\frac{r_1 r_2}{r_2 - r_1}$.

(05marks)



For A

$P_1 - H = \frac{4\gamma}{r_1}$ .... (i)

For B

$P_2 - H = \frac{4\gamma}{r_2}$ .... (ii)

From equations (i) and (ii)

$P_1 - P_2 = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$ .... (iii)

$P_1 - P_2 = \frac{4\gamma}{r}$ .... (iv)

From equation (iii) and (iv)

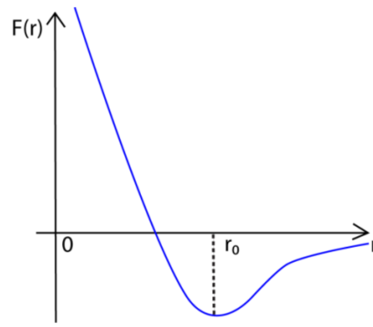
$$\frac{4\gamma}{r} = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$$

$$\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$$

$$\frac{1}{r} = \frac{r_2 - r_1}{r_2 r_1}$$

$$r = \frac{r_2 r_1}{r_2 - r_1}$$

(c) (i) Sketch a graph of potential energy against separation of two molecules of a substance. (01mark)

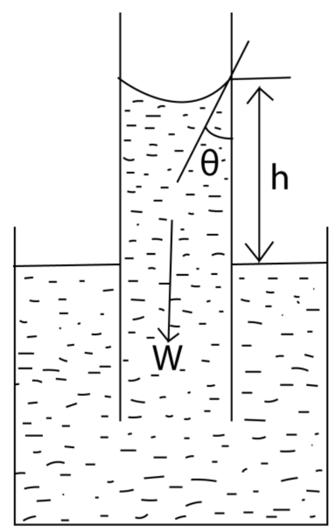


(ii) Explain the main features of the graph in (d)(i). (03marks)

At $r = r_0$, the resultant force is zero and the corresponding potential energy is minimum. So r_0 is the equilibrium separation

For $r < r_0$, the net force is repulsive, whereas $r > r_0$, the net force is attractive in order to restore the e separation to the equilibrium separation of r_0.

(iii) Describe an experiment to measure surface tension of a liquid by capillary method. (06marks)



The liquid rises until the vertical component of the upward forces due to surface tension is equal to the weight of the liquid column.

Fγcosθ = W

γ = $\frac{F}{L}$

F = γL

L = 2πr

But W = mg and m = Vρ (where ρ is the density of the liquid in kg/m^3)

W = vρg = $2\pi r^2 h\rho g$

Fγcosθ = $2\pi r^2 h\rho g$

γ.2πrcosθ = $2\pi r^2 h\rho g$

$$h = \frac{2\gamma\cos\theta}{r\rho g}$$

γ – coefficient of surface tension

θ – angle of contact

r – radius of capillary tube

ρ – density of the liquid

6. (a) Define surface tension and derive its dimensions (03marks)

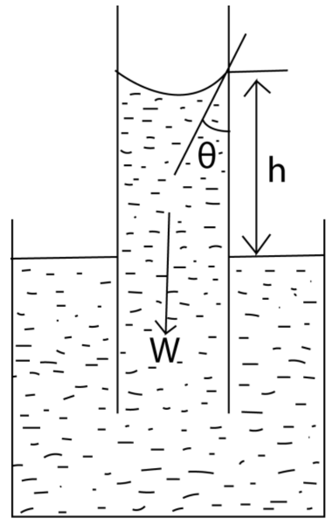
Surface tension is the force per unit length acting perpendicularly to one side if an imaginary line.

$$\gamma = \frac{Force}{length} = \frac{[Ma]}{[L]} = \frac{MLT^{-2}}{L} = MT^{-2}$$

(b) Explain using the molecular theory the occurrence of surface tension. (04 marks)

Molecules at the liquid surface have greater molecular separation than the equilibrium separation. These molecules experience greater attraction from their neighbours. This puts them in a state of tension. Thus the liquid surface behaves like a stretched elastic skin, a phenomenon called surface tension

(c) Derive an experiment to measure surface tension of a liquid by the capillary tube method. (06marks)



A capillary tube of radius, r, is vertically placed in a liquid. The liquid rises until the vertical component of the upward forces due to surface tension is equal to the weight of the liquid column.

Fγcosθ = W

γ = $\frac{F}{L}$

F = γL

L = 2πr

But W = mg and m = Vρ (where ρ is the density of the liquid in kg/m^3)

W = vρg = $2\pi r^2 h\rho g$

Fγcosθ = $2\pi r^2 h\rho g$

γ.2πrcosθ = $2\pi r^2 h\rho g$

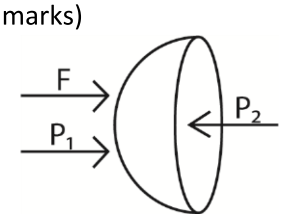
$$\gamma = \frac{hr\rho g}{2\cos\theta}$$

γ – coefficient of surface tension

θ – angle of contact

ρ – density of the liquid

(d) (i) Show that the excess pressure in a soap bubble is given by P = $\frac{4\gamma}{r}$, (03marks)



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The force, F, on one half of the bubble due to surfaces is thus
$= \gamma \times 2\pi r \times 2$
$= 4\pi r\gamma$

For equilibrium of A, it follows that.
$4\pi r\gamma + \pi r^2 \times P_1 = \pi r^2 \times P_2$

where P_2, and P_1 are pressure inside and outside the bubble respectively
Simplifying

$$P_2 - P_1 = \frac{4\gamma}{r}$$

Therefore, excess pressure, $P = \frac{4\gamma}{r}$

(ii) Calculate the total pressure within a bubble of air of radius 0.1mm in water if the bubble is formed 10cm below the water surface and surface tension of water is $7.27 \times 10^{-2}Nm^{-1}$. [Atmospheric pressure = 1.01×10^5Pa] (05marks)

Excess pressure $= \frac{2\gamma}{r}$

$$= \frac{2 \times 7.27 \times 10^{-3}}{0.1 \times 10^{-2}}$$

$$= 1454Pa$$

Total pressure within the bubble

= atmospheric pressure + hρg + excess pressure

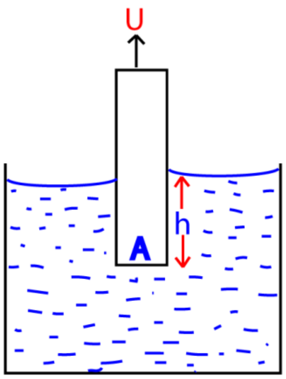
$= 1.01 \times 10^5 + (0.1 \times 10^3 \times 9.81) + 1454$

$= 1.034 \times 10^5Pa$

Pressure in liquids

This is the force exerted normally per unit area. The SI units of pressure of liquids are Nm^{-2} or Pascals (pa)

The pressure in liquids is independent of the shape and cross sectional are as shown below



Volume of a liquid displaced
= Ah (A = cross section area)

Mass of the liquid displaced
= Ahρ (ρ = density of a liquid)

Weight of the liquid displaced
= Ahρg (g = acceleration due to gravity)
= upthrust

$$\text{Pressure} = \frac{Force}{Area}$$
$$= \frac{upthrust}{area}$$
$$= \frac{\text{Ahρg}}{A}$$
$$= \text{hρg}$$

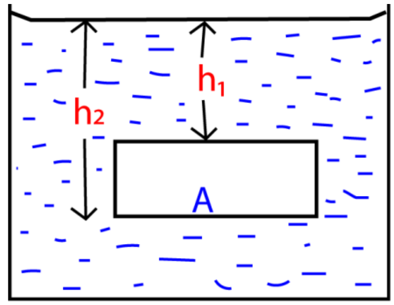
Since, ρ and g are constant; $P \propto h$

Floating objects

Archimedes principle

When a body is partially or fully immersed in a fluid, it experiences an upthrust which is equal to the weight of a fluid displaced.

Consider a solid of cross section area A immersed in a liquid of density, ρ.



Total pressure at the top = $H + h_1\rho g$
Force on top surface = $(H + h_1\rho g)A$

Total pressure at the bottom = $H + h_2\rho g$
Force on bottom surface = $(H + h_2\rho g)A$

Resultant upward force
= upthrust
= $(H + h_2\rho g)A - (H + h_1\rho g)A$
= $(h_2 - h_1)\rho gA$

But $(h_2 - h_1)A$ = volume of the solid
= volume of the liquid displaced
$(h_2 - h_1)A\rho g$ = weight of liquid displaced

Hence, Upthrust = weight of the liquid displaced

Conclusion

Since there is no side way movement, the resultant horizontal force is zero. Therefore, up thrust is equal to the weight of fluid displaced.

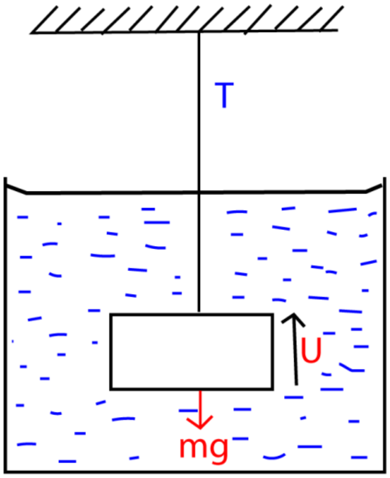
Law of floatation

States that a floating object displaces a liquid equal to its own weight of the liquid in which it floats

Example 4

A string supports a solid of mass 5kg totally immersed in a liquid of density $800kgm^{-3}$. Find the tension in the string if the object has a density of $2575kgm^{-3}$.

Solution



mg = T + U

T = mg – U

U = weight of fluid displaced.

Volume of solid = volume of liquid displace

$= \frac{Mass}{density} = \frac{5}{2575}$

Mass of the liquid displaced

= volume x density

$= \frac{5}{2575} \; x\; 800$

Weight of the liquid displaced, U

$= \frac{5}{2575} \; x\; 800 \; x\; 9.81$

= 15.25N

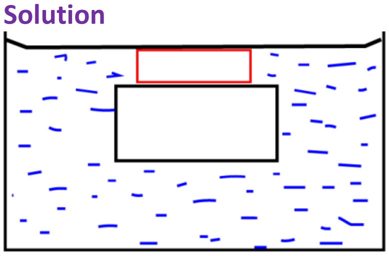
Tension T = 5 x 9.81 – 15.24

= 33.81N

Example 5

A piece of metal of mass 2.60 x 10^{-3} kg and density 8.4 x 10^3kgm^{-3} is attached to the block of mass of 1.0x 10^{-2}kg and density 9.2 x 10^2kgm^{-3}. When the system is placed in a fluid, it floats with wax just submerged. Find the density of the fluid.

Solution



Volume = $\frac{mass}{density}$

Volume of liquid displaced = volume of the metal + volume of wax

$= \frac{2.60\; x\; 10^{-3}}{8.4\;\; x\; 10^{3}} + \frac{1.0\; x\; 10^{-2}}{9.2\;\; x\; 10^{2}}$

=1.118 x 10^{-5}m^3

Mass of the liquid displaced = mass of metal + mass of wax

= 2.60 x 10^{-3} + 1.0x 10^{-2}

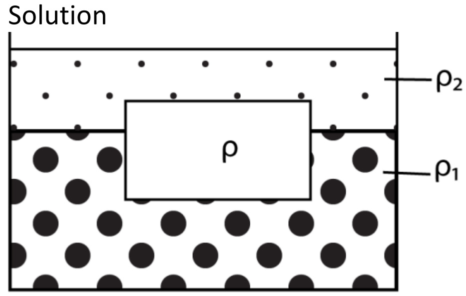
= 0.0126kg

Density of a liquid = $\frac{mass}{volume} = \frac{0.0126}{1.118\; x\; 10^{-5}}$ = 1,127kgm^{-3}

Example 6

A solid of density, ρ, floats at the interface of two liquids of densities ρ_1 and ρ_2 with 80% of its volume in liquid of density ρ_1. Show that $\frac{\rho - \rho_2}{\rho_1 - \rho} = 4\; (\rho_1 > \rho_2)$

Solution



Let the volume of the solid be V

Volume of the solid in liquid of density, ρ_1 = $\frac{8}{10}V$

Volume of the solid in liquid of density, ρ_2 = $\frac{2}{10}V$

Mass of the solid = Vρ

$V\rho = 0.8V\rho_1 + 0.2V\rho_2$

$\rho = 0.8\rho_1 + 0.2\rho_2$

$0.8\rho + 0.2\rho = 0.8\rho_1 + 0.2\rho_2$

$0.2(\rho - \rho_2) = 0.8(\rho_1 - \rho)$

$\frac{(\rho - \rho_2)}{(\rho_1 - \rho)} = 4$

Relative density (R.D)

This is the ratio of mass of a substance to the mass of equal volume of water.

$$R.d = \frac{mass\ of\ a\ substance}{mass\ of\ equal\ volume\ of\ water}$$

$$R.d = \frac{Weight\ of\ a\ substance}{Weight\ of\ equal\ volume\ of\ water}$$

$$= \frac{weight\ of\ substance}{upthrust}$$

$$= \frac{weight\ in\ air}{apparent\ losss\ in\ weight}$$

Example 7

A solid of mass 0.2kg is suspended from a spring balance when the block is immersed in water the spring reads 0.84N. When the block is immersed in a liquid of unknown density, the spring balances reads 0.95N. Find

(i) Density of the block

(ii) Density of the liquid

Solution

$$R.D = \frac{weight\ in\ air}{apparent\ losss\ in\ weight}$$

$$= \frac{0.2\ x\ 9.81}{(0.2\ x\ 9.81) - 0.84}$$

= 1.749

Density of the solid = 1.749 x 1000

= 1749kgm^{-3}

(ii) For liquids

$$R.D = \frac{loss\ in\ weight\ of\ solid\ in\ liquid}{loss\ in\ weight\ of\ solid\ in\ water}$$

$$= \frac{1.962 - 0.95}{1.962 - 0.84}$$

$$= \frac{1.012}{1.122}$$

$= 0.902$

Density of the liquid = 0.902 x 10000

= 902kgm^{-3}

Experiment to determine the relative density of a substance that floats in water

Apparatus: thread, spring balance, object, sinker and water

By means of a thread tied to the object, determine the weight, W_1, of an object in air using a spring balance.

Attach a sinker to the object and immerse the two in water and determine the weight, W_2, of the body and the sinker.

Determine the weight of the sinker, W_3.

$$R.D = \frac{weight\ in\ air}{apparent\ losss\ in\ weight} = \frac{W_1}{W_1 - (W_2 - W_3)}$$

Density of the substance, ρ = R.D x 1000kgm^{-3}

Experiment to determine the relative density of a liquid

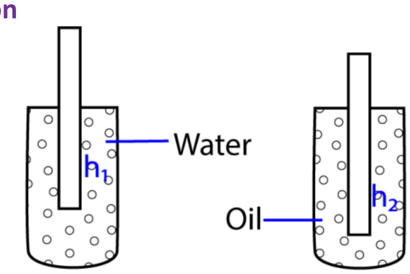
By means of a thread, determine the weight of solid in air, liquid, and water = W_1, W_2, and W_3 respectively.

$$R.D = \frac{W_1 - W_2}{W_1 - W_2}$$

Density rod

This is a rod used to compare densities of two liquids. The higher the rod floats, the higher the density of the liquid. A density rod floats with height h_1, submerged in water. In oil, it floats with height, h_2, submerged. Show that the relative density of oil is $\frac{h_1}{h_2}$.

Solution



Mass of the rod = mass of water displaced

= mass of oil displaced

Let the rod have a cross sectional area = A

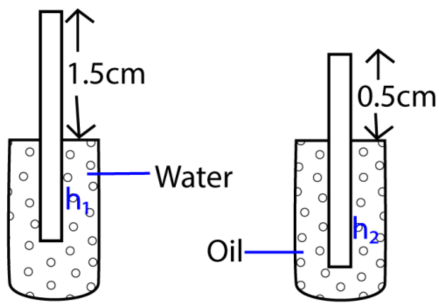
$Ah_1\rho_w = Ah_2\rho_o$

$$R.D = \frac{\rho_o}{\rho_w} = \frac{h_1}{h_2}$$

Example 8

An object of mass 30g and density $2g/cm^3$ has a uniform cross section area of $3cm^2$ floats in water and oil leaving a height of 1.5 and 0.5cm respectively above the surfaces. Calculate the relative density of oil.

Solution



$Ah_1\rho_w = Ah_2\rho_o$

$R.D = \frac{\rho_o}{\rho_w} = \frac{h_1}{h_2}$

But $V = \frac{m}{\rho} = \frac{30}{2} = 15cm^3$

V = Ah

15 = 3h

h = 5cm

$h_1 = 5 - 1.5 = 3.5cm$

$h_2 = 5 - 0.5 = 4.5cm$

$R.D = \frac{3.5}{4.5} = 0.7$

Hygrometer

It consists of a bulb that floats upright above a liquid surface. Lead shots are placed in the bulb to ensure that the system floats upright.

Example 9

A hygrometer floats on water with 72% of its volume submerged in a liquid. It floats with 68% of its volume submerged. Find the relative density of the liquid

Mass of hygrometer = mass of water displaced

= mass of liquid displaced

Let the volume of hygrometer = V

$\frac{72V\rho_l}{100} = \frac{68V\rho_w}{100}$

$\frac{\rho_l}{\rho_w} = \frac{68}{72} = 0.94$

Types of flow

(i) Laminar flow

Laminar flow occurs when the fluid flows in tiny parallel layers with no disruption between them. The successive particles passing a given point have the same velocity.

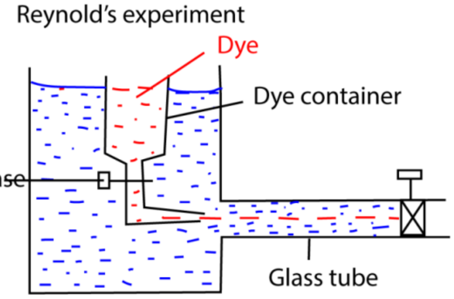
The velocity of particles may change from one streamline to another

(ii) Turbulence/turbulent flow/non uniform flow

In turbulent flow the speed of the fluid at a point is continuously undergoing changes in both magnitude and direction.

Common examples of turbulent flow are blood flow in arteries, oil transport in pipelines, lava flow,
atmosphere and ocean currents,
the flow through
pumps and turbines, and the flow in boat wakes and around aircraft-wing tips.

Experiment to demonstrate laminar and turbulent flow



Water is kept flowing at a constant velocity from a constant water tank.

The rate of flow of a dye is controlled by a tap A.

At low water velocity a streamline of a dye is observed flowing through water. This is laminar flow

A turbulent flow is observed when the velocity of water is increased here the dye mixes with water.

Viscosity

This is the frictional force that opposes the relative motion between different fluid layers. It is the result of intermolecular forces between particles within a fluid which necessitates work to be done when layer move over one another.

Factors affecting the magnitude of viscosity

(i) Temperature: increase in temperature reduces intermolecular forces due to increased kinetic energy. This reduces viscosity.

In gases viscosity increases as temperature increases due to molecular diffusion from one layer to another of different velocities. As the temperature increases, the rate of diffusion also increases and the drag exerted on each layer by the other increases.

(ii) Chemical composition
The viscosity of liquids generally depends upon the size, shape and chemical nature of their molecules.
It is greater with larger than with smaller molecules; with elongated than with spherical molecules.
Large amounts of dissolved solids generally increase viscosity. Small amounts of electrolytes lower the viscosity of water slightly.

(iii) Colloid Systems:
The viscosity of lyophilic colloid solution is generally relatively high.

(iv) Suspended Material:
Suspended particles cause an increase in the viscosity. The viscosity of blood is important in relation to the resistance offered to the heart in circulating the blood. The heart muscle functions best while working against a certain resistance. The viscosity of blood is due largely to the emulsoid colloid system present in plasma and to the great proportion of suspended corpuscles.

Velocity gradient

This is the change in velocity per unit length

Velocity gradient = $\frac{\Delta V}{L} = \frac{V}{L} = \frac{LT^{-1}}{L} = T^{-1}$

Newton's law of viscosity

The frictional force between different fluid layers is directly proportional to the area of molecular layer.

$F \propto A$ …………………………………… (i)

The frictional between different fluid layers is directly to velocity gradient

$F \propto \frac{\Delta V}{L}$ …………………………………… (ii)

Combining (i) and (ii)

$F \propto \frac{\Delta V}{L} A$

$F = \eta \frac{\Delta V}{L} A$

$\eta = \frac{F}{\frac{\Delta V}{L} A}$

If $A = 1m^2$, $\frac{\Delta V}{L} = 1s^{-1}$, $F = 1N$

$\eta = 1Nsm^{-2}$

Coefficient of viscosity of a liquid

This is the frictional force per unit area exerted on the fluid in the region of a unit velocity gradient

Or

It is the ratio of tangential stress exerted on layers of fluid to velocity gradient. Units are $NM^{-2}s$ or Ns/m^2. Other units are $kgm^{-1}s^{-1}$.

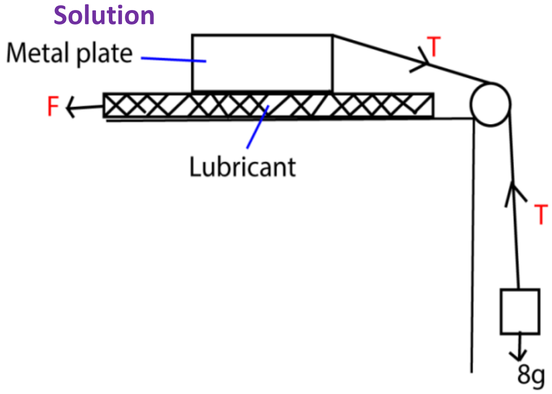
Example 10

A metal plate of area $0.25m^2$ is connected to 8g mass via a light string that passes over a frictionless pulley. A lubricant with a film of thickness 0.6mm is placed between the plate and the horizontal surface. When released, the plate moves with a speed of $87ms^{-1}$

(i) Find the coefficient of viscosity of lubricant

(ii) State any assumptions made

Solution



F = mg

$F = \eta \frac{\Delta V}{L} A$

$\eta = \frac{F}{\frac{\Delta V}{L} A} = \frac{8 \times 10^{-3} \times 9.81}{\frac{(0.087-0)}{0.6 \times 10^{-3}} \times 0.25}$

$= 2.16 \times 10^{-3} Nsm^{-2}$

(ii) the top layer of the film is assumed to move with the same velocity as the metal plate while the bottom layer is stationary.

Viscous drag

This is the frictional force that oppose relative motion between a solid and a viscous fluid

Stokes' law

The viscous drag experienced by an object depends on the velocity, viscosity constant and the radius of an object.

$F \propto v^x \eta^y r^z$

$F = k\, V^x \eta^y r^z$

$[F] = [v]^x [\eta]^y [r]^z$

$MLT^{-2} = (LT^{-1})^x (MLT^{-1}T^{-1})^y (L)^z$

For M: y = 1

For T: -2 = -x – y

x = 1

For L: 1 = x – y +z

Z = 1

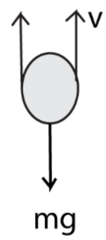
From repeated experiments, k = 6π

⇨ F = 6πηvr

Example 11

The water drop of mass 10g falls through air of viscosity constant 1.0×10^{-5}Pa. Calculate the viscous drag, experienced by the droplet when it attains a terminal velocity of $2mms^{-1}$

Solution



F = 6πηvr

m = volume x density

$10 \times 10^{-3} = \frac{4}{3}\pi r^3 \rho$

$r^3 = \frac{3 \times 10 \times 10^{-3}}{4\pi \times 1000}$

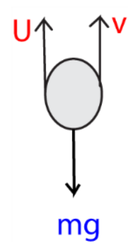
r = 0.0134m

$F = 6\pi\eta vr = 6\pi \times (1.0 \times 10^{-5}) \times (2 \times 10^{-3}) \times 0.0134$

$= 5.04 \times 10^{-4}$N

Terminal velocity

Consider a spherical object dropped in a viscous fluid



As the object drops, it is acted on by three forces, U= up thrust up, viscous drag up and weight, (mg) down.

F = mg – (U+v)

But U = $\frac{4}{3}\pi r^3 \rho g$ and v= 6πηvr

Where r = radius of the ball, ρ = density of fluid, η= viscous constant, v = velocity of the falling ball.

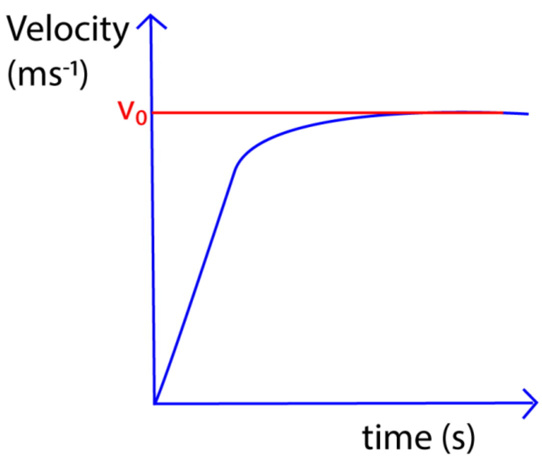
As the velocity increases, the viscous drag force increases. At a certain velocity v_0, known as terminal velocity, the resultant force acting at the body is zero.

mg = U + v

Definition

Terminal velocity is the maximum constant velocity attained by an object falling through a viscous fluid.

A graph of the velocity of an object falling through a viscous fluid against time



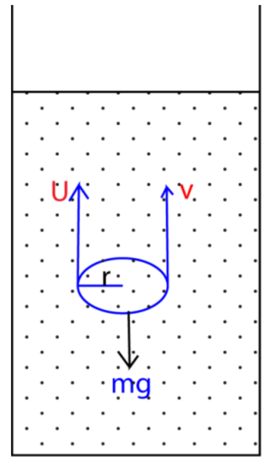
Example 12

Explain why raindrops hit the ground with less force than they should.

The drag force and up thrust reduce the force by which raindrops would hit the grounds

Derivation of terminal velocity

Consider a spherical object of radius r and density, σ, falling through a viscous fluid of density, ρ, viscous constant, η, and v_0 = terminal velocity



mg = U + v

mg = $\frac{4}{3}\pi r^3 \sigma g$

U = weight of fluid displaced

= $\frac{4}{3}\pi r^3 \rho g$

$\frac{4}{3}\pi r^3 \sigma g = \frac{4}{3}\pi r^3 \rho g + 6\pi\eta v_0 r$

$6\pi\eta vr = \frac{4}{3}\pi r^3(\sigma - \rho)g$

$v_0 = \frac{2r^2(\sigma-\rho)g}{9\eta}$

Example 13

A spherical ball of radius 2.5mm and density 900kgm^{-3} fall through air of viscosity constant 1.88 x 10^{-3}.calculate the terminal velocity if

(i) Density of air is 1.29kgm^{-3}.

(ii) Density of air is negligible.

Solution

$v_0 = \frac{2r^2(\sigma-\rho)g}{9\eta}$

(i) $v_0 = \frac{2(2.5 \, x \, 10^{-3})^2 \, x \, 9.81 \, x \, (900-1.29)}{9 \, x \, 1.88 \, x \, 10^{-3}}$

= 6.513ms^{-1}

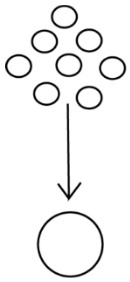
(ii) $v_0 = \frac{2(2.5 \, x \, 10^{-3})^2 \, x \, 9.81 \, x \, 900}{9 \, x \, 1.88 \, x \, 10^{-3}}$

= 6.523ms^{-1}

Example 13

Eight similar water drops fall with a terminal velocity of 5mms^{-1}. And when mid-way, they coalesce forming a big droplet. Calculate the terminal velocity of a big droplet if the density of air is negligible.

Solution



$v_0 = \frac{2r^2(\sigma-\rho)g}{9\eta}$

When density of air is negligible, terminal velocity v_r for small droplets is given by

$V_r = \frac{2r^2\sigma g}{9\eta} = 5.0 \times 10^{-3}$ ……………………….. (i)

Terminal velocity for big droplet v_R is given by

$V_R = \frac{2R^2\sigma g}{9\eta}$

By conserving volume

$\frac{4}{3}\pi r^3 \ x\ 8\sigma = \frac{4}{3}\pi R^3 \ x\ \sigma$

$R^3 = 8r^3$

$R = 2r$

$V_R = \frac{2(2r)^2\sigma g}{9\eta}$ ………………………………. (ii)

Dividing Eqn (ii) with Eqn (i)

$\frac{v_R}{5 \times 10^{-3}} = \frac{2(2r)^2\sigma g}{9\eta} \ x\ \frac{9\eta}{2r^2\sigma g}$

$$v_R = 2 \times 10^{-2} ms^{-1}$$

Example 14

A metallic ball of mass 0.9g and diameter 8mm is dropped in oil of density 780kgm^{-3} attaining a terminal velocity of 0.07ms^{-1}. A ball falls with terminal velocity of 0.03ms^{-1} when oil is replaced with water of density 1000kgm^{-3}. Find the ratio of the coefficient of viscosity of oil to that of water at the same temperature.

Solution

Density of the ball, $\sigma = \frac{mass}{volume} = \frac{0.9 \times 10^{-3}}{\frac{4}{3}\pi(4 \times 10^{-3})^3} =$ 3.35x10^{3}kgm^{-3}

$\eta = \frac{2r^2(\sigma-\rho)g}{9v}$

For oil,

$\eta_o = \frac{2\,(4 \times 10^{-3})^2 \times 9.81(3.36 \times 10^3 - 780)}{9 \times 0.07}$

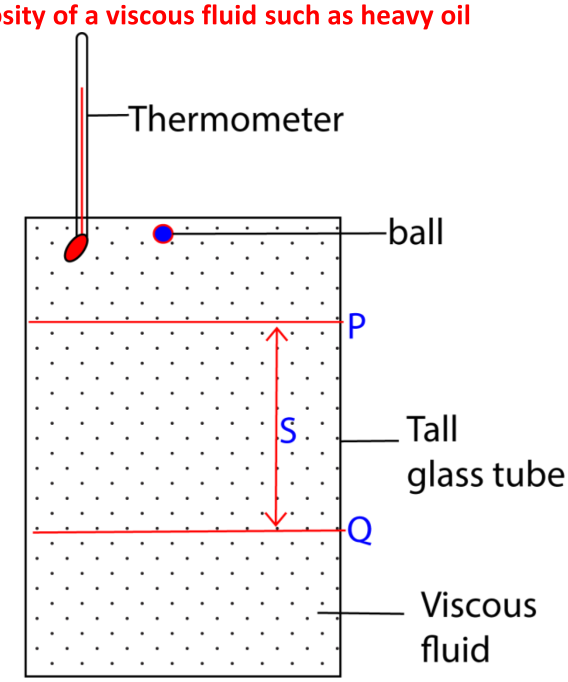
For water,

$\eta_w = \frac{2\,(4 \times 10^{-3})^2 \times 9.81(3.36 \times 10^3 - 1000)}{9 \times 0.03}$

$\frac{\eta_o}{\eta_w} = \frac{2\,(4 \times 10^{-3})^2 \times 9.81(3.36 \times 10^3 - 780)}{9 \times 0.07} \times \frac{9 \times 0.03}{2\,(4 \times 10^{-3})^2 \times 9.81(3.36 \times 10^3 - 1000)}$

$\frac{\eta_o}{\eta_w} = \frac{2850 \times 3}{7 \times 2360} = \frac{12}{28}$

Experiment to determine the coefficient of viscosity of a viscous fluid such as heavy oil



1. A viscous fluid of density, ρ, at constant temperature is put in a tall glass jar with reference marks P and Q a distance, S, apart.
2. A ball bearing of density, σ, and radius, r, is dropped into the fluid.
3. Time taken, t, taken for the ball bearing to drop from P to Q is noted
4. Assuming the ball bearing travels with a terminal velocity, v, between P and Q, then

$v = \frac{S}{t} = \frac{2r^2(\sigma-\rho)g}{9\eta}$

$\eta = \frac{2r^2(\sigma-\rho)gt}{9\,x\,S}$

Experiment to determine the coefficient of viscosity of a viscous liquid using the graphical method

(i) A viscous fluid of density, ρ, at constant temperature is put in a tall glass jar with reference marks P and Q a distance, S, apart.

(ii) A ball bearing of density, σ, and radius, r, is dropped into the fluid.

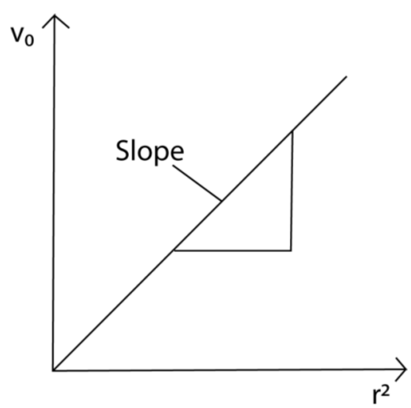
(iii) Time taken, t, taken for the ball bearing to drop from P to Q is noted

(iv) Assuming the ball bearing travels with a terminal velocity, v_0, between P and Q, then, $v_0 = \frac{S}{t}$

(v) The procedure is repeated for different ball bearing having various radii.

(vi) The results of t, r, v_o, r^2 are tabulated.

(vii) A graph of v^0 against r^2 is plotted.



Slope $= \frac{2(\sigma-\rho)g}{9\eta}$

$\eta = \frac{2(\sigma-\rho)g}{9\,x\,slope}$

Experiment to compare the coefficient of viscosity of two viscous fluids

(i) A viscous fluid 1 of density, ρ_1, at constant temperature is put in a tall glass jar with reference marks P and Q a distance, S, apart.

(ii) A ball bearing of density, σ, and radius, r, is dropped into the fluid.

(iii) Time taken, t_1, taken for the ball bearing to drop from P to Q is noted

(iv) Assuming the ball bearing travels with a terminal velocity, v_0, between P and Q, then, $v_0 = \frac{S}{t}$.

(v) Procedure 1, 2, 3, 4 are repeated for viscous fluid 2 of density ρ_2 and time t_2 to fall from P to Q is determined

$v_2 = \frac{S}{t_2}$

$\eta_1 = \frac{2r^2(\sigma-\rho_1)g}{9\,x\,v_1}$

$\eta_1 = \frac{2r^2(\sigma-\rho_2)g}{9\,x\,v_2}$

$\frac{\eta_1}{\eta_2} = \frac{2r^2(\sigma-\rho_1)g}{9\,x\,v_1}\ x\ \frac{9\,x\,v_2}{2r^2(\sigma-\rho_2)g}$

$\frac{\eta_1}{\eta_2} = \frac{2r^2(\sigma-\rho_1)g}{9\,x\,v_1}\ x\ \frac{9\,x\,v_2}{2r^2(\sigma-\rho_2)g}$

$\frac{\eta_1}{\eta_2} = \frac{(\sigma-\rho_1)v_2}{(\sigma-\rho_2)v_1}$

Poissulle's law

During steady flow, the rate of flow of a liquid through a pipe depends on;

(i) Coefficient of viscosity of the fluid

(ii) Radius of the pipe

(iii) The pressure gradient across the pipe

$\frac{v}{t} = k\eta^x r^y \left(\frac{P}{L}\right)^z$

$L^3T^{-3} = (ML^{-1}T^{-1})^x\ L^y\ x\ \left(\frac{MLT^{-2}}{L}\right)^z$

Solving

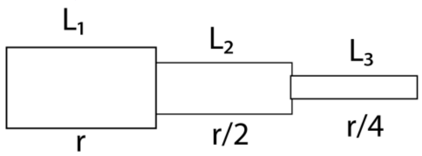
x = -1

z = 1

y= 4

$\frac{v}{t} = \frac{\pi r^4 P}{8\eta l}$, k = $\frac{\pi}{8}$

Example 15



Three pipes are arranged in some area as shown above. If the pressure in the first pipe is P1, deduce the pressure in the second and third pipe assuming there is a steady flow and $2l_1 = 3l_2 = \frac{1}{2} l_3$

Solution

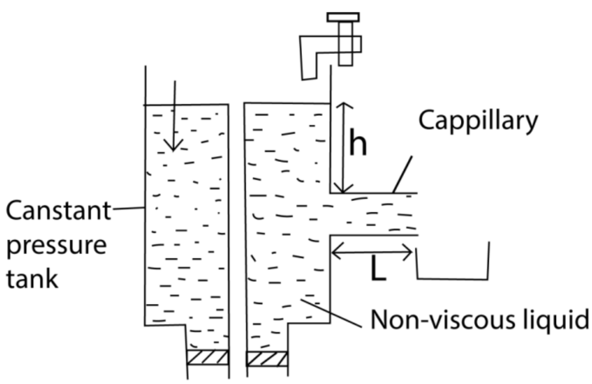
$$\frac{v_1}{t} = \frac{v_2}{t} = \frac{v_3}{t}$$

$$\frac{\pi r^4 P_1}{8\eta l} = \frac{\pi\left(\frac{r}{2}\right)^4 P_2}{\frac{2x\,8\eta}{3} l_1}$$

$$P_2 = \frac{3P_1}{32}$$

$$P_3 = 1024 P_1$$

Experiment to determine coefficient of viscosity of non-viscous liquid



(i) One end of capillary tube whose diameter, r, is known is connected to constant pressure apparatus

(ii) The liquid is allowed to flow in a capillary tube until a steady state is reached when height, H, is stable

(iii) Volume of liquid V flowing out in time t is measured.

(iv) The pressure height, h, in the capillary tube is measured.

For steady flow, $\frac{V}{t} = \frac{\pi r^4 P}{8\eta l}$

But P = hρg

$r = \frac{d}{2}$

$$\frac{V}{t} = \frac{\pi\left(\frac{d}{2}\right)^4 P}{8\eta l}$$

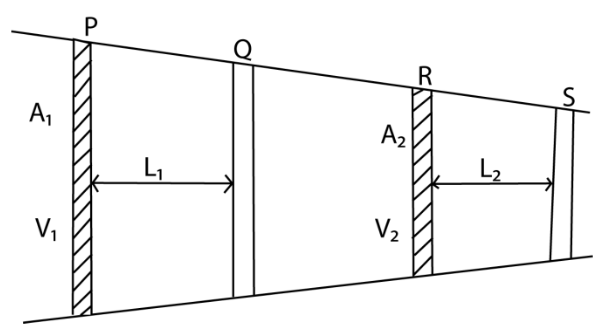
$$\eta = \frac{\pi\left(\frac{d}{2}\right)^4 h\rho g}{8\left(\frac{v}{t}\right) l}$$

Incompressible liquid

This is the liquid whose density does not change with change in pressure.

Continuity equation

Consider a non-viscous incompressible liquid flowing through a tube of non-uniform cross sectional area.



Volume of a liquid between P and Q = volume of a liquid between R and S

$A_1L_1 = A_2L_2$

But $L_1 = V_2\Delta t$

$A_1V_1\Delta t = A_2V_2\Delta t$

AV = constant

$A_1V_1 = A_2V_2$

$\frac{A_1}{A_2} = \frac{L_2}{L_1}$

$A_1 > A_2 \Rightarrow L_2 > L_1$

$A_1V_1 = A_2V_2$

$\frac{A_1}{A_2} = \frac{V_2}{V_1}$

$A_1 > A_2$, ∴ $V_2 > V_1$

Liquids travel longer distances with high velocity in pipes of small diameter compared to those of large diameters.

$\frac{A\,x\,l}{T} = \frac{V}{T}$

V $\propto \frac{1}{A}$

For a streamline, the velocity of a liquid at any section of the pipe is inversely proportional to the area of cross section at that point

Bernoulli's principle

For a streamline flow, the sum of pressure, kinetic energy per unit volume and potential energy per unit volume is constant at all points for a non-viscous incompressible fluid.

Bernoulli's equation

A moving liquid has 3 types of energies

(i) Kinetic energy: energy possessed by a liquid due to motion

(ii) Potential energy: energy possessed by a liquid due to its position in the field of force

(iii) Pressure energy: energy posed by a liquid due to its pressure at particular point.

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

Derivation of Bernoulli's expression

- Considering a moving incompressible liquid, if the viscosity is negligibly small, there are no frictional forces to overcome.
- In this case the work done by the pressure difference per unit volume of a fluid flowing along a pipe steadily is equal to the gain of kinetic energy per unit volume plus the gain in potential energy per unit volume.
- Assuming the area is constant at a particular place for a short time of flow; the work done by a pressure in moving a fluid through a distance

  = force x distance moved

  = (pressure x area) x distance moved

  = pressure x volume moved,
- At the. beginning of the pipe where the pressure is P_1, the work done per unit volume on the fluid is thus P_1;
- At the other end, the work done per unit volume by the fluid is likewise P_2
- Hence the net work done on the fluid per unit volume = $P_1 - P_{21}$
- The kinetic energy per unit volume

  = $\frac{1}{2}$ mass per unit volume x velocity^2

  = $\frac{1}{2}\rho$ x velocity^2, where ρ is the density of the fluid.

  .= $\frac{1}{2}\rho$ x velocity^2
- Thus if v_2 and v_1 are the final and initial velocities respectively at the end and the beginning of the pipe, the kinetic energy gained per unit volume = $\frac{1}{2}\rho({v_2}^2 - {v_{12}}^2)$.
- Further, if h_2 and h_1, are the respective heights measured from a fixed· level at the end and· beginning of the pipe, the potential energy gained per unit volume

  = mass per unit volume x g x (h_2 - h_1)

  = pg(h_2 - h_1).

- Thus, from the conservation of energy

$$P_1 - P_2 = \frac{1}{2}\rho({v_2}^2 - {v_{12}}^2) + \rho g(h_2 - h_1).$$

$$P_1 + \frac{1}{2}\rho {v_1}^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho {v_2}^2 + \rho g h_2$$

$$\therefore P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

Hence for streamline motion of an incompressible non-viscous fluid

"The sum of the pressure at any part plus the kinetic energy per unit volume pus potential energy per unit volume is always constant."

Example 16

Explain why Bernoulli's equation only applies for a non-viscous incompressible fluid.

Solution

For the viscous fluid, energy is not constant while for compressible liquids, the density keeps on changing.

Example 17

Water enters a house through a supply pipe of diameter 2cm at a velocity of $0.1ms^{-1}$. The internal house connection pipe has the diameter of 1.0cm.

Calculate

(i) Speed of water as it enters the house

(ii) The rate of mass flow of water it has a density of $1000kgm^{-3}$.

Solution

(i) $A_1V_1 = A_2V_2$

$\pi(0.01)^2 \times 0.1 = \pi(0.005)^2V_2$

$V_2 = 0.4ms^{-1}$

(ii) Volume per second = AV

Mass per second = AVρ

$= \pi(0.01)^2 \times 0.1 \times 1000$

$= 0.0314kgs^{-1}$

Example 18

A compound sprinkler has 8 holes each of cross sectional area of $0.05cm^2$ is connected to a supply pipe of area $2.5cm^2$. If the speed of water in the pipe is $4ms^{-1}$, calculate the speed with which water jets out of the sprinkler into the grass.

Solution

$A_1V_1 = A_2V_2$

$(2.5 \times 10^{-4}) \times 4 = 8\ (0.05 \times 10^{-4})V_2$

$V2 = 25ms^{-1}$

Example 19

Water flows along a horizontal pipe of cross sectional area $48cm^2$ with a pressure of 10^5Pa. The pipe has a constriction of area $12cm^2$ at one point. If the speed of water at the constriction is $4ms^{-1}$. Calculate

(i) speed of water in the pipe in the pipe

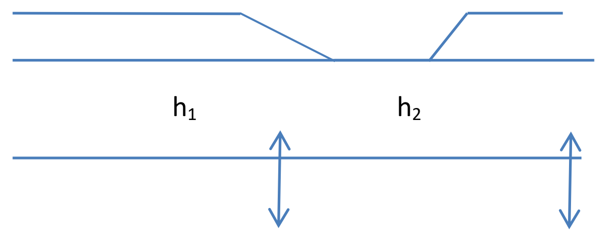
(ii) The pressure at the constriction

Solution

$A_1 = 48cm^2$ $A_2 = 12\ cm^2$

$V_1 = ?$ $V_2 = 4ms^{-1}$

$P_1 = 10^5Pa$ $P_2 = ?$



(i) $A_1V_1 = A_2V_2$

$(48 \times 10^{-4})V_1 = (12 \times 10^{-2}) \times 4$

$V_1 = 1ms^{-1}$

(ii) From $P + \frac{1}{2}\rho v^2 + \rho gh = constant$

But $h_1 = h_2 = h$

$10^5 + \frac{1}{2} \times 1000 \times 1^2 + 1000 \times 9.81h$

$= P + \frac{1}{2} \times 1000 \times 4^2 + 1000 \times 9.81h$

$P = 10^5 - 8000 + 500$

$= 9.25 \times 10^4Pa$

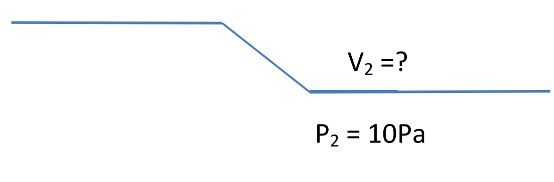
Example 20

Water flows through a horizontal pipe with a velocity of $10ms^{-1}$ and pressure of 10^4Pa. the water flows out through a jet with pressure of 10Pa. Calculate the active velocity

Solution

$V1 = 10ms^{-1}$

$P1 = 10^4Pa$

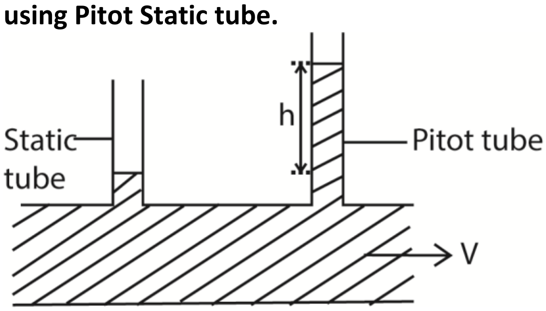


From P + $\frac{1}{2}\rho v^2 + \rho gh = constant$
But $h_1 = h_2 = h$
$10^4 + \frac{1}{2} x\ 1000\ x\ 10^2$ + 1000 x 9.81h
= 10 + $\frac{1}{2} x\ 1000\ x\ v^2$ + 1000 x 9.81h
$V_2 = 10.95ms^{-1}$

Static pressure
This is the pressure exerted by a fluid at rest

Dynamic pressure
This is the pressure exerted by a fluid due to its velocity. Dynamic pressure is given by $\frac{1}{2}\rho v^2$

Experiment to determine static and dynamic pressure, hence velocity of the liquid in a pipe using Pitot Static tube.



It consists of a static tube which measure the static pressure and the pitot tube that measures the total pressure. Total pressure is the sum of static and dynamic pressure.

From P + $\frac{1}{2}\rho v^2 + \rho gh = constant$
Static pressure = P + ρgh
Dynamic pressure = $\frac{1}{2}\rho v^2$
Total pressure, P_y
= static pressure (P_x) + dynamic pressure
= P + $\frac{1}{2}\rho v^2 + \rho gh$
For horizontal tube, h is constant
But, Total pressure, P_y
= static pressure (P_x) + dynamic pressure
$P_y = P_X + \frac{1}{2}\rho v^2$
$(P_y - P_x) = \frac{1}{2}\rho v^2$
V = $\sqrt{\left(\frac{2(P_y - P_{x)}}{\rho}\right)}$

The assumption when using Pitot static tube

- The fluid is non-compressible
- The pitot tube is placed in the center of the fluid because velocity is highest in the middle of lamina flow
- velocity is low and pressure difference is small.

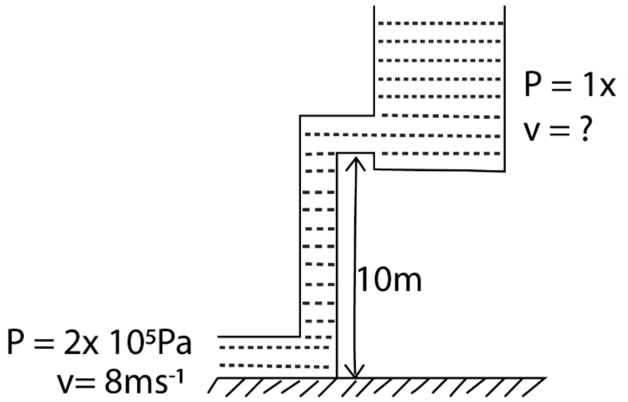
Application of pitot tube
It can be used to measure the airspeed of aircraft, the speed of boats, or the flow of gases and liquids in pipes.

Example 21
A pitot static tube fitted with a pressure gauge is used to measure the speed of the boat at the sea. Given that the speed of the boat does not exceed $10m^{-1}$ and the density of seawater is $1050kgm^{-3}$. Calculate the maximum pressure of the gauge.

Solution
Dynamic pressure = $\frac{1}{2}\rho v^2$
= $\frac{1}{2} x\ 1050\ x\ 10^2$ = 52500Pa

Example 22
Water flowing in a pipe on aground with velocity $8ms^{-1}$ and a gauge pressure of 2 x 10^5Pa is pumped in a water tank 10m above the ground. Calculate the velocity with which water enters the tank at pressure of 1 x 10^5Pa

Solution



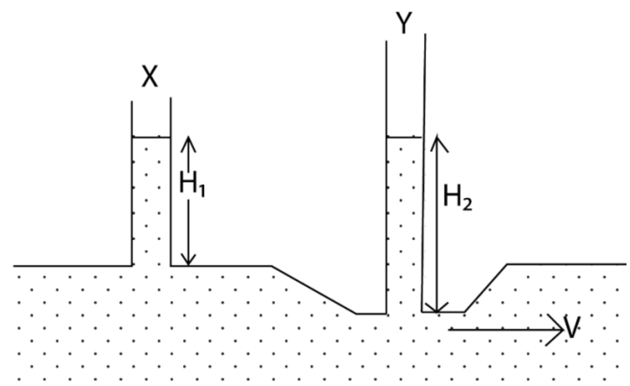
From P + $\frac{1}{2}\rho v^2 + \rho gh = constant$

2 x 10^5 + $\frac{1}{2}$ $x1000\ x\ 8^2$ + 1000 x 9.81 x 0

= 1 x 10^5 + $\frac{1}{2}$ $x1000\ x\ v^2$ + 1000 x 9.81 x 10

v = 8.23ms^{-1}

Venturimeter



This consist of the horizontal tube with a constriction at one point. Vertical manometers are inserted into the tube and at the constriction to measure respective pressures. Then

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

Since $h_1 = h_2$

$P_1 = (H + H_1)\rho g$

$P_2 = (H + H_2)\rho g$

$(H + H_1) + \frac{1}{2}\rho v_1^2 = (H + H_2) + \frac{1}{2}\rho v_2^2$

$A_1V_1 = A_1V_2$

$V_1 = \frac{A_2V_2}{A_1}$

$(H + H_1) + \frac{1}{2}\rho\left(\frac{A_2V_2}{A_1}\right)^2 = (H + H_2) + \frac{1}{2}\rho v_2^2$

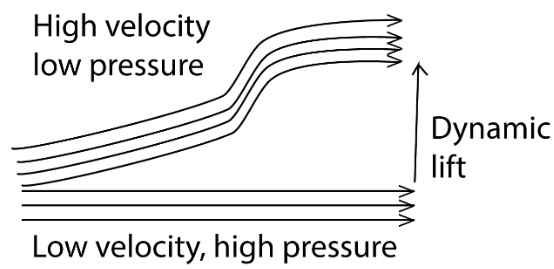
$(H_1 - H_2) = \frac{1}{2}\left(v_2^2 - \left(\frac{A_2V_2}{A_1}\right)^2\right)$

$(H_1 - H_2) = \frac{1}{2}\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)$

$V = \sqrt{\frac{2(H_1-H_2)g}{\left(1-\left(\frac{A_2}{A_1}\right)^2\right)}}$

Application of Bernoulli's principle

(i) Origin of the lift force on wings of an aero plane



The curved nature of the wings of an aero plane ensures that at the takeoff, the air above the wing has higher velocity than that below. From Bernoulli's principle, the pressure above the wing is less than that below. This difference in pressure creates a net upward force on the wings.

Example 23

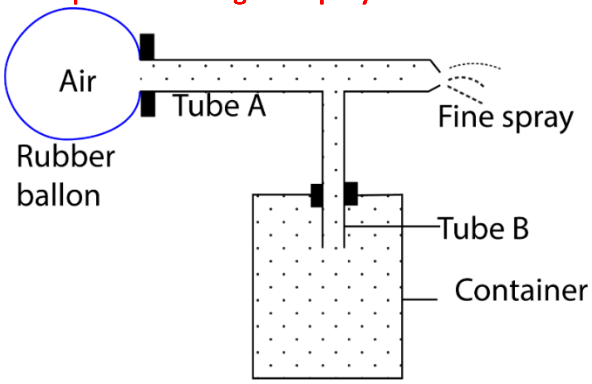
Explain why a spinning ball takes a curved path.

Solution

Air on the upper side is in opposite direction to that of the spin. The resultant velocity of the air is reduced. The air below is in the direction of the spin which increases velocity.

From Bernoulli's principle, the resultant pressure above the spinning ball is higher than that below it. The difference in pressure creates a net downward force on the ball making it to take on a curve path.

Principle of working of a spray



(i) Pressing the rubber balloon forces the air out of the horizontal tube A at high velocity.

(ii) From Bernoulli's principle, this decreases the pressure in the horizontal tube to below atmospheric pressure.

(iii) The liquid rises up in the vertical tube B from the container.

(iv) The liquid collides with the fast speeding molecules of air sucked and breaks into fine spray particles.

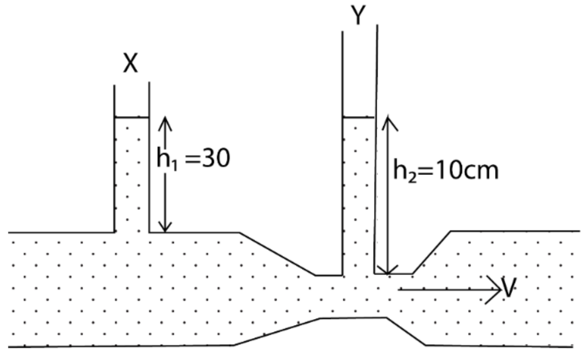
Example 24

(a) (i) Define coefficient of viscosity and determine its dimensions.

(ii) the resultant, F, on a steel ball bearing of radius, r, falling with speed, V, a liquid of viscosity, η, is given by F = kηrv, where K is a constant. Show that K is dimensionless.

(b) Write down Bernoulli's equation for fluid flow, defining all symbols used.

(c) A venturimeter consists of a horizontal tube with a constriction which replaces part of the piping system as shown below.



If the cross sectional area of the main pipe is 5.81 x 10^{-3} m^2 and that of the constriction is 2.58 x 10^{-3} m^2. Find the velocity V_1 of the liquid in the main pipe

(d) Explain the origin of the lift on an aero plane at takeoff.

Solution

From P + $\frac{1}{2}\rho v^2 + \rho gh = constant$

Since h is constant

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

But $P_1 =+ \rho g h_1$, P2 = $+ \rho g h_2$

$\rho g h_1 + \frac{1}{2}\rho v_1^2 = \rho g h_2 + \frac{1}{2}\rho v_2^2$

$v_2^2 - v_1^2 = 2g(h_1 - h_2)$

$A_1V_1 = A_1V_2$

$V_2 = \frac{A_1V_1}{A_2}$

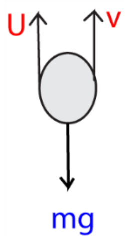
$v_1^2\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right) = 2g(h_1 - h_2)$

$V_1 = \sqrt{\frac{2(h_1 - h)g}{\left(\left(\frac{A_2}{A_1}\right)^2 - 1\right)}} = \sqrt{\frac{2 \ x \ 9.81 \ (0.3 - 0.1)}{\left(\left(\frac{5.81 \ x \ 10^{-3}}{2.58 \ x \ 10^{-3}}\right)^2 - 1\right)}}$ = 0.983ms^{-1}

Example 25

Explain why the acceleration of a ball bearing falling through a liquid decreases continuously until it becomes zero.

Solution



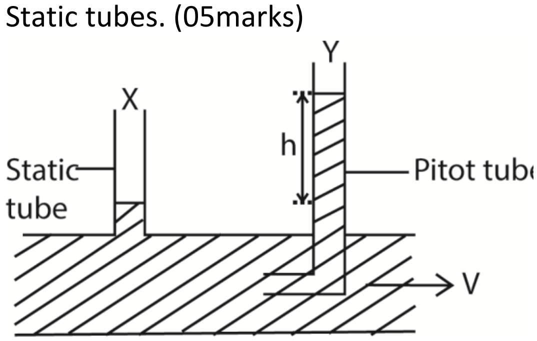
For a ball bearing falling through a liquid, it has three forces acting on it namely the up thrust, U, the weight of the bearing, mg, and viscous force, v, acting as shown above.

Viscous force, v, increases with velocity reducing the accelerating force, F = mg – (U+v) which reduces the acceleration to zero when mg = (U +v).

Revision exercise

1. (a) (i) Define the term static pressure and dynamic pressure as applied to fluid flow. (02marks)
   - **Static pressure** at a point is the pressure which the fluid would have if was at rest.
   - **Dynamic pressure** is the pressure due to fluid motion.

   (ii) Describe how the speed of flowing water can be obtained using Pitot-Static tubes. (05marks)



Pitot-static tube consists of a static tube which measures the static pressure and the pitot tube that measures the total pressure. Total pressure is the sum of static and dynamic pressure.

From $P + \frac{1}{2}\rho v^2 + \rho gh = constant$

Static pressure = P + ρgh (where P is atmospheric pressure)

Dynamic pressure = $\frac{1}{2}\rho v^2$

Total pressure, P_y = static pressure (P_x) + dynamic pressure

$= P + \frac{1}{2}\rho v^2 + \rho gh$

For horizontal tube, h is constant

But, Total pressure, P_y

= static pressure (P_x) + dynamic pressure

$P_y = P_x + \frac{1}{2}\rho v^2$

$(P_y - P_x) = \frac{1}{2}\rho v^2$

$V = \sqrt{\left(\frac{2(P_y - P_x)}{\rho}\right)}$

(iii) State the assumption made in (a)(ii) (01mark)
   - The fluid is non-compressible
   - The pitot tube is placed in the center of the fluid because velocity is highest in the middle of lamina flow
   - velocity is low and pressure difference is small.

(b) (i) State Bernoulli’s principle. (01mark)

Bernoulli’s Principle states that for a streamline motion of an incompressible non viscous fluid, the sum of pressure at any point and kinetic energy per unit volume is always constant.

(ii) Explain why gas in a Bunsen burner does not escape from the base of the burner but continues burning at the top. (03marks)

- **Upward Flow**: The gas is forced to flow upward through the burner tube due to the high pressure from the gas supply. This upward flow ensures that the gas does not escape from the base.
- **Venturi Effect**: The design of the burner creates a Venturi effect, where the gas velocity increases as it passes through a narrow section, drawing in air through the side holes and mixing with the gas. The Fast flow of the gas **creates** low pressure causing in flow of air thus preventing gas escape through the air holes

(iii) A large tank contains water to depth of 1.0m. If a small hole is drilled in the tank at a depth of 20cm below the top surface of the water, calculate the speed at which the water emerges from the hole. (03marks)

Using Torricelli's theorem

It statesthat thespeed v ofinflux of a fluid under gravity through an orifice at a depth h below the surface of the fluid is given by

$v = \sqrt{2gh}$

Where g is the acceleration due to gravity ($9.81ms^{-2}$)

h is the depth of the hole below the surface of the water = 20cm =0.2m

substitution

$v = \sqrt{2x9.81\ x\ 0.2}$

$= 1.98ms^{-1}$

(c)(i) State Archimedes' principle (01 mark)

When a body is partially or fully immersed in a fluid, it experiences an up thrust which is equal to the weight of a fluid displaced.

(ii) An alloy of mass 588g and volume $100cm^3$ is made of iron of relative density 8.0 and alumimium of relative density 2.7. Find the proportion by volume of the alloy. (04marks)

Data given

- Total mass of the alloy (m) = 588g
- Totalvolume of the alloy (V): $100cm^3$
- Relativedensity of iron (ρ_1) =8.0
- Relative density of aluminium (ρ_2) =2.7

Converting relative density to actual density (since relative density is the ratio of the density of of substance to density of water, which is 1g/cm)

- Density of iron (ρ_1) =8.0 g/cm^3
- Density of aluminium (ρ_2) =$2.7g/cm^3$

Let the volume of iron be V_i and the volume of aluminium be V_a

$V = V_i + V_a$

Expressing the mass of iron and aluminium in terms of their densities and volumes

Mass of iron (mi) = $\rho_i.V_i$

Mass ofaluminium (ma) = $\rho_a.V_a$

The total mass of the alloy is the sum of the masses iron and aluminum

m = mi + ma

$588 = 8.0\ xV_i + 2.7\ x\ V_a$………………… (i)

Total volume = volume of iron + volume of aluminium

$V = V_i + V_a$

$100 = V_i$ and V_a ……………………(ii)

Solving equations (i) and (ii)

$Vi = 60cm^3$

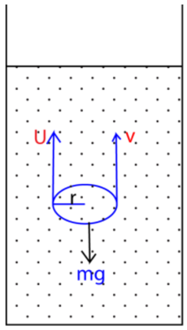
$Va = 40cm^3$

Hence the volume of Iron in the alloy is $60cm^3$ and that of aluminium is $40cm^3$

2. A sphere of radius, **r**, and of material of density**, ρ**, falls vertically through a liquid of density, **σ**, and viscosity, **η**. Derive an expression for the terminal velocity in terms of the quantities given

and acceleration due to gravity, **g**. (04 marks)

Solution



mg = U + v

mg = $\frac{4}{3}\pi r^3 \sigma g$

U = weight of fluid displaced

= $\frac{4}{3}\pi r^3 \rho g$

v = drag force = $6\pi\eta v_0 r$

$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta v_0 r$

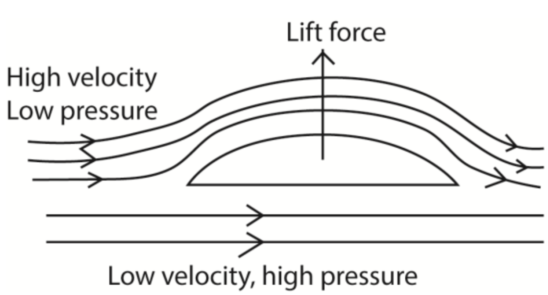
$6\pi\eta v_0 r = \frac{4}{3}\pi r^3 (\rho - \sigma) g$

$v_0 = \frac{2r^2(\rho-\sigma)g}{9\eta}$

3. (a) (i) State **Bernoulli's principle**. (02 marks)

For non- viscous incompressible fluid, flowing steadily, the sum of the pressure, kinetic energy and potential energy per unit volume is constant.

(ii) Explain with the aid of a diagram, why air flows over the wings of an aircraft causes a lift. (02 marks)



- Air flows above the wing of a plane at high velocity hence low pressure.
- Below the wings, air flows at low velocity and hence high pressure.
- The difference in pressure causes a lift force, therefore net upward force.

(b) Air flows over the upper surface of an aircraft's wings at a speed of $135ms^{-1}$ and passed the lower surfaces of the wing at a speed of $120ms^{-1}$.

(i) Calculate the pressure difference due to the flow. (02marks)

From P + $\frac{1}{2}\rho v^2 + \rho g h$

Assuming the difference in height is negligible;

Difference in pressure =

$\frac{1}{2}x1.2(135^2 - 120^2)$ = 2,295Pa

(ii) Determine the lift force on the aircraft if the total wing area is $28m^2$. (*Assume the density of air is $1.2kgm^{-3}$*) (02marks)

F = pressure x area

= 2.295 x 28

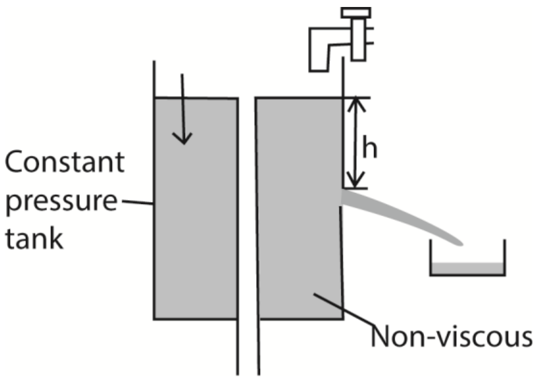
= 64, 260N

(c) (i) What is meant by **streamline flow**? (02marks)

Laminar/streamline flow occurs when the fluid flows in tiny parallel layers with no disruption between them. The successive particles passing a given point have the same velocity.

(ii) With the aid of a labeled diagram, describe how the velocity of a fluid can be measured. (05 marks)

Experiment to determine the velocity of fluid

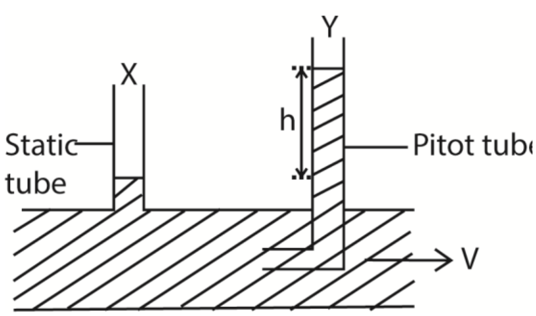


(i) A hole of diameter, r, is known is made through constant pressure apparatus
(ii) Volume of liquid V flowing out of the hole in time t is measured.
(iii) Velocity of the fluid $=\frac{V}{t}(cm^3s^{-1})$

Or

Using pitot static tube

Pitot static tube is placed in the center of moving uncompressible liquid moving in horizontal direction



Pitot-static tube consists of a static tube which measures the static pressure and the pitot tube that measures the total pressure. Total pressure is the sum of static and dynamic pressure.

From $P + \frac{1}{2}\rho v^2 + \rho gh = constant$

Static pressure = P + ρgh (where P is atmospheric pressure)

Dynamic pressure $= \frac{1}{2}\rho v^2$

Total pressure, P_y = static pressure (P_x) + dynamic pressure

$= P + \frac{1}{2}\rho v^2 + \rho gh$

For horizontal tube, h is constant

But, Total pressure, P_y

= static pressure (P_x) + dynamic pressure

$P_y = P_x + \frac{1}{2}\rho v^2$

$(P_y - P_x) = \frac{1}{2}\rho v^2$

$V = \sqrt{\left(\frac{2(P_y - P_x)}{\rho}\right)}$

(d) The depth of water in a tank of a large cross-sectional area is maintained at 2.0m. If the water emerges out of the tank continuously through a hole of diameter 5mm drilled at a height of 10cm above the base of the tank, calculate the;

(i) Speed at which water emerges out from the hole. (03marks)

Height of water above the hole

= 2 – 0.1 = 1.9m

Pressure of water at the hole

= hρg

= 1.9 x 1000 x 9.81

$= \frac{1}{2}\rho v^2$

$= \frac{1}{2}x1000v^2$

$v = 61.1ms^{-1}$

(ii) rate of mass flow of water from the hole. (02marks)

Volume per second =

Area of hole x speed of water

$= \pi\left(\frac{5\,x\,10^{-3}}{2}\right)^2 x61.1$

$= 1.2\ x10^{-3}m^3s^{-1}$

4. (a) Define the following:

(i) Pressure. (01 mark)

Pressure is force over area

(ii) Relative density (01 mark)

Relative density is the ratio of the mass of any volume of a substance to the mass of an equal volume of water

(b) (i) State Archimedes Principle

(01 mark)

When a body is partially or fully immersed in a fluid, it experiences an up thrust which is equal to the weight of a fluid displaced.

(ii) Describe an experiment to determine the relative density of a liquid. (04marks)

By means of a thread, determine the weight of solid in air, liquid, and water = W_1, W_2, and W_3 respectively.

R.D = $\frac{w_1 - W_2}{W_1 - W_2}$

(c) (i) Derive the expression for Bernoulli's equation. (05marks)

Derivation of Bernoulli's expression

- Considering a moving incompressible liquid, if the viscosity is negligibly small, there are no frictional forces to overcome.
- In this case the work done by the pressure difference per unit volume of a fluid flowing along a pipe steadily is equal to the gain of kinetic energy per unit volume plus the gain in potential energy per unit volume.
- Assuming the area is constant at a particular place for a short time of flow; the work done by a pressure in moving a fluid through a distance

= force x distance moved

= (pressure x area) x distance moved

= pressure x volume moved,

- At the. beginning of the pipe where the pressure is P_1, the work done per unit volume on the fluid is thus P_1
- At the other end, the work done per unit volume by the fluid is likewise, P_2
- Hence the net work done on the fluid per unit volume = $P_2 - P_1$
- The kinetic energy per unit volume

  = $\frac{1}{2}$ mass per unit volume x velocity^2

  = $\frac{1}{2}\rho$ x velocity^2,

  where ρ is the density of the fluid.

- Thus if v_2 and v_1 are the final and initial velocities respectively at the end and the beginning of the pipe, the kinetic energy gained per unit volume

  $= \frac{1}{2}\rho(v_2{}^2 - v_1{}^2)$.
- Further, if h_2 and h_1, are the respective heights measured from a fixed· level at the end and· beginning of the pipe, the potential energy gained per unit volume

  = mass per unit volume x g x (h_2 - h_1)

  $= \rho g(h_2 - h_1)$.

- Thus, from the conservation of energy

$$P_1 - P_2$$
$$= \frac{1}{2}\rho(v_2{}^2 - v_{12}{}^2) + \rho g(h_2 - h_1).$$
$$P_1 + \frac{1}{2}\rho v_1{}^2 + \rho g h_1$$
$$= P_2 + \frac{1}{2}\rho v_2{}^2 + \rho g h_2$$
$$\therefore P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

Hence for streamline motion of an incompressible non-viscous fluid

"The sum of the pressure at any part plus the kinetic energy per unit volume pus potential energy per unit volume is always constant."

(ii) Explain why a person standing by the road side may be pulled towards the road when a very fast moving bus passes by. (03marks)

- Air close to fast moving bus has high velocity and hence lower pressure compared to the still air around a stationary person
- The difference in air pressure between the low-pressure area near the bus and the higher-pressure area around the person causes a net force that pulls the person towards the bus.

(d) A water tight drum tied to a cable anchored on the sea-bed floats 500m beneath the sea surface as shown in figure 1.

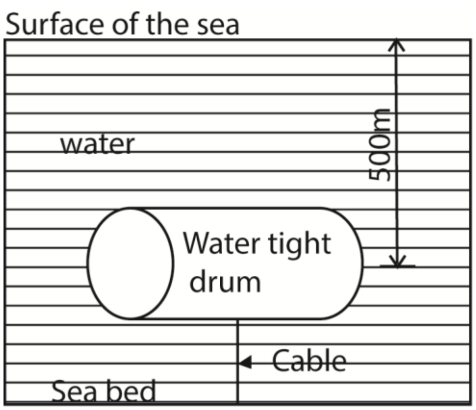


Fig. 1

If the weight of the drum is 500N and its volume $25m^3$, calculate the;

(i) pressure on the drum due to sea water. (02 marks)

Upthrust = weight of displaced water

= mg

= vρg

= 25 x 1000 x 9.81

= 245,250N

(ii) tension in the cable assuming it is vertical. (03 mark)

T = upthrust – (weight of the drum)

= 245,250N – 500N

= 244,**750N**

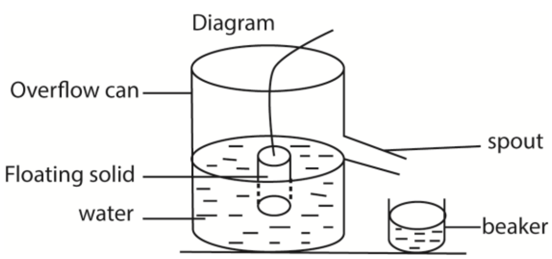
5. (a) State and illustrate Archimedes' principle. (05marks)

Archimedes' Principle: when a body is wholly or partially immersed in a fluid, it experiences an up thrust force equal to the weight of the fluid displaced.

(b)(i) State the law of flotation (01 marks)

A floating body displaces its own weight of fluid in which it floats.

(ii) Describe an experiment to verify the law in (b)(i). (05marks)



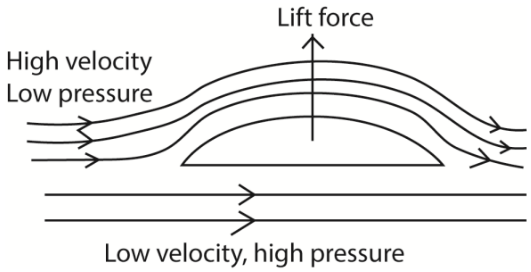
- Overflow can is filled with water up to spout level
- An object is lowered in the can until it floats.
- The displaced water is collected in the beaker.
- The weight of the displaced water is determined and found to be equal to the weight of the object.

(c) (i) Write Bernoulli's equation and define each term in the equation. (02marks)

$P + \frac{1}{2}\rho v^2 + h\rho g$ = constant

Where P= pressure, $\frac{1}{2}\rho v^2$ = kinetic energy, hρg = potential energy per unit volume.

(ii) Explain the origin of lift force on the wings of a plane. (03marks)



- Air flows above the wing of a plane at high velocity hence low pressure.
- Below the wings, air flows at low velocity and hence high pressure.
- The difference in pressure cause a lift force, therefore net upward force.

(i) Air flows over the upper surfaces of the wings of an aeroplane at a speed of $120ms^{-1}$, and past the lower surface of the wings at $110ms^{-1}$.Calclate the lift force on the aeroplane if it has a total wing area of $20m^2$. Density of air = $1.29kgm^{-3}$)

P = $\frac{1}{2}\rho(v_2^2 - v_1^2)$

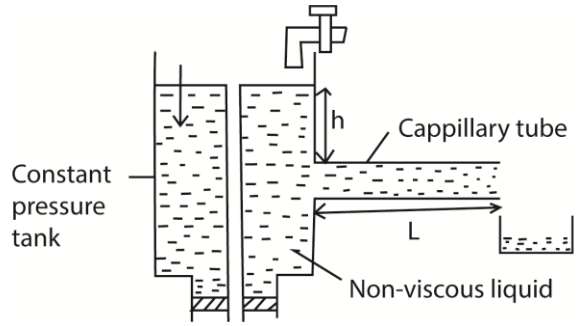
F = PA

= $\frac{1}{2}x1.29(120^2 - 110^2)\ x\ 20$

= 2.97×10^4N

6. (a) (i) State two factors on which the rate of flow of a fluid through a tube depends. (02marks)
   - viscosity of fluid
   - diameter/radius or cross sectional area of the tube
   - pressure difference between its end

   (ii) Describe an experiment to measure the coefficient of viscosity of a liquid using Ponselle's formula



- the liquid of density, ρ, passes slowly from a constant head tank through a capillary tube of length, l and radius r.
- for a height, h, of the tube, the volume V is collect in time t.
- the flow rate R = $\frac{V}{t}$ is calculated
- the experiment is repeated for different value of V and h.
- A graph of R against h is plotted and slope S is obtained

For steady flow, slope $S = \frac{\pi r^4 P}{8\eta l}$

But P = hρg

r = $\frac{d}{2}$

$$S = \frac{\pi\left(\frac{d}{2}\right)^4 P}{8\eta l}$$

$$\eta = \frac{\pi\left(\frac{d}{2}\right)^4 \text{h}\rho\text{g}}{8Sl}$$

(d) Find the time take for an oil drop of diameter 6.0×10^{-3}mm to fall through a distance of 4.0 cm in air of coefficient of viscosity 1.8×10^{-5}Pa.

[The density of oil and air are $8.0 \times 10^3kgm^{-3}$ and $1kgm^{-3}$ respectively]

v = $\frac{2r^2(P-\sigma)g}{9\eta}$

= $\frac{2(3.0\ x\ 10^{-6})^2(800-1)x\ 9.81}{9\ x\ 1.8\ x\ 10^{-5}}$

= $8.7 \times 10^{-4}ms^{-1}$

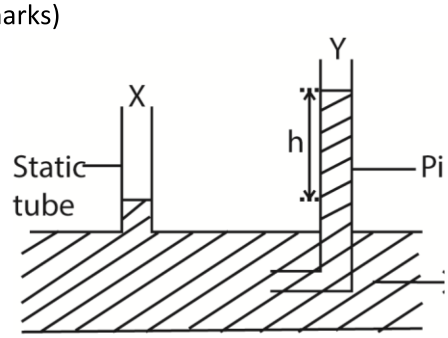
time = $\frac{distance}{speed} = \frac{4\ x\ 10^{-2}}{8.7\ x\ 10^{-4}}$ = 45.98s

7. (a) (i) What is meant by fluid element and flow line as applied to fluid flow? (02mark)
   - A fluid element is a molecule (the smallest volume) of the fluid which follows the flow.
   - A flow line is the path which individual molecule in a fluid element describes.

   (ii) Explain why some fluids flow more easily than others (03marks)
   Fluid flow involves different parts of a fluid moving at different velocities. Different parts of the fluid therefore slide past each other in layers. There exists frictional force between the layers which affects the flow rate. Liquids with low friction or viscosity flow faster than those with high viscosity.

   (b) (i) State Bernoulli's Principle. (01mark)
   Bernoulli's Principle states that for a streamline motion of an incompressible non viscous fluid, the sum of pressure at any point, the kinetic energy per unit volume is always constant.

   (ii) Explain how a Pitot-static tube works (04marks)



Pitot-static tube consists of a static tube which measures the static pressure and the pitot tube that measures the total pressure. Total pressure is the sum of static and dynamic pressure.

From P + $\frac{1}{2}\rho v^2 + \rho gh = constant$

Static pressure = P + ρgh (where P is atmospheric pressure)

Dynamic pressure = $\frac{1}{2}\rho v^2$

Total pressure, P_y

= static pressure (P_x) + dynamic pressure

= P + $\frac{1}{2}\rho v^2 + \rho gh$

For horizontal tube, h is constant

But, Total pressure, P_y

= static pressure (P_x) + dynamic pressure

$P_y = P_X + \frac{1}{2}\rho v^2$

$(P_y - P_x) = \frac{1}{2}\rho v^2$

$V = \sqrt{\left(\frac{2(P_y - P_x)}{\rho}\right)}$

(c) Air flowing over the upper surface of an air craft's wing causes a lift force of 6.4 x 10^5N. The air flows under the wing at a speed of 120ms^{-1} over an area of 28m^2. Find the seed of air flow over an equal area of the upper surface of the air craft's wings. [Assume density of air = 1.2kgm^{-3}) (04marks)

$$P_b + \frac{1}{2}\rho V_b^2 = P_u + \frac{1}{2}\rho V_u^2$$

$$P_bA - P_uA = \frac{1}{2}\rho\left(V_u^2 - V_b^2\right)$$

$$6.4\ x\ 10^4 = \frac{1}{2}\ x\ 1.2(V_u^2 - 120^2)$$

$$V_u = 121.6ms^{-1}$$

(d) (i) What is meant by surface tension and angle of contact of a liquid? (02marks)
   - Surface tension is the force per metre length acting in the surface at right angles to one side of the line drawn in the surface.
   - Angle of contact is the angle between the solid surface and the tangent plane to the liquid surface measured through the liquid.

   (ii) A water drop of radius 0.5cm is broken up into other drops of water each of radius 1mm. assuming isothermal conditions, find the total work done to break up the water drop. (04marks)

Number of drops formed = $\frac{\frac{4}{3}\pi(0.5)^3}{\frac{4}{3}\pi(0.1)^3}$ = 125

Total surface area = 125 x 4πr^2

= 125π x 4 x (0.1 x 10^{-2})^2

= 1.57 x 10^{-3}m^2

Initial surface area = 4πR^2

= 4π(0.5 x 10^{-2})^2

= 3.14 x 10^{-4}m^2

Work done = γ x change in surface area

= 7.0 x 10^{-2} (1.57 x 10^{-3} – 3.14 x 10^{-4})

= 8.8 x 10^{-}5J

8. (i) State Bernoulli's principle (0marks)

Bernoulli's Principle states that for a streamline motion of an incompressible non viscous fluid, the sum of pressure at any point and kinetic energy per unit volume is always constant.

(ii) Explain how wind at a high speed over the roof of a building can cause the roof to be ripped off the building. (03marks)

Wind blowing at a high speed over the roof of a building causes pressure above the roof to decrease below the pressure in the building where the wind is slow. This difference in pressure causes a resultant force that pushes the roof off the building.

(iii) An aeroplane has a mass of 8,000kg and wing area of 8.0m^2. When moving through still air, the ratio of its velocity to that of the air above its wings is 0.25. At what velocity will the aeroplane be able to just lift off the ground? (Density of air = 1.3kgm^{-3})

- Minimum force to lift an aeroplane = 8000 x 9.81 = 78480N
- If v is the velocity of the aeroplane, then velocity of air below the wings v_b = v.
- Velocity of the air above the aeroplane ,
- = $v_a = \frac{v}{0.25} = \frac{v_b}{0.25}$

  or v_a = 4v
- From Bernoulli's Principle,

$P_b + \frac{1}{2}\rho v_b^2 = P_a + \frac{1}{2}\rho v_a^2$

$P_b - P_a = \frac{1}{2}\rho(v_a^2 - v_b^2)$

$= \frac{1}{2}\ x\ 1.3\ ((4v)^2 - v^2)$ = 9.75v^2

- But force = pressure x area

⇨ 78480 = 9.75v^2 x 8

v = 31.72ms^{-1}

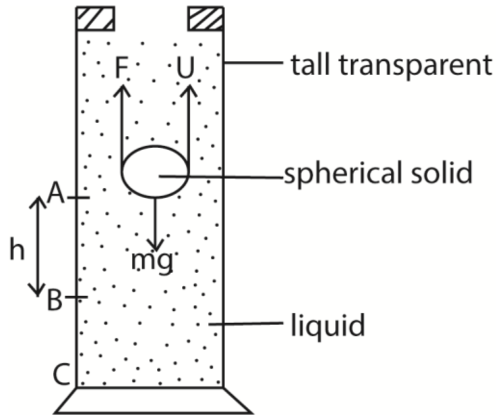
9. (a) Define coefficient of viscosity and state its units. (02marks)

Coefficient of viscosity is the frictional force on unit area of a liquid in a region of unit velocity gradient. Units Pas or Nsm^{-2}

(b) Explain the origin of viscosity in air and account for the effect of temperature on it. (05marks)

In air, molecules are further apart and have negligible intermolecular forces. Therefore the molecules move randomly colliding with one another and continuously transferring momentum to the neighbouring layers. The transfer of momentum constitute viscosity of air. When the temperature increases the molecules move faster, their kinetic energy increases and make more frequent collision. This increases the transfer of momentum and lead to increase in viscosity.

(c) Describe, stating the necessary precautions an experiment to measure the coefficient of viscosity of a liquid using Stroke's law. (07marks)



- A liquid of known density, ρ, is put in a tall transparent glass with reference marks A and B, h metres apart
- A spherical solid of radius a and density, σ, is dropped into the liquid and time t taken to drop from A to B is determined.
- Terminal velocity, $v_0 = \frac{h}{t}$
- The coefficient viscosity, $\eta = \frac{2r^2(\sigma-\rho)g}{9v_0}$

Assumptions

- The spherical solid moves with terminal velocity by the time it reaches A

Precautions

- The glass tube should be very wide compared to the diameter of the ball.
- The point C should be far away from the top of the tube
- Temperature is constant

(d) A steel ball bearing of diameter 8.0mm falls steadily through oil and covers a vertical height of 20.0cm in 0.56s. If the density of steel is7800kgm^{-3} and that of oil is 900kgm^{-3}, calculate the

(i) up thrust on the ball (03marks)

$U = \frac{4}{3}\pi r^3 \rho g$

$= \frac{4}{3}\ \pi\ x\ (4\ x\ 10^{-3})^3 x\ 900\ x\ 9.81$

= 2.37 x 10^{-3}N

(ii) viscosity of the oil (03marks)

From $\eta = \frac{2r^2(\sigma-\rho)g}{9v_0}$

$\eta = \frac{2(4\ x\ 10^{-3})^2\ x\ 9.81(7800-900)}{9\ x\ 0.357}$

= 0.674Nsm^{-2}

10. (a) Define terminal velocity (01mark)

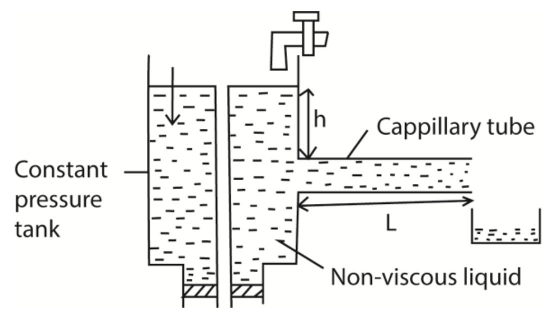
Terminal velocity is the maximum velocity attained by a body falling through a viscous fluid.

(b) Explain laminar flow and turbulent flow (03marks)

Laminar/streamline flow occurs when the fluid flows in tiny parallel layers with no disruption between them. The successive particles passing a given point have the same velocity.

Turbulent flow is the type of flow where equidistant layers of a fluid from the axis of flow have varied velocities. The flow lines are not parallel and the flow is disorderly.

(c) Describe an experiment to measure the coefficient of viscosity of water using Ponselle's formula. (07marks)



- the liquid of density, ρ, passes slowly from a constant head tank through a capillary tube of length, l and radius r.
- for a height, h, of the tube, the volume V is collect in time t.
- the flow rate $R = \frac{V}{t}$ is calculated
- the experiment is repeated for different value of V and h.
- a graph of R against h is plotted and slope S is obtained

For steady flow, $S = \frac{\pi r^4 P}{8\eta l}$

But P = hρg

$r = \frac{d}{2}$

$$S = \frac{\pi\left(\frac{d}{2}\right)^4 P}{8\eta l}$$

$$\eta = \frac{\pi\left(\frac{d}{2}\right)^4 \text{h}\rho\text{g}}{8Sl}$$

(d) (i) State Bernoulli's principle (01 mark)

For non-viscous incompressible fluid flowing steadily, the sun of pressure plus kinetic energy per unit volume Plus potential energy per unit volume is constant.

(ii) Explain why a person standing near a railway line is sucked towards the railway line when a fast moving train passes. (03marks)

Between the man and the train, the velocity of air is increased due to the motion of the train resulting in a decrease in pressure according to Bernoulli's Principle. Behind the man, the flow velocity is lower and pressure is higher. This results in a resultant force towards the train, hence the man is sucked in

(e) A horizontal pipe of cross sectional area $0.4m^2$, tapers to a cross section area of $0.2m^2$. The pressure at the large section of the pipe is $8.0 \times 10^4 Nm^{-2}$ and the velocity of water through the pipe is $11.2ms^{-1}$. If the atmospheric pressure is $1.01 \times 10^5 Nm^{-2}$, find the pressure at the small section of the pipe. (05marks)

$A_1V_1 = A_2V_2$

$0.4 \times 1.2 = 0.2 \times V_2$

$V_2 = 2.4ms^{-1}$

$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$

Since h1 = h2

$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$

$8.0 \times 10^4 + \frac{1}{2} x\ 10^3\ x\ 1.2^2$

$= P_2 + \frac{1}{2} x\ 10^3 x\ 2.4^2$

$P_2 = 7.784 \times 10^4 Pa$

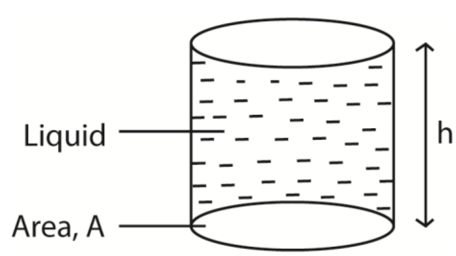
11. (a)(i) What is meant by the following terms, steady flow and viscosity? (02marks)

- Steady flow is the flow where the velocity of the liquid past any given point is constant.
- Viscosity is the friction between layers of a fluid in motion.

(ii) Explain the effect of increase in temperature on viscosity of a liquid. (03marks)

- When temperature rises, the molecular separation of liquid molecules increases and the intermolecular forces decrease. The resistance to flow then decreases hence a decrease in viscosity.

(b) (i) Show that the pressure, P, exerted at a depth, h, below the free surface of a liquid of density, ρ, is given by P =hρg. (03marks)



Weight of a liquid above A = mg

But = Ahρ

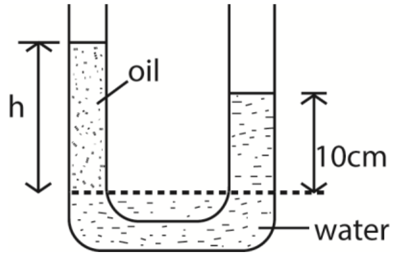
⇨ Weight = Force = Ahρg

Pressure = $\frac{F}{A} = \frac{\text{Ah}\rho\text{g}}{A}$ = hρg

(ii) Define relative density. (01mark)

Relative density is the ratio of the mass of any volume of a substance to the mass of an equal volume of water

(ii) A U-tube whose ends are open to atmosphere, contains water and oil as shown in the figure below



Given the density of oil is $800kgm^{-3}$, find the value of h.

Pressure of oil and that of water at the dotted line are equal, if the density of oil = σ and that of water = ρ, then

$h\sigma g = 0.1\rho g$

$h = \frac{0.1\rho}{\sigma} = \frac{0.1 \, x \, 1000}{800}$

= 0.125m

(c) A metal ball of diameter 10mm is timed as it falls through oil at a steady speed. It takes 0.5s to fall through a vertical distance of 0.3m. Assuming that the density of metal is $7500kgm^{-3}$ and that of oil is $900kgm^{-3}$, find

(i) the weight of the ball (02marks)

Weight

= volume x density of the ball x acceleration due to gravity

$= \frac{4}{3}\pi r^3 \sigma g$

$= \frac{4}{3}\pi(5 \, x \, 10^{-3})^3 x \, 7500 \, x \, 9.81$

$= 3.85 \times 10^{-2}$N

(ii) the up thrust on the ball (02marks)

Up thrust = volume of water displaced x density of water x acceleration due to gravity

$= \frac{4}{3}\pi r^3 \rho g$

$= \frac{4}{3}\pi(5 \, x \, 10^{-3})^3 x \, 1000 \, x \, 9.81$

$= 4.62 \times 10^{-3}$N

(iii) the coefficient of viscosity of the oil (03marks)

W = U + F

$mg = U + 6\pi r\eta v_0$

But $v_0 = \frac{0.30}{0.6} = 0.6ms^{-1}$

$\eta = \frac{mg - U}{6\pi r v_0}$

$= \frac{3.85 \, x \, 10^{-2} - 4.62 \, x \, 10^{-3}}{6\pi \, x \, (5 \, x \, 10^{-3}) \, x \, 0.6}$

= 0.6Pa (or $0.6Nm^{-2}s$)

[Assume the viscous force = $6\pi\eta r v_0$ where η is the coefficient of viscosity, r is the radius of the ball, v_0 is the terminal velocity]

12. (a) (i) State Archimedes' Principle. (01mark)

Archimedes' Principle states that when an object is wholly or partially immersed in a fluid, it experiences an up thrust equal to the weight of the fluid displaced.

(ii) Use Archimedes' Principle to derive an expression for resultant force on a body of weight, W, and density, σ, totally immersed in a fluid of density, ρ. (04marks)

Up thrust (U) = weight of fluid displaced

= mg

= Vρg

Resultant force Fr = W – Vρg

but $V = \frac{W}{\sigma g}$

$$\therefore Fr = W - \frac{W\rho g}{\sigma g}$$

$$= W\left(1 - \frac{\rho}{\sigma}\right)$$

(b) A tube of uniform cross sectional area of $4 \times 10^{-3}m^2$ and mass 0.2kg is separately floated vertically in water of density $1.0 \times 10^3 kgm^{-3}$ and in oil of density $8.0 \times 10^2 kgm^{-3}$. Calculate the difference in the lengths immersed (04marks)

m = PV = ρAL

For water, $L_w = \frac{m}{A\rho}$

$$= \frac{0.2}{1000 \, x \, 4 \, x \, 10^{-3}}$$

$$= 0.05m$$

For oil, $L_o = \frac{m}{A\rho}$

$$= \frac{0.2}{800 \, x \, 4 \, x \, 10^{-3}}$$

$$= 0.0625m$$

Difference in length = 0.0625 – 0.05

= 0.0125m

(c) (i) Define surface tension in terms of work (01mark)

Surface tension is the work done to increase the surface area of a liquid under isothermal conditions.

(ii) Use the molecular theory to account for surface tension of a liquid. (04marks)

- Liquid molecules attract each other.
- The molecules within the body of the liquid (bulk) molecules is attracted equally by neighbors in all direction, hence, the force on the bulk molecules is zero,
- For a surface molecules, there is a net inward force because there are no molecules above the surface to attract them equally.
- To the surface, work must be done against the inward attractive force, hence, a molecule in the surface of a liquid has a greater potential energy than a molecule in the bulk. The potential energy stored inmolecules at the surface is called free surface energy or surface tension.
- Due to the attractive forces experienced by surface molecules due to their neighbours put in a state of tension; the liquid surface behave as a stretched skin.

(iii) Explain the effect of increasing temperature of a liquid on its surface tension. (04marks)

When a liquid is heated, the average kinetic energy of its molecules increase. So the intermolecular of attraction decrease because molecules spend less time in the neighbourhood of a given molecule. At the same time, more molecule enter the liquid surface which lowers the potential energy of the surface thereby lowering the surface tension.

(iv) Calculate the excess pressure inside a soap bubble of diameter 3.0cm if the surface tension of the soap solution is $2.5 \times 10^{-2}Nm^{-1}$. (02marks)

Excess pressure = $\frac{4\gamma}{r} = \frac{4 \, x \, 2.5 \, x \, 10^{-2}}{1.5 \, x \, 10^{-2}}$

= 6.67Pa

13. (a)(i) State Archimedes' Principle. (01mark)

Archimedes' Principle states that when a body is fully or partially immersed in a fluid, it experiences an up thrust equal to the weight of the fluid displaced.

(ii) Describe an experiment to determine the relative density of an irregular solid which floats in water

- Weigh the solid in air = W
- Attach a sinker to irregular solid and weigh them when the solid is outside but the sinker immersed in water = W_1
- Weigh the solid and the sinker when the both completely immersed in water= W_2.
- Upthrust on irregular solid in water = $W_1 – W_2$

  Relative density = $\frac{W}{W_1 - W_2}$

(iii) A block of wood floats at an interface between water and oil with 0.25 of its volume submerged in oil. If the density of the wood is 7.3 x 10^2kgm^{-2}, find the density of oil. (04marks)

From mass = volume x density

Mass of water displaced

= 0.75V x 1000

where V = volume of wood

Mass of oil displaced = 0.25Vρ

where ρ = density of oil

Mass of water displaced + mass of oil displaced = mass of wood

0.75V x 1000 + 0.25Vρ

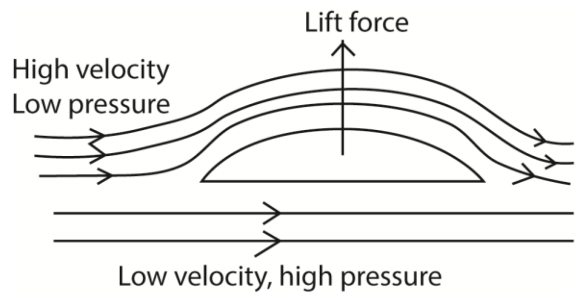
= 7.3 x 10^2V

ρ = -80kgm^{-3}

(b) (i) State Bernoulli's Principle. (04marks)

For non- viscous incompressible fluid, flowing steadily, the sum of the pressure, kinetic energy and potential energy per unit volume is constant.

(ii) Explain the origin of the lift force on the wings of an aeroplane at take-off. (04marks)



- Air flows above the wing of a plane at high velocity hence low pressure.
- Below the wings, air flows at low velocity and hence high pressure.
- The difference in pressure cause a lift force, therefore net upward force.

(c) Water flowing in a pipe on the ground with a velocity of 8ms^{-1} and at gauge pressure of 2.0 x 10^5Pa is pumped into a water tank 10m above the ground. The water enters the tank at a pressure of 1.0 x 10^5Pa. Calculate the velocity with which the water enters the tank. (03marks)

From P + $\frac{1}{2}\rho v^2 + \rho g h = constant$

where P = atmospheric pressure+ gauge pressure

$3\ x\ 10^5 + \frac{1}{2} x\ 10^3\ x\ 8^2 +\ 0$

$=$

$1x\ 10^5 + \frac{1}{2} x\ 10^3\ x\ v^2 +\ 10^3\ x\ 9.81\ x\ 10$

V = 16.4ms^{-1}

(d) Describe how terminal velocity can be measured. (04marks)

- A viscous fluid is filled in a tall jar

- A spherical ball is dropped centrally into the fluid and time t taken by the ball to fall through known distance, d, between known points is determined.
- Terminal velocity, $v_0 = \frac{d}{t}$
- The experiment is repeated to obtain average value.

Thank you
Dr. Bbosa Science