SENIOR FIVE TERM 2

TOPIC 2/7: MECHANICAL PROPERTIES OF MATTER

Competency: The learner explores the effects of force on the strength of different materials in order to guide on the selection of appropriate materials for construction work.

Mechanical property of matter and Hooke's law

These are properties of material under the action of force

(a) **Strength:** this is the ability of a material to resist breaking when a force is applied. E.g. metals

(b) **Stiffness:** this is the ability of a material to resist change in shape or size when a force is applied to it e.g. glass, dry wood.

(c) **Elasticity** is the ability of a material to regain its original shape and size after its deforming force has been removed. E.g. rubber, nullified spring.

(d) **Plasticity:** This is the ability of a material to remain permanently deformed when a deforming force has been removed e.g. plasticine, wet clay

(e) **Ductility:** this is the ability of a material to be changed into various shapes without breaking or is the ability of a material to be deformed without breaking e.g. metals

Ductile materials undergo both elastic and plastic deformation.

(f) **Brittleness:** this is the ability of a material to break easily or suddenly when a force is applied to it e.g. dry clay, glass, chalk. Brittle materials bend very little and break. Therefore they undergo only elastic deformation and their elastic region is very small.

(g) **Hardness/toughness:** this is the ability of a material to resist wearing e.g. rubber and metals

Hooke's law

This states that the extension produced in a wire is directly proportional to the force applied provided the elastic limit is not exceeded.

$F \propto e$

$F = ke$

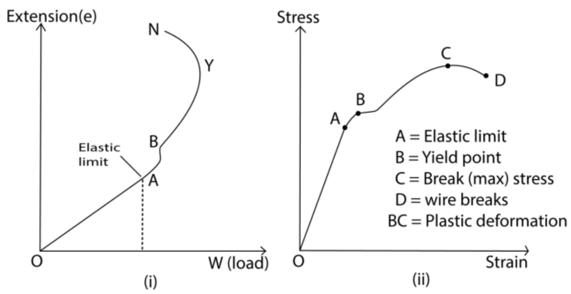
$k = \frac{F}{e}$ where k = force constant

Units of k

$k = \frac{MLT^{-2}}{L} = MT^{-2}$ or ks^{-1} or Nm^{-1}

Force constant, k, is the force required to change length of material by 1m.

Graph of force against extension versus load



Description of section of stress- strain curve for ductile material above

Elastic limit: This is the stress/load/point beyond which a material stops undergoing elastic deformation.

Yield point: Is the stress/load/point beyond which a material stops undergoing plastic deformation

NB. At yield point, there is a sudden increase in extension even though a small force is used.

Region OA

Extension is directly proportional to the force applied (stress is directly proportional to strain).

The material regains its original length and shape when a force/stress is removed. Extension produced is due to the molecules being displaced from their equilibrium position.

Region AB

Material regains its original shape when the force /stress are removed.

Stress is not proportional to strain or extension is not directly proportional to applied force therefore the material does not obey Hooke's law.

Region BC

The material does not fully regain its original shape and length when the force/stress is removed. Therefore Hooke's law is not obeyed.

The extension produced is due to the atoms of molecules being pulled apart breaking the bond between them. When the stretching forces are removed, these bonds are never recovered therefore; the material does not fully regain its original length.

Region CD

The material remains permanently stretched or deformed when the force/stress is removed. The bonds between the molecules are broken completely.

Region DE

The wire breaks in this region with any further increase in force or stress.

At breaking point the wire thins out, become hot. i.e. heat is given out.

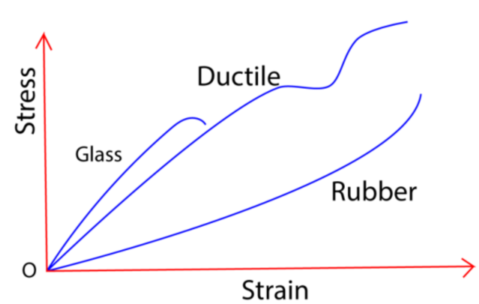
Elastic deformation

During elastic deformation, molecular separation increases. This increases elastic potential energy when the force/stress is removed; the molecules regain their equilibrium position. Initial elastic potential energy is regained and stability is restored.

Plastic deformation

During plastic deformation, molecular separation increases leading to gain in elastic potential energy. Some of this energy is then lost in form of heat. When the force/stress is removed, the lost heat is never regained and therefore stability is never restored.

Stress-strain curve for non-ductile material



Glass: has he smallest elastic region and no plastic deformation regions. Glass is brittle due to small cracks on its surface. Any concentration of tensile stress/force on any of these cracks makes the glass break.

Rubber

Stretches easily without breaking and has a greatest range of elasticity. It does not undergo plastic deformation

Unstretched rubber consist of coiled molecules when a tensile force is applied, they uncoil, become straight and hard. Any further increase in tensile force makes the rubber to break.

Work – hardening

When a metal is repeatedly deformed, it becomes brittle and its resistance to plastic deformation increases. This is called work-hardening of a metal.

Stress

This is the force acting on an area of 1m^2 of a material or it is force per unit area

$$Stress = \frac{Force}{Area}$$

$$[s] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

Units = kgm^{-1}s^{-2}, Nm^{-2} or Pascal (Pa)

Strain

This is the change in length per unit original length or is the change in length per 1m of original length

Strain = $\frac{Extension}{original\ length} = \frac{e}{L}$ (no units)

Young's modulus of elasticity

Young's modulus, E, is the ratio of tensile stress to tensile strain

E = $\frac{Tensile\ stress}{Tensile\ strain}$

[Y] = $\frac{MLT^{-2}}{L^2} = ML^{-1}T^{-1}$

Units = kgm^{-1}s^{-2}, Nm^{-2} or Pascal (Pa)

Or

E = $\frac{Tensile\ stress}{Tensile\ strain} = \frac{F}{A} \div \frac{e}{L} = \frac{FL}{eA}$

Then

F = $\frac{EeA}{L}$

Example 1

A mass of 2kg attached to the end of a wire 2m and diameter 0.64mm causes an extension of 0. 6mm. Find the Young's modulus.

F = 2 x 9.81 = 19.62N, A = $2\pi r^2 = \pi\ x\ (0.32\ x\ 10^{-3})^2 =$ 3.22 x 10^{-7}m^2

$Stress = \frac{19.62}{3.22x\ 10^{-5}}$ = 6.1 x 10^{7}Nm^{-2}

And

$Strain = \frac{e}{L} = \frac{0.6\ x\ 10^{-3}}{2}$ = 3 x 10^{-4}

E = $\frac{Tensile\ stress}{Tensile\ strain} = \frac{6.1\ x\ 10^7}{3\ x\ 10^{-4}}$ = 2.03 x 10^{11} Nm^{-2}

Alternatively

E = $\frac{Tensile\ stress}{Tensile\ strain} = \frac{F}{A} \div \frac{e}{L} = \frac{FL}{eA}$

E = $\frac{2\ x\ 9.81\ x\ 2}{\pi\ 0.32\ x\ 10^{-6}\ x\ 0.3\ x\ 0.3\ x\ 10^{-3}}$ = 2.03 x 10^{11}Nm^{-2}

It should be notes that Young's modulus, E, is calculated from ratio stress/ strain with the elastic limit of the material.

Example 2

Find the maximum load in kg in which may be placed on a steel wire of diameter 0.10cm if the permitted strain must not exceed 0.001 and Young's modulus for steel is 2.0 x 10^{11} Nm$^{-2.}$

E = $\frac{Tensile\ stress}{Tensile\ strain}$

Maximum stress = maximum strain x Young's modulus

= 0.001 x 2 x 10^{11}

= 2 x 10^{8}Nm^{-2}

Area of cross-section in m^2

$= \frac{\pi d^2}{4}$

$= \frac{\pi\, x\, (0.1\, x\, 10^{-2})^2}{4}$

$= 7.85 \times 10^{-7} m^2$

Stress = $\frac{Force}{Area}$

Force = stress x area

$= 2 \times 10^8 \times 7.85 \times 10^{-7}$

= 157N

Mass = $\frac{F}{g} = \frac{157}{9.81} = 16kg$

Force in bar due to contraction or expansion

When a bar is heated, and then prevented from contracting as it cools, a considerable force is exerted at the end of the bar.

For a bar which is L m having Young's modulus, E, a cross-sectional area, A, a linear expansivity of magnitude, α, and decrease in temperature, t^0 C; the decrease length e if were free to contract will be = αLt

$E = \frac{Tensile\ stress}{Tensile\ strain}$

$= \frac{F}{A} \div \frac{e}{L}$

$= \frac{FL}{eA}$

$F = \frac{EA\alpha Lt}{L}$

$= EA\alpha t$

Example 3

A steel rod of cross section area $2.0cm^2$ is heated to 100^0C, and then prevented from contracting when it is cooled to 10^0C. the linear expansivity of steel = $12 \times 10^{-6}K^{-1}$ and Young's modulus, E = $2.0 \times 10^{11}Nm^{-2}$. Find the force exerted.

Solution

A = $2cm^2 = 2 \times 10^{-4}\ m^2$, t = 100 – 10 = 90^0C

$F = EA\alpha t$

$= 2 \times 10^{11} \times 2 \times 10^{-4} \times 12 \times 10^{-6} \times 90$

= 43200N

Energy stored in a stretching wire

When a wire is stretched by an amount, e, by applying a force, F without exceeding elastic limit. The average force = $\frac{(0+F)}{2} = \frac{1}{2}F$

Work done/ work stored in a wire = force x distance = $\frac{1}{2}F.e$

Work done per unit volume of a wire

The volume of the wire = AL

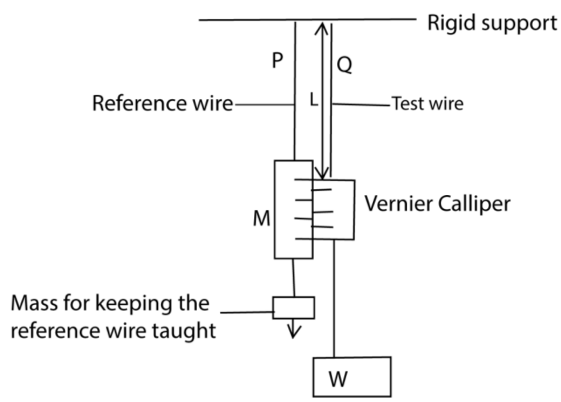
Energy per unit volume = $\frac{1}{2}F.e\ \div AL$

$= \frac{1}{2}\, x\, \frac{F}{A}\, x\, \frac{e}{L}$

Energy stored per unit volume

$= \frac{1}{2}\ x\ stres\ x\ strain$

Experiment to determine Young's Modulus for a metal wire



(i) Two thin, long wires of the same material and length P and Q are suspended from a rigid support.

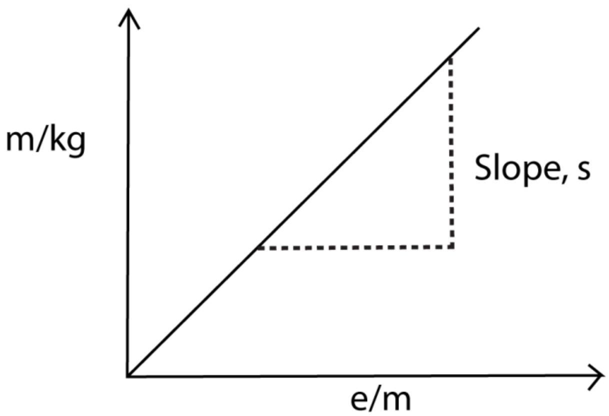
(ii) P carries a scale M in mm and its straightened by attaching a weight at its end.

(iii) Q carries a vernier scale which is alongside scale M

(iv) Various loads are added to the test wire and corresponding extensions caused are read off from a vernier scale.

(v) The diameter (2r) of the wire is obtained by a micrometer screw gauge, and the cross section area of the wire A = $4\pi r^2$

(vi) A graph of mass (m) of the load against extension e is plotted



Young's modulus, Y = $\frac{gsL}{A}$

Precaution in the experiement above

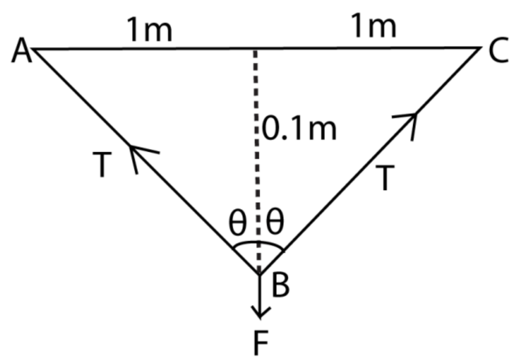
1. After each reading, the load is removed to check that the wire returns to its original length, to ensure that elastic limit is not exceeded.
2. Long wires are used to achieve measurable expansion
3. Thin wires are used to produce high tensile stress
4. Identical wires are used to eliminate error of expansion or contraction due to changes in temperature.

Example 4

A metal wire of diameter 2.0×10^{-4}m and length 2m is fixed horizontally between two points 2m apart. Young's modulus for the wire is $2 \times 10^{11}Nm^{-2}$.

(i) What force should be applied at the midpoint of the wire to depress it by 0.1m

(ii) Find the work done



cos θ = $\frac{0.1}{AB}$ but $AB = \sqrt{(1^2 + 0.1^2)}$ = 1.005m

cos θ = $\frac{0.1}{1.005}$

Length ABC = 2AB = 2 x 1.005 = 2.01

Extension, e = 2.01 – 2 = 0.01m

T = $\frac{YAe}{l}$ and A = $A = \pi r^2 = \frac{\pi d^2}{4}$

Resolving vertically

F = 2Tcosθ

F = $\frac{2YAecos\,\theta}{L} = \frac{2Y\pi d^2 ecos\theta}{4L}$

F = $\frac{2 \times 2 \times 10^{11} x\, \pi\, x\, (2 \times 10^{-4})^2 \times 0.01 \times 0.1}{1 \times 4 \times 1.005} = 12.5N$

(iii) Work done = $\frac{1}{2}Fe = \frac{1}{2} \times 12.5 \times 0.01 = 0.0625J$

Example 5

A uniform bar of length 1.0m and diameter 2.0cm is fixed between two rigid supports at 25^0C. If the temperature of the rod is raised to 75^0C.

Find

(i) The force exerted on the supports.

(ii) The energy stored in the rod at 75^0C.

(Young's modulus for the metal = 2.0×10^{11}Pa, coefficient of linear expansion = $1.0 \times 10^{-5}K^{-1}$)

Solution

(i) F = YAαΔθ

= 2.0×10^{11} x $(\pi \times 0.01^2)$ x 1.0×10^{-5} (75 – 25)

= 31400N

(ii) Energy stored = $\frac{1}{2}Fe$ but e = αLΔθ

$= \frac{1}{2}\, x\ 31400\ x\ 1\ (75 - 25)$
$= 7.85\text{J}$

Revision Exercise

1. (a) (i) What is an elastic material? (01mark)

An **elastic material** is a type of material that can undergo significant deformation when subjected to an external force but will return to its original shape and size once the force is removed.

(ii) Show that the energy,E, stored in a stretched elastic material of elastic constant,k, is given by $E = \frac{1}{2}k({e_2}^2 - {e_1}^2)$, where e_2 and e_1 are extensions produced. (04marks)

- Elastic Potential Energy: The elastic potential energy stored in a material when it is stretched or compressed by an extension e is given by $E = \frac{1}{2}ke^2$ (where k is the elastic constant of the material)
- Energydifference,E: If the material is stretched from an initial $e_1$to a final extension e_2, the energy difference is given by

$E = E_2 - E_1$
$= \frac{1}{2}k{e_2}^2 - \frac{1}{2}k{e_1}^2$
$= \frac{1}{2}k({e_2}^2 - {e_1}^2)$ as required

(iii) Explain the energy transforation which occur during elastic deformation. (03marks)

- When external force is applied to an elastic material, it deforms and the work done is stored as elastic potential energy or on molecular level; the atoms and molecules within the material are displaced from their equilibrium positions. The potential energy is stored in the form of elastic bonds between these molecules when external force is applied.
- For small deformations, the relationship between the force and the deformation follows Hooke's Law, where the force is proportional to the displacement.
- As long as the deformation is within the elastic limit of the material, the energy is stored without causing permanent changes to the material's structure.
- When external force is released, the stored elastic potential energy is released and the material returns to its initial state/shape.

(b)(i) State the easurements taken in an experieent to determine Young's Modulus of a mateiralof a material in form of a wire. (02marks)

- Initil length
- Diameter of the wire
- Final length
- Applied force/load

(ii) Explain why in the experient in (b)(i), two identical wires are used. (02marks)

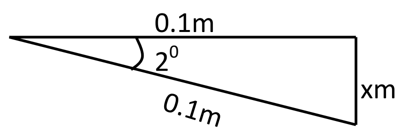
- To eliminate errors due to (i) the yielding of the support when loads are added to the test wire, (ii) changes of temperature.
- (c) Two wires 2.0 m long, one of steel and the other of brass are suspended vertically from two points 0.10m apart in the same horizontal plane. Their lower ends ae fixed to a light horizontal bar at points 0.10m apart. When a force

of 100N is applied vertically downwards to the centre of the bar, the bar tilts by 2^0 to the horizontal due to the brass wire of the brass wire stretching more than the steel.

Assuming tht the wires are vertical and the diameter of each wire is 0.80mm, calculate the;

(i) Difference between the extensions in the wires. (02marks)

Let the difference in extension be x



Using cosine rule

$x^2 = 0.1^2 + 0.1^2 - 2 \times 0.1 \times 0.1\cos 2$

x = 0.0035m

(ii) Extension produced in the brass wire

[Young Modulus for steel = $2.0 \times 10^{11}nm^{-2}$] (06marks)

Force on the steel wire

$$= \frac{100}{2}$$
$$= 50N$$

Crossection areaof steel wire ,

$$A = \pi\left(\frac{d}{2}\right)^2$$

$$= \pi\left(\frac{0.8x10^{-3}}{2}\right)^2$$

$$= 5.0x10^{-7}m^2$$

Extension = $\frac{F.L}{A.E}$

$$= \frac{50\ x\ 2}{5.0x10^{-7}.2\ x\ 10^{11}}$$

= 0.001m

Hence extension of brass = 0.001+0.0035

= 0.0045m

2. (a) Define the following

(i) Brittleness (01 mark)

A brittle material is a hard substance that breaks easily when a force is exerted on it e.g. glass

(ii) Elasticity (01mark)

Elasticity is **ability of a deformed material body to return to its original shape and size when the forces causing the deformation are removed**.

(b) Stat Hooke's law (01mark)

Hooke's law states that the extension of a material is proportional to the stretching force provided the elastic limit is not exceeded.

(c) Figure 1 chows graphs of stress against strain for two metals X and Y.

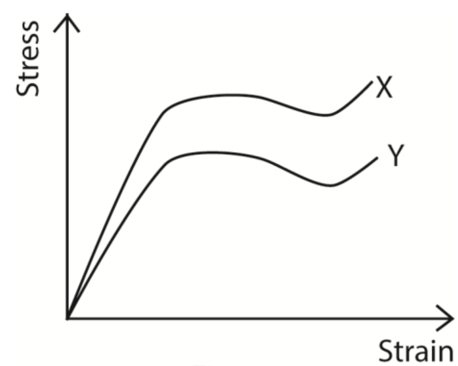


Fig. 1

State and explain which metal;

(i) has a greater Young's modulus, (02marks)

X has high stress per unit of strain

(ii) is more ductile, (02 marks)

Y stretches more per given stress leading to low Young's modulus; often materials with low Young's modulus are ductile like aluminium, although low Young's modulus does not guarantee ductility; for instance, Glass fibers are flexible but brittle.

(iii) is stronger than the other. (02marks)

X resists deformation. Usually stiff materials are strong, although ceramics is not. While rubber is strong but with low Young's modulus

(d) Two wires P and Q of the same material have equal length but the radius of P is twice that of Q. Which wire;

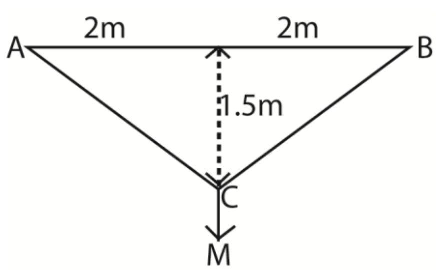
(i) can withstand the greater load before breaking? (02marks)
P because the maximum load a wire can withstand is directly proportional to its cross-sectional area. i.e. F_{max} = kA

(ii) has the greater strain for a given load? (02 marks)
Q because it experienced larger increase in length because it experiences large increase in length per unit load for unit length

(e) A copper wire of length 4m and cross sectional area 1.0 x 10^{-3}mm^2 is fixed between two rigid supports A and B, 4m apart. What mass, when suspended at the middle of the wire will produce sag of 1.5m at that point?

(*Young's modulus of copper = 1.2 x 10^{11}Pa*) (04marks)

Let the mass be M kg



Length of the of the stretched wire

= $2\sqrt{2^2 + (1.5)^2}$

= 5m

Extension = 5 – 4 = 1m

$$E = \frac{Tensile\ stress}{Tensile\ strain} = \frac{F}{A} \div \frac{e}{L} = \frac{FL}{eA}$$

$$1.2\ x\ 10^{11} = \frac{M\ x\ 9.81\ x\ 4}{1\ x\ 1x10^{-3}x10^{-6}}$$

$$M = 3.0581kg$$

(f) Explain why water flowing out of a small hole at the bottom of a wide tank results in a backward force on the tank. (03marks)

Water flowing out of a small hole creates a backward force on the tank due to the **conservation of momentum**, where the water gains forward momentum as it exits, causing the tank to gain equal and opposite backward momentum (recoil), and also because of **Newton's Third Law** (action-reaction), as the water exerts a force forward, the tank pushes back equally and oppositely, similar to a rocket. The small hole forces the water out at high velocity, creating significant momentum in the water that must be countered by the tank.

3. A uniform wire of upstretched length 2.49m is attached to two points A and B which are two meters apart and in the same horizontal line. When 5kg mass is attached to the midpoint, c, of the wire, the equilibrium point of c is 0.75m between the line AB.

(Young's modulus for the wire =2.0 x 10^{11}Pa)

Find

(a) Strain and stress in the wire

[strain = 0.00402, stress = 8.04 x 10^8Pa]
(ii) Energy stored in the wire [Ans. 2.02 x 10^3J]

(b) A thin steel wire initially 1.5m long and diameter 0.5mm is suspended from a rigid support. Calculate
(i) extension (Ans. 3.53 x 10^{-3}m)
(ii) the energy stored in the wire, when a mass of 3kg is attached to the lower end. [5.19 x 10^{-2}J] (Young's modulus = 2.0 x 10^{11}Nm^{-2}]

(c) Two thin wires, one of steel and the other of bronze each 1.5mg and the diameter 0.2cm are joined end to end to form a composite wire of 3m. What tension will produce a total extension of 0.064cm? (Young's modulus for steel = 2.0 x 10^{11}Pa, Young's modulus of bronze = 1.2 x 10^{11}Pa) {Ans. 1009N]

4. (a) Define the following as applied to materials
(i) Stress (01 mark)
Stress is force per unit area.
(ii) Young's Modulus (01 mark)
Young's modulus is the ratio of tensile stress to tensile strain

(b) The velocity of compressional waves travelling along a rod made of material of Young's Modulus, E, and density, ρ, is given by V = $\left(\frac{E}{\rho}\right)^{\frac{1}{2}}$. Show that the formula is dimensionally consistent. (02 marks)

LHS, [V] = LT^{-1} …………………… (i)
Since [E] = $ML^{-1}T^{-2}$ and [ρ] = ML^{-3}

⇨ RHS = $\left[\frac{ML^{-1}T^{-2}}{ML^{-2}}\right]^{\frac{1}{2}}$ = LT^{-1} ……. (ii)

From eqn. (i) and eqn. (ii) the relation is dimensionally consistent.

(c) Derive an expression for the energy stored in a stretched wire within the elastic limit. (03marks)

Energy stored in the rod = $\frac{1}{2}Fe$

∴ Energy stored per unit volume = $\frac{\frac{1}{2}Fe}{AL}$

But F = $\frac{YAe}{L}$

Energy store per unit volume = $\frac{1}{2}\ x\ \frac{YAe.e}{AL^2}$

$= \frac{1}{2}Y\left(\frac{e}{L}\right)^2$

Or

For a small extension, dx

Work done, dw = Fdx

From Hooke's law, F = kx

Since, $dw = kxdx$

=> Total work done, w =$\int dw$

$w = \int_0^e kxdx$

Energy store =$\left|\frac{kx^2}{2}\right|_0^e$, but k = $\frac{YA}{L}$

=> Energy stored = $\frac{1}{2}\ x\ \frac{YAe^2}{L}$

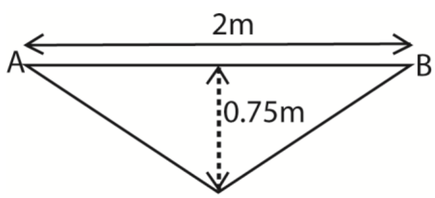
Energy stored per unit volume = $\frac{1}{2}\ x\ \frac{YAe.e}{AL^2}$

$= \frac{1}{2}Y\left(\frac{e}{L}\right)^2$

Where is Y is Young's modulus, e = extension, L=initial length of the rod, A = cross section 'area

(d) A uniform wire of length 2.49m is attached to two fixed points A and B, a horizontal distance 2m apart. When a 5kgmass is attached to mid-point C of the wired, the equilibrium position of C is 0.75m below the line AB. Neglecting the weight of the wire and taking Young's Modulus for the material to be 2 x $10^{11}Nm^{-2}$, find the

(i) strain in the wire (04 marks)



Final length of the wire

$= 2\sqrt{1^2 + (0.75)^2}$

=2.5m

Extension, e = 2.5 – 2.49

= 0.01m

Strain = $\frac{e}{L} = \frac{0.01}{2.49}$= 0.004

(ii) stress in the wire (02 marks)

Young's modulus = $\frac{stress}{strain}$

Stress = Young's modulus x strain

= 2 x 10^{11} x 0.25

= 8 x 10^8 Nm^{-2}

(iii) energy stored in the wire (04 marks)

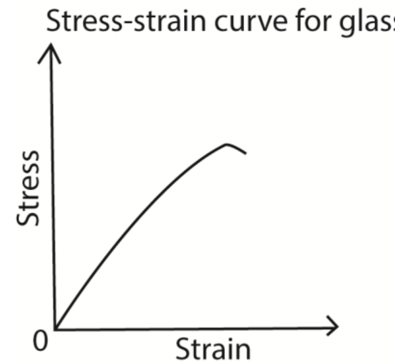
Energy stored in the rod

$= \frac{1}{2}Fe$

$= \frac{1}{2}x\ 5\ x\ 9.81\ x\ 0.01$

= 2.4525J

(e) (i) Sketch the stress-strain curve for glass and explain its shape. (02 marks)



Glass: has the a very small elastic region and no plastic deformation regions.

(ii) Why does glass break easily? (01 marks)

Glass is brittle due to small cracks on its surface. Any concentration of tensile stress/force on any of these cracks makes the glass break.

5. (a) What is meant by a
   (i) Brittle material (01mark)
   A brittle material is a substance that breaks easily when a force is exerted on it e.g. glass
   (ii) Ductile material (01marks)
   A ductile material is one that can be hammered, rolled or moulded into different shapes.

(b) Give one example of each of the materials in (a) (01mark)

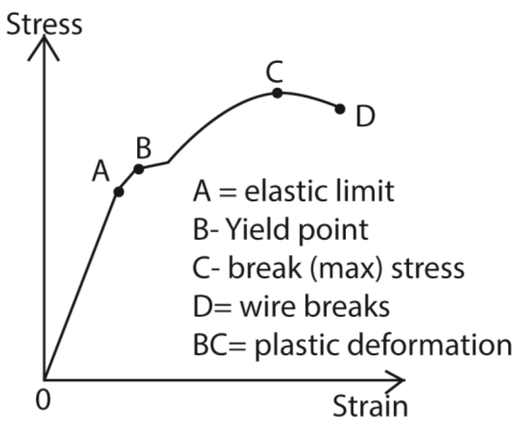
Brittle material: glass, clay, cast iron, stone

Ductile material: copper, aluminium

(c) Explain why bicycle frame are hollow (02mark)

Bicycle frame is hollow to reduce weight while maintain strength. A tube is significantly tougher to bend than a rod of the same material.

(d) (i) Sketch a labeled graph of stress against strain for a ductile material (02marks)



(ii) Explain the main features of the graph in (d)(i) (04marks)

- OA – stress is proportional to strain and the material regains its length
- AB - stress is not proportional to strain but the material regains its length
- Beyond B the material becomes permanently stretched
- CD the material undergoes plastic deformation
- Beyond D the material breaks

(e) Derive the expression for the energy stored per unit volume in a rod of length, L, Young's Modulus, Y, when stretched through distance, e. (04marks)

Energy stored in the rod = $\frac{1}{2}Fe$

∴ Energy stored per unit volume = $\frac{\frac{1}{2}Fe}{AL}$

But F = $\frac{YAe}{L}$

Energy store per unit volume

= $\frac{1}{2}\ x\ \frac{YAe.e}{AL^2}$

= $\frac{1}{2}Y\left(\frac{e}{L}\right)^2$

Or

For a small extension, dx

Work done, dw = Fdx

From Hooke's law, F = kx

$\therefore dw = kxdx$ => Total work done, w =$\int dw$

$w = \int_0^e kxdx$

Energy store =$\left|\frac{kx^2}{2}\right|_0^e$, but k = $\frac{YA}{L}$

=> Energy stored = $\frac{1}{2}\ x\ \frac{YAe^2}{L}$

Energy stored per unit volume

= $\frac{1}{2}\ x\ \frac{YAe.e}{AL^2}$

= $\frac{1}{2}Y\left(\frac{e}{L}\right)^2$

(f) A load of 5kg is placed on top of a vertical brass rod of radius 10mm and length 50cm. find

(i) decrease in length (03marks)

F = mg = 5 x 9.81

A = πr^2 = π(10 x 10^{-3})^2

e = $\frac{FL}{AY}$

= $\frac{5\ x\ 9.81\ x\ 0.5}{\pi(10\ x\ 10^{-3})^2\ x\ 3.5\ x\ 10^{10}}$

= 2.23 x 10^{-6}m

(ii) energy stored in the rod. (02marks)

$E = \frac{1}{2}Fe$

$= \frac{1}{2} \text{ x } 5 \text{ x } 9.81 \text{ x } 2.23 \text{ x } 10^{-6}$

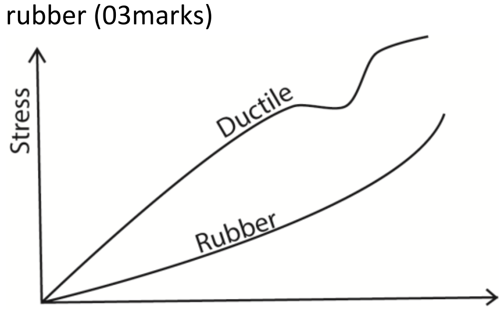
$= 5.46 \text{ x } 10^{-5}\text{J}$

6. (a)(i) Define elastic deformation and plastic deformation (02marks)
   - Elastic deformation is when a material is deformed and it regains its original shape and size when the deforming force is removed.
   - Plastic deformation occurs when a force is applied and the material does not regain its original shape and size when the force is removed

   (ii) Explain what is meant by work hardening. (02marks)

   Work hardening is the strengthening of a material by repeatedly deforming it. Atomic planes slide over each other; this increases plane dislocations which prevent further sliding of planes.

   (b) (i) Sketch using the same axes, stress-strain curves for ductile material and for rubber (03marks)



   (ii) Explain the features of the curve for rubber. (03marks)

   Rubber does not obey Hooke's law except for a very small range; it stretches easily without breaking and has a greatest range of elasticity. It does not undergo plastic deformation

   Unstretched rubber consist of coiled molecules when a tensile force is applied, they uncoil, become straight and hard. Any further increase in tensile force makes the rubber to break.

7. (a)(i) What is meant by Young's modulus? (03marks)

   Young's Modulus is the ratio of tensile stress to tensile strain of a material

   (ii) State Hooke's law (01mark)

   Hooke's law states that the extension of a material is proportional to the stretching force provided the elastic limit is not exceeded.

   (iii) Derive an expression for energy released in a unit volume a stretched wire in terms of stress and strain. (04marks)

   Suppose a force, F, stretches the wire by extension x

   Work done,

   W = average force x extension

   $= \frac{1}{2}Fx$ = stored energy

   Energy stored per unit volume $= \frac{W}{AL}$; where AL is the volume of the wire

   Hence energy store

   $= \frac{1}{2}\ x \left(\frac{F}{A}\right)\left(\frac{x}{L}\right)$

   $= \frac{1}{2}\ x\ stress\ x\ strain$

   (b) A steel wire of length 0.6m and cross-section area $1.5 \text{ x } 10^{-6}\text{m}^2$ is attached at B to a copper wire BC of length 0.39m and cross section area $3.0 \text{ x } 10^{-6}\text{m}^2$. The combination is suspended vertically from a fixed point at A and supports a weight of 250N at C. find the extension in each of the wires, given that Young's Modulus for steel is $2.0 \times 10^{11}Nm^{-2}$ and that of copper is $1.3 \times 10^{11}Nm^{-2}$. (05marks)

From $e = \frac{FL}{AE}$

For steel, $e_1 = \frac{250 \times 0.6}{1.5 \times 10^{-6} \times 2.0 \times 10^{11}}$

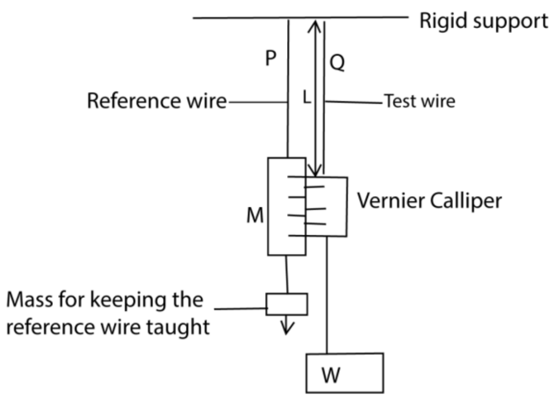
$= 5.0 \times 10^{-4}m$

For copper, $e_2 = \frac{250 \times 0.39}{3.0 \times 10^{-6} \times 1.3 \times 10^{11}}$

$= 2.5.0 \times 10^{-4}m$

(c) With the aid of a labelled diagram, describe an experiment to determine the Young's Modulus of a steel wire (07marks)

Experiment to determine Young's Modulus for a metal wire



(i) Two thin, long wires of the same material and length P and Q are suspended from a rigid support.

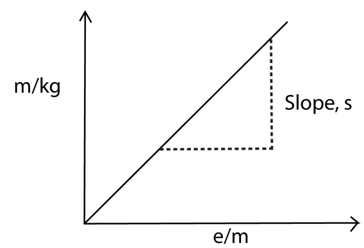
(ii) P carries a scale M in mm and it's straightened by attaching a weight at its end.

(iii) Q carries a Vernier scale which is alongside scale M

(iv) Various loads are added to the test wire and corresponding extensions caused are read off from a vernier scale.

(v) The diameter (2r) of the wire is obtained by a micrometre screw gauge, and the cross section area of the wire $A = 4\pi r^2$

(vi) A graph of mass (m) of the load against extension e is plotted



Young's modulus, $Y = \frac{gsL}{A}$

(d) Explain the term plastic deformation in metals (02marks)

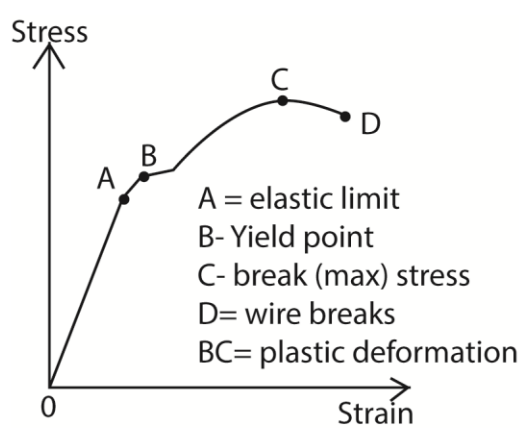
During plastic deformation, some crystal planes slide over each other. The movement of dislocation takes place and on removing the stress, the original shape and size are not recovered due to energy loss in form of heat.

8. (a) State Hooke's law. (01mark)

Hooke's law state that extension is proportional to the load provided the proportionality limit is not exceeded.

(b) A copper wire is stretched until it breaks.

(i) Sketch a stress- strain graph for the wire and explain the main features of the graph. (04marks)



- OA – stress is proportional to strain and the material regains its length
- AB - stress is not proportional to strain but the material regains its length
- Beyond B the material becomes permanently stretched
- CD the material undergoes plastic deformation
- Beyond D the material breaks

(ii) Explain what happens to the energy used to stretch the wire at each stage. (04marks)

- During elastic deformation, atoms are slightly displaced from equilibrium position. When the load is applied to the wire, energy used to stretch wire becomes elastic potential energy
- When the wire is stretched beyond elastic limit, permanent displacement of atoms occurs. The energy is used to break some interatomic bond and some is released as heat.
- At the breaking point, energy is used to break the interatomic bonds.

(iii)Derive the expression for the work done to stretch a spring of force constant, k, by a distance, 2. (03marks)

dw = Fdx

Work done = $\int_0^e Fdx$

But F = kx

$\therefore W = \int_0^e kxdx = \frac{1}{2}ke^2$

(c)(i) Define Young's Modulus. (01mark)

Young's Modulus is the ratio of tensile stress to tensile strain

(ii) Two identical steel bars A and B of radius2.0mm are suspended from the ceiling. A mass of 2.0kg is attached to the free end of bar A. Calculate the temperature to which B should be raised so that the bars are again equal in length.

[Young's Modulus of steel = 1.0×10^{11}Nm^{-2}, Linear expansivity of steal = 1.2×10^{-5}K^{-1}] (05mark)

e $= \frac{Fl}{AE}$, also e = αlΔθ where Δθ is temperature change

=> $\Delta\theta = \frac{F}{AE\alpha}$

$= \frac{2 \times 9.81}{\pi(2.0 \times 10^{-3})^2 \times 1.0 \times 10^{11} \times 1.2 \times 10^{-3}}$

= 1.4K

(d) Why does an iron roof make cracking sound at night? (02marks)

During the day, the roof is heated, it expands and buckles since it is fixed. At night, the roof contracts due to fall in temperature. As it straightens again sound is produced.

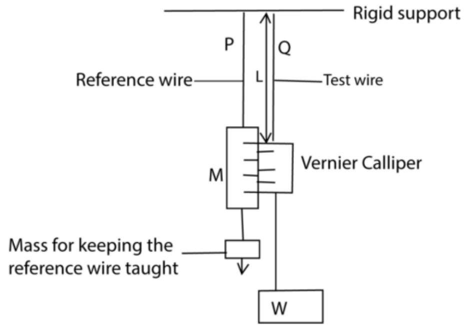
9. (a) (i) Describe the terms tensile stress and tensile strain as applied to a stretched wire. (02marks)

- Tensile stress is the ratio of the stressing force applied on the wire to the cross sectional area of the wire.
- Tensile strain is the ratio of extension in a wire to original length of the wire.

(ii) Distinguish between elastic limit and proportional limit. (02marks)

- Elastic limit s the point beyond which a material does not regain its original length and shape when the load is removed.
- Proportional limit is a point beyond which the extension of a material is not proportional to the applied force or load.

(b) With the aid of a labeled diagram, describe an experiment to investigate the relationship between tensile stress and tensile strains of a steel wire. (07marks)



- Two thin, long wires of the same material and length P and Q are suspended from a rigid support.
- P carries a scale M in mm and it's straightened by attaching a weight at its end.
- Q carries a Vernier scale which is alongside scale M
- Various loads are added to the test wire and corresponding extensions caused are read off from a vernier scale.
- The diameter (2r) of the wire is obtained by a micrometre screw gauge, and the cross section area of the wire A = $4\pi r^2$
- The strain, $\frac{e}{l_0}$ and stress, $\frac{F}{A}$ are obtained
- The procedure is repeated for different values of F and e
- A graph of stress against strain is plotted. From the graph it is found that in the first part stress is proportional to strains up to a certain point beyond which it not proportional.

(c)(i) A load of 60N is applied to a steel wire of length 2.5m and cross sectional area of 0.22mm^2. If Young's Modulus for steel is 210GPa, find the expansion produced. (03marks)

$\Delta t = \frac{FL}{AE} = \frac{60 \, x \, 2.5}{(0.22 \, x \, 10^{-6} x \, 210 \, x \, 10^{9})} =$ 3.25 x 10^{-3}m

(ii) If the temperature rise of 1K causes a fractional increase of 0.001%, find the change in length of a steel wire of length 2.5 when the temperature increases by 4K. (03marks)

1K causes an extension of $\frac{2.5 \, x \, 0.001}{100}$

4K causes an extension of $\frac{2.5 \, x \, 0.001}{100} \, x \, \frac{4}{1} = 1 \, x \, 10^{-4}$m

(d) The velocity, V, of a wave in a material of Young's Modulus, E and density, ρ, is given by

V = $\sqrt{\frac{E}{\rho}}$. Show that the relationship is dimensionally correct. (03marks)

LHS, [V] = LT^{-1} .... (i)

Since [E] = ML^{-1}T^{-2} and [ρ] = ML^{-3}

⇨ RHS = $\left[\frac{ML^{-1}T^{-2}}{ML^{-2}}\right]^{\frac{1}{2}}$ = LT^{-1} .... (ii)

From eqn. (i) and eqn. (ii) the relation is dimensionally consistent.

10. (a) (i) Define stress and strain (02marks)

Stress is force per unit area.

Strain is force per unit length

(ii) Determine the dimensions of Young's modulus. (03marks)

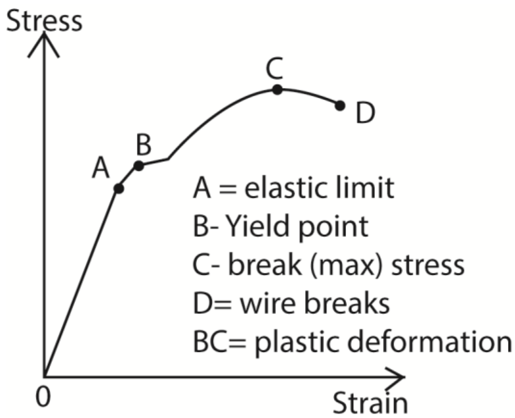
$Y = \frac{stress}{strain}$

$= \frac{F}{A} \div \frac{e}{e}$

$= \frac{MLT^{-2}}{L^2} \, x \, \frac{L}{L}$

= ML^{-1}T^{-2}

(b) Sketch a graph of stress versus strain for a ductile material and explain its features. (06marks)



- OA – stress is proportional to strain and the material regains its length
- AB - stress is not proportional to strain but the material regains its length
- Beyond B the material becomes permanently stretched
- CD the material undergoes plastic deformation
- Beyond D the material breaks

(c) A steel wire of cross section area $1mm^2$ is cooled from a temperature of 60^0C to 15^0C. Find the:

(i) strain (02marks)

Strain = αΔθ

= 1.1x 10-5 x (60 – 15)

= 4.95×10^{-4}

(ii) force needed to prevent it from contracting. (03marks)

[Young's Modulus= 2.0×10^{11}Pa, Coefficient of linear expansion of steel = $1.1 \times 10^{-5}K^{-1}$]

Force = AYstrain

= $10^{-6} \times 2 \times 10^{11} \times 4.95 \times 10^{-4}$

= 99N

(d) Explain the energy changes which occur during plastic deformation (04marks)

During plastic deformation, molecular separation increase leading to a gain in elastic potential energy and heat is evolved. This heat is not recoverable when stress is reduced to zero.

11. (a) Explain the term

(i) Ductility (01mark)

Ductility is the ability of a material to be transformed into different shapes without crumbling

(ii) Stiffness (01mark)

Stiffness is the ability of a material to oppose change in shape

(b) A copper wire and steel wire each of length 1.0m and diameter 1.0mm are joined end to end to form a composite wire 2.0m long. Find the strain in each when the composite stretches by 2.0×10^{-3}m.

[Young's Modulus for copper and steel are 1.2×10^{11}Pa and 2.0×10^{11}Parespectively] (07marks)

$F_1 = k_1e_1$; $F_2 = k_2e_2$

But $F_1 = F_2$

$\therefore k_1e_1 = k_2e_2$

=> $e_1 = \frac{k_2e_2}{k_1}$

$= \frac{Y_2e_2}{Y_1}$

$= \frac{2 \times 10^{11}}{1.2 \times 10^{11}} e_2$

$= 2 \times 10^{-3}$

$e_2 = 0.75 \times 10^{-3}$ m

$strain\ in\ steel\ wire = \frac{e_2}{l_2}$

$= \frac{0.75\ x\ 10^{-3}}{1}$

$= 0.75\ x\ 10^{-3}$

$e_1 = 1.25\ x\ 10^{-3}m$

Strain in copper wire = $\frac{e_1}{l_1}$

$= \frac{1.25\ x\ 10^{-3}}{1}$

$= 1.25\ x\ 10^{-3}$

Thank you
Dr. Bbosa Science