SENIOR FIVE TERM 2

TOPIC 6/7: BEHAVIOUR OF GASES

Competency: The learner investigates modes of heat transfer in nature and their application in industry and society.

Definition of Gases

A gas is a term applied to a substance which is in the gaseous phase above its critical temperature.

A critical temperature is a temperature above which a gas cannot be liquefied no matter how great the pressure is.

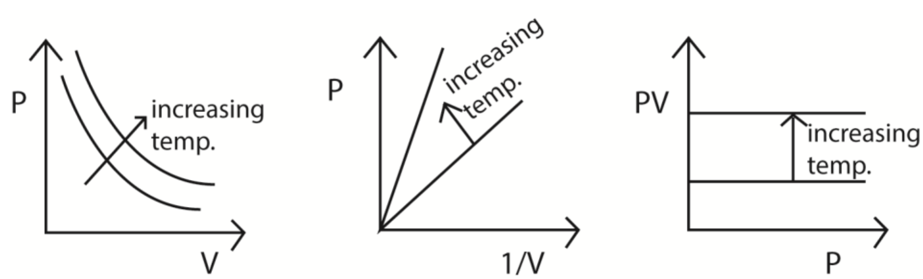
Gas laws

(a) Boyle's law

States that the pressure of a fixed mass of a gas is inversely proportional to volume. If P stand for pressure and V for volume; then

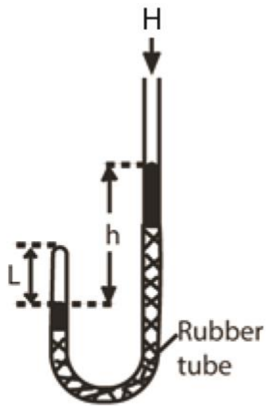
$P \propto \frac{1}{V}$ $or\ P = k\frac{1}{V}$ $or\ PV = k$ where k is a constant that depend on the mass of a gas and temperature.

Graphically



In general, when the pressure and volume of a gas change from P_1 and V_1 to P_2 and V_2 respectively at constant temperature, then $P_1V_1 = P_2V_2$.

An experiment that can be used to verify Boyles' law.

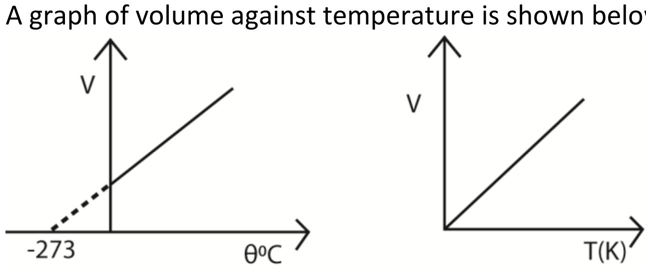


- air in the closed limb of a U-tube barometer as shown above
- Mercury is poured to a height, h, and the length of the air column, L is noted.
- the length h is varied to obtain different sets of values of h and L
- Pressure of the gas is calculated from P = (H + h)ρg where H= height of barometer corresponding to atmospheric pressure, ρ = density of mercury, g = acceleration due to gravity. Note that h can be positive or negative.
- If A is the cross section area, V =AL
- Values of h, L, P, V and 1/V are tabulated
- A plot of P against 1/V gives a straight line through the origin which verifies Boyle's law.

(b) Charles' law

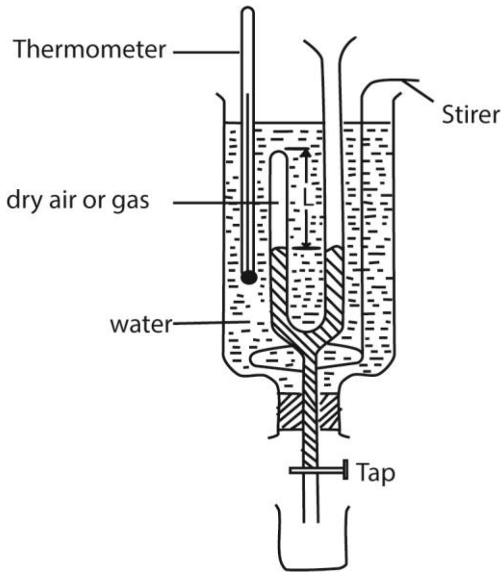
Charles's states that the volume of a fixed mass of a gas is directly proportional to its absolute temperature at constant pressure

A graph of volume against temperature is shown below



In general when the volume and temperature of a gas change from V_1 and T_1 to V_2 and T_2 respectively at constant pressure, then, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

Experiment to verity Charles' law



- Dry air is trapped in the closed limb as shown above
- The level of mercury in the two limbs is maintained at the same level by adding or removing mercury at each temperature of the bath to ensure that pressure of the air is equal to atmospheric temperature.
- The length L of the air column and temperature of the water bath θ are recorded.
- Several values of L and θ are obtained by passing steam
- A graph of L against θ gives a straight line showing that the volume of the gas is proportional to temperature.

(c) Pressure law

It states that the pressure of a fixed mass of a gas is proportional to temperature provided the volume is constant. i.e. , $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

Ideal gas equation or equation of state

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

Or

PV = kT

Example 1

A gas cylinder has volume of $0.040m^3$ and contains air at pressure of 2.0MPa. Assuming that the temperature remain constant, calculate

(a) The equivalent volume of air at atmospheric pressure of 1.0×10^5Pa
From PV = constant
$0.04 \times 2 \times 10^6 = V \times 1.0 \times 10^5$
$V = 0.8m^3$

(b) The volume of air, at atmospheric pressure, which escapes from the cylinder when it is opened to atmosphere.

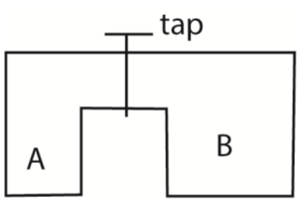
$0.8 - 0.04 = 0.76m^3$

i.e. air escapes from the cylinder until it contains $0.04m^3$ of air at atmospheric pressure

(d) Dalton's law of partial pressures

States that the total pressure of a mixture of gases, which do not react chemically, is equal to the sum of the partial pressures of the gases.

Example 2



Two cylinders A and B of volumes V and 3V respectively are separately filled with a gas. The cylinders are connected as shown above with the tap closed. The pressures of A and B are P and 4P respectively. When the tap is opened the common pressure becomes 60Pa. Assuming isothermal conditions find the value of P. (04marks)

Solution

From PV = nRT

Moles n_1 of the gas in A before mixing= $\frac{PV}{RT}$

Moles n_2 of the gas in B before mixing = $\frac{4P\ x\ 3V}{RT} = \frac{12PV}{RT}$

Moles n_3 of the gas when tap is opened = $\frac{60\ x\ 4V}{RT}$

But moles of the gas before mixing = mole of the gas after mixing

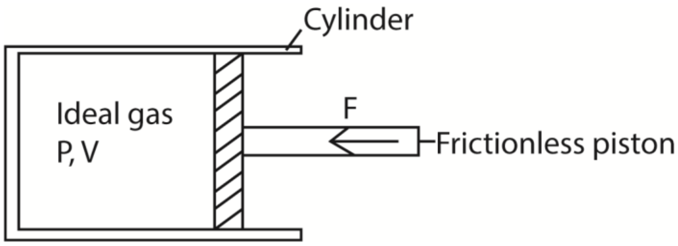
⇨ $n_1 + n_2 = n_3$

$\frac{PV}{RT} + \frac{12PV}{RT} = \frac{60\ x\ 4V}{RT}$

$13P = 240$

P = 18.46Pa

External work done by expanding gas.



Suppose the gas expands by dv so that the piston moves out through a small distance dx.

Work done by the gas, dW = Fdx
= PAdx
= Pdv

Total work done during expansion from v_1 to v_2 is given by

$$W = \int_{v_1}^{v_2} P dv$$

Example 3

When 1.5kg of water is converted to steam at (100^{0}C) at standard pressure (1.01 x 10^{5}Nm^{-2}) 3.39MJ of heat is required. During the transformation from liquid to vapour, the increase in volume of water is 2.5m^3.

(i) Calculate the work done against the external pressure during the process of evaporation.
From $\Delta W = P(V_2 - V_1)$
External work done = 1.01 x 10^5 x 2.5 = 2.53 x 10^{5}J

(ii) Explain what happens to the rest of the energy.
The difference in energy is the increase in internal energy of water molecules.
i.e. 3.39 x 10^6 – 2.53 x 10^5 = 3.24 x 10^{6}J = increase in internal energy of water molecules.

Molar Heat capacities

Molar heat capacity of a substance is the amount of heat required to raise the temperature of one mole of it by 1K. It is expressed in Jmol^{-1}K^{-1}.

Molar heat capacity at constant volume, c_v

Molar heat capacity at constant volume of a substance is the amount of heat required to raise the temperature of one mole of it by 1K at constant volume. It is expressed in Jmol^{-1}K^{-1}.

Molar heat capacity at constant pressure, c_p

Molar heat capacity at constant pressure of a substance is the amount of heat required to raise the temperature of one mole of it by 1K at constant pressure. It is expressed in Jmol^{-1}K^{-1}.

The relationship between the principal molar heat capacities C_p and C_v for an deal gas.

From dQ = dU + dW…………… (i)
But dQ = C_pdT, dU = C_vdT and dW= PdV = RdT
Substituting in (i)

C_pdT = C_vdT + RdT
∴$C_p - C_v = R$
Where R is the universal gas constant per unit mass.

Example 4

The temperature of 1mole of helium gas at a pressure of 1.0 x 10^5Pa increases from 20^0C to 100^0C when the gas is compressed adiabatically.

Find the final pressure of the gas. (Take $^{C_p}/_{C_v}$ =γ =1.67) (04 marks)

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$
$$but\ V = \frac{nRT}{P} => \frac{P_1T_1^{\gamma}}{P_1} = \frac{P_2T_2^{\gamma}}{P_2}$$
$$=> \frac{T_1^{\gamma}}{P_1^{\gamma-1}} = \frac{T_2^{\gamma}}{P_2^{\gamma-1}}$$
$$\frac{(293)^{1.67}}{(1.0\ x\ 10^5)^{0.67}} = \frac{(373)^{1.67}}{(P)^{0.67}}$$
$$P = 1.87\ x\ 10^5 Pa$$

Example 5

Nitrogen gas is trapped in a container by a movable piston. If the temperature of the gas is raised from 0^0C to 50^0C at constant pressure of 4.0 x 10^5Pa and the total heat added is 3.0 x 10^4J, calculate the work done by the gas. [The molar heat capacity of nitrogen at constant pressure is 29.1Jmol^{-1}K^{-1}, C_p/C_v= 1.4]

ΔQ = ΔU + Δw …………………… (i)
Cv = $\frac{C_p}{1.4} = \frac{29.1}{1.4}$ = 20.79Jmol^{-1}
ΔQ = nC_pΔT
n = $\frac{\Delta Q}{C_p \Delta T} = \frac{3\ x\ 10^4}{29.1\ x\ 50} = 20.62$
From equation (i)
3 x 10^4 = 20.62 x 20.79 (50-0) + Δw
Δw = 8.57 x 10^3J

Example 6

Ten moles of a gas, initially at 27^0C are heated at constant pressure of 1.01 x 10^5Pa and volume increased from 0.25m^3 to 0.375m^3. Calculate the increase in internal energy.
[Assume C_p = 28.5Jmol^{-1}K^{-1}] (06marks)
T_1 = 27^0C = 300K
Using $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$\frac{0.250}{300} = \frac{0.375}{T_2}$; T_2 = 450K
$\Delta T = 450 - 300 = 150K$
ΔQ = ΔU + Δw
$nC_p\Delta T = nC_v\Delta T + nR\Delta T$
$nC_v\Delta T = nC_p\Delta T - nR\Delta T$
= 10 x 28.5 x 150 – 10 x 8.31 x 150
= 3.03 x 10^4J

Example 7

An ideal gas at 18^0C is compressed adiabatically until the volume is halved. Calculate the final temperature of the gas. [Assume specific heat capacities of the gas at constant pressure and volume are 2100Jkg^{-1}K^{-1} and 1500Jkg^{-1}K^{-1}] respectively. (04marks)

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$
$\gamma = \frac{2100}{1500} = 1.40$
⇨ $291(V)^{1.4-1} = T_2\left(\frac{V}{2}\right)^{1.4-1}$
T_2 = 384K

Dalton's Law

Dalton's law states that the total pressure of a mixture of gases which do not interact chemically is equal to the sum of partial pressures of the individual gases.

Definition

Partial pressure is the pressure of an individual gas in a mixture or partial pressure of a gas in a mixture is the pressure which it would exert if it were allowed to occupy the volume of the mixture at the same temperature as the mixture

The kinetic theory of matter

- Gases are composed of molecules which are in continuous random motion.
- The molecules collide elastically with one another and also with the walls of the container.
- The pressure of a gas is due to the molecules bombarding the walls of its container. Whenever a molecule bounces off a wall, its momentum at right-angles to the wall is reversed; the force which it exerts on the wall is equal to the rate of change of its momentum. The average force exerted by the gas on the whole of its container is the average rate at which the momentum of its molecules is changed by collision with the walls.

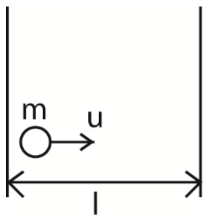
Calculation of pressure

To find the pressure of the gas we must find this force, and then divide it by the area of the walls.

Assumptions

(a) The attraction between the molecules is negligible.
(b) The volume of the molecules is negligible compared with the volume occupied by the gas.
(c) The molecules are like perfectly elastic spheres.
(d) The duration of a collision is negligible compared with the time between collisions.

Consider a molecule of mass, m, moving in a cube of length, l and velocity, u.



Change in momentum = mu - $^{-}$mu = 2mu

Rate of change of momentum = $\frac{2mu}{t}$

Time, t, between collision = $\frac{2l}{u}$

$F_1 = 2mu \div \frac{2l}{u} = \frac{mu^2}{l}$

For N molecules, force on the wall,

F = $\frac{mu_1^2}{l} + \frac{mu_2^2}{l} + \frac{mu_3^2}{l} + \cdots + \frac{mu_N^2}{l}$

Pressure, P = $\frac{F}{A} = \frac{m}{l^3}(u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2)$ since A = l^2

$\overline{u^2} = \frac{u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2}{N}$

$N\overline{u^2} = u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2$

$\therefore P = \frac{Nm\overline{u^2}}{l^3} = \rho\overline{u^2};\ since\ \rho = \frac{Nm}{l^3}$

$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$ and $\overline{u^2} = \overline{v^2} + \overline{w^2}$

$\therefore \overline{c^2} = 3\overline{u^2}$ => $\overline{u^2}$ = $\frac{1}{3}\overline{c^2}$

$\therefore P = \frac{1}{3}\rho\overline{c^2}$

Definition

Mean square speed is the average square speed of the gas molecules at a particular temperature, T

Root-mean-square speed, $\sqrt{\overline{c^2}}$ is the square root of mean of square velocities of gas molecules.

Since $P = \frac{1}{3}\rho\overline{c^2}$

$\sqrt{\overline{c^2}} = \sqrt{\frac{3P}{\rho}}$ ms^{-1}

From PV = nRT and $\rho = \frac{Nm}{V}$

P = $\frac{N}{3V}m\overline{c^2}$

PV = $\frac{N}{3}m\overline{c^2}$

⇨ $\frac{N}{3}m\overline{c^2} = RT$

This shows that **mean square speed s proportional to temperature.**

Also

RT = $\frac{N}{3}m\overline{c^2} = \frac{2N}{3}\left(\frac{1}{2}m\overline{c^2}\right)$

$\frac{1}{2}m\overline{c^2} = \frac{3R}{2N}T$

$= \frac{3}{2}kT$

Where k = $\frac{R}{N}$ is Boltzmann's constant numerically equal to 1.38 x $10^{-2}JK^{-1}$.

Therefore, the average kinetic energy of translation of the random motion of the molecule of a gas is proportion to the kinetic energy.

Example 8

If the mass of 1mole of hydrogen is 2.0g and this occupies a volume of 0.022m^3 at 273K and pressure of 10^5Nm^{-2}. Calculate the r.m.s speed of hydrogen at 546K.

$P = \frac{1}{3}\rho\overline{c^2}$

$\rho = \frac{M}{V} = \frac{2 \, x \, 10^{-3}}{0.022} = 0.0909$$kgm^{-3}$

$\sqrt{\overline{c^2}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \, x \, 10^5}{0.0909}}$ = 1.8167 x 10^3ms^{-1}

Let r.m.s speed at 546K be $\sqrt{\overline{c_1^2}}$

$$\sqrt{\frac{\overline{c^2}}{\overline{c_1^2}}} = \sqrt{\frac{T}{T_1}}$$

$$\sqrt{\overline{c_1^2}} = \sqrt{\frac{T_1}{T}} = \sqrt{\overline{c^2}} = \sqrt{\frac{546}{273}\ x\ 1.8167\ x\ 10^3} = 2569.2\text{ms}^{-1}$$

Example 9

Derive the expression P = $\frac{1}{3}\rho\overline{c^2}$ for the pressure, P, of an ideal gas of density ρ and mean square speed, $\overline{c^2}$. State any assumptions made (07marks)

Example 10

Derivation of Dalton's law from kinetic theory expression, p = $\frac{1}{3}\rho\overline{c^2}$, where ρ is the pressure of a gas of density ρ and mean square speed c^2

P = $\frac{1}{3}N\frac{m}{V}\overline{c^2}$ = $\frac{2}{3}N\left(\frac{1}{2}m\overline{c^2}\right)$

For gas 1, P_1V_1 =$\frac{2}{3}N_1\left(\frac{1}{2}m_1\overline{c_1^2}\right)$

=> $N_1 = \frac{3}{2}P_1V_1.\frac{1}{K_1}$

Similarly for gas 2

$N_2 = \frac{3}{2}P_2V_2.\frac{1}{K_2}$

For a mixture of gases, N = $\frac{3}{2}PV.\frac{1}{K}$; but N = $N_1 + N_2$

$\frac{3}{2}PV.\frac{1}{K} = \frac{3}{2}P_1V_1.\frac{1}{K_1} + \frac{3}{2}P_2V_2.\frac{1}{K_2}$

Since temperature is constant, $K_1 = K_2 = K$

- $PV = P_1V_1 + P_2V_2$
- But $V = V_1 = V_2$
- $\therefore P = P_1 + P_2$

Example 11

Explain why the pressure of a fixed mass of a gas rises if its temperature is increased. (02marks)

When the temperature of a fixed mass of a gas is increased, at constant volume, the velocities and kinetic energy of molecules is increased. They bombard the walls of the container more frequently with increased force. This increases pressure since pressure is proportional to force.

Example 12

Explain the following observations using the kinetic theory.

(i) A gas fills any container in which is it placed and exerts pressure on its walls. (03marks)

A gas contains molecules with negligible intermolecular forces that are free to move in all directions. As they move, they collide with each other and with the walls of the container. The unrestricted movements make them to fill the available space and collisions with the walls contribute to the pressure exerted on the wall.

(ii) The pressure of a fixed mass of a gas rises when temperature is increased at constant volume. (02 marks)

When the temperature of a gas increases, the kinetic energy of the gas molecules increases. This increases the frequency and force of collision against the walls of a container leading to increase in pressure.

Example 13

Explain why the pressure of a fixed mass of a gas in a closed container increases when the temperature of the container is raised. (02marks)

When the temperature of the container increases, the average velocity of the molecules increases. So the number and force of collisions with the walls of the container per second increase. Consequently the momentum change per second increases as they bombard the walls. This leads to increase in the impulsive force exerted on the walls causing increase in pressure

Graham's law

States that the rate of diffusion of a gas is inversely proportional to the square root of its density.

From $\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$ where k is a universal constant

At the same Kelvin temperature T, the mean kinetic energies of the molecules of different gases are equal.

If subscript 1 and 2 denote gases 1 and 2 respectively

$$\frac{1}{2}m_1\overline{c_1^2} = \frac{1}{2}m_2\overline{c_2^2}$$

$$\frac{\overline{c_1^2}}{\overline{c_2^2}} = \frac{m_2}{m_1}$$

At a given temperature and pressure, the density of a gas, ρ, is proportional to the mass of its molecule, m, since equal volumes contain equal number of molecules

Therefore $\frac{m_1}{m_2} = \frac{\rho_2}{\rho_1}$

Then $\frac{\overline{c_1^2}}{\overline{c_2^2}} = \frac{\rho_2}{\rho_1}$

Hence, $\frac{\sqrt{\overline{c_1^2}}}{\sqrt{\overline{c_2^2}}} = \frac{\sqrt{\rho_2}}{\sqrt{\rho_1}}$

The equation

(i) shows that the average molecular speeds are inversely proportional to the square roots of the densities of the gases.

(ii) explains why the rates of diffusion-which depend on the molecular speeds-are also inversely proportional to the square roots of the densities.

Real gases

As opposed to ideal gases, in real gas;

- The volume of the molecules may not be negligible in relation to the volume V occupied by the gas.
- The attractive forces between the molecules may not be negligible.

Real gas equation

To account for molecular volume and intermolecular forces J.H. Vander Waal proposed the following equation

$$(P+\frac{a}{V^2})(V-b) = RT$$

For 1mole of a gas

$\frac{a}{V^2}$ corrects deficit in pressure due to intermolecular attractions of gas molecules and

b, called the co-volume accounts for the finite volume of molecules themselves

NB: (i) A real gas obeys ideal gas equation above the critical temperature

Differences between ideal and real gases

(i) Ideal gases obey Boyle's law while real gases do not.

(ii) The volumes of the molecules of ideal gases are negligible compared to the container while for the real gases the volume is not negligible.

(iii) Ideal gases have negligible intermolecular forces while the intermolecular forces of real gases are not negligible

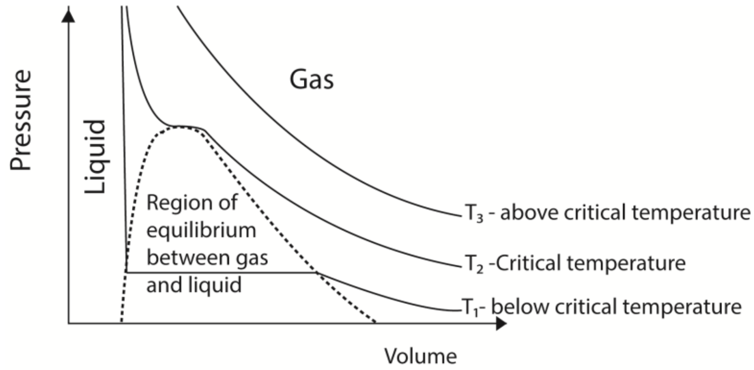
(iv) The velocity of ideal gas molecules is constant in between collision while real gases molecules do not have constant velocity due to intermolecular forces.

Vapours

A gas is the gaseous state of a substance above its critical temperature, T_C.

A vapour is the gaseous state of a substance below its critical temperature.

A graph of a real gas below and above the critical temperature



- Above the critical temperature a gas obeys Boyle's law.
- Below the critical temperature a gas exist as unsaturated vapour at low pressure when the pressure is increase it condenses until all the gas is turned into a liquid.

A critical temperature is a temperature above which a gas cannot be turned into a liquid by compression.

Saturated and unsaturated vapour

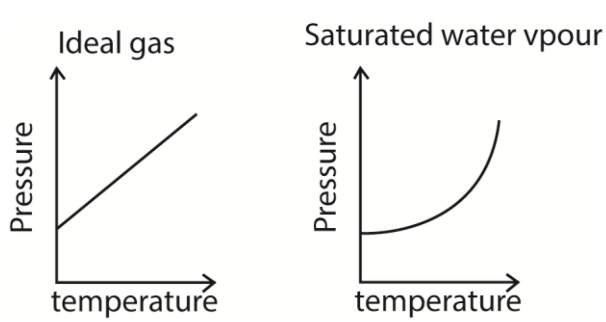
Unsaturated vapour is a vapour that is not in dynamic equilibrium with its own liquid while saturated vapour is a vapour that is in dynamic equilibrium with its own liquid

Differences between saturated and unsaturated vapors

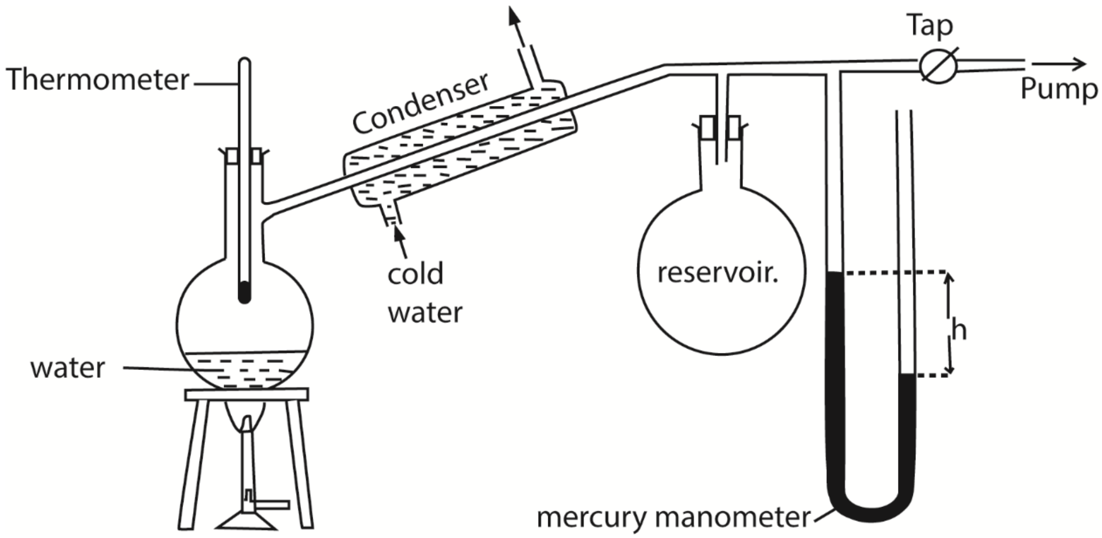
- A saturated vapour pressure is one which is in dynamic equilibrium with its own liquid and an unsaturated vapour is one that is not in dynamic equilibrium with its own liquid.
- A saturate vapour pressure does not obey gas laws whereas unsaturated vapour approximately obeys gas laws.

Example 14

Sketch graphs of pressure against temperature for an ideal gas and for saturated water vapour originally at 0^0C (03marks)



An experiment to determine saturated vapour pressure at a given temperature



- The pressure of the air in Reservoir is shown by the mercury manometer; if its height is h, the pressure in mm mercury is P = H-h, where H is the barometer height.
- The tap is opened and the pressure above water varied using the pump to a suitable value.
- The tap is closed and water in the flask in heated until it boil.
- The temperature θ and difference in mercury levels, h, are noted and recorded.
- The saturated vapour pressure, P = $(H \pm h)$ is calculated
- The procedure is repeated other values of θ and h
- A graph of P versus θ is plotted and the saturated vapour pressure at a particular temperature is obtained.

Example 15

With the aid of a labelled diagram, describe an experiment to determine standard saturated vapour pressure of water. (05marks)

Example 16

Use the kinetic theory to explain the following observations

(i) Saturated vapour pressure of a liquid increases with temperature. (03marks)
If a liquid is in dynamic equilibrium with its vapour, an increase in temperature increases mean kinetic energy of the molecules and hence the rate at which molecules escape from the liquid.
The density of the vapour increases implying increase in the rate of condensation until dynamic equilibrium is restored. There are now more molecules in the vapour phase than previously that are moving faster and hence higher pressure.

(ii) Saturated vapour pressure is not affected by decrease in volume at constant pressure. (03marks)

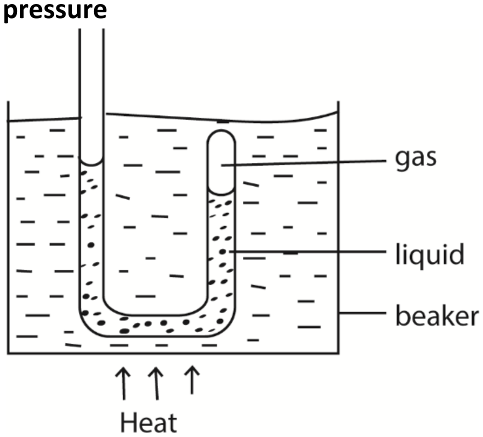
A decrease in volume leads to a momentary increase in vapour density. Consequently, the rate of condensation increases while the rate of evaporation rate is constant. When the vapour density reduces, the condensation rate also reduces. So the dynamic equilibrium is restored to the initial value.

Example 17

Two similar cylinders P and Q contain different gases at the same pressure. When gas is released from P the pressure remains constant for some time before it starts dropping. When gas is released from Q the pressure continuously drops. Explain the observation above. (05marks)

- The gas in P is in form of a saturated vapour; that is, in dynamic equilibrium with a liquid. As the gas is released, more liquid turns into a gas to restore pressure until the gas becomes unsaturated and the pressure begins to drop as the moles of the gas decrease
- The gas in Q is unsaturated, and thus pressure reduces as the moles of the gas reduce up on release.

Experiment to show that a liquid boils off when its saturated vapour pressure equals the external pressure



- Air is trapped in the closed limb of the tube by water column.
- The tube is heated in water bath.
- When the water bath begins to boil, the water in the tube comes to the same level in each limb.
- This shows that the vapor pressure in closed limb is equal to external pressure.

Evaporation

During evaporation, a liquid changes to vapour low the boiling point.

Reasons why evaporation causes cooling

During evaporation liquid molecules with high kinetic energy escape from inter molecular attraction in the liquid leaving molecules of low kinetic energy behind. Since temperature s proportional average kinetic energy of molecules, the liquid cools.

Factors affecting the rate of evaporation

(i) **Surface area.** Evaporation increases with surface area of a liquid due to increase in the number of exposed molecules.

(ii) **Temperature:** increase in temperature increases the rate of evaporation because it increases the average kinetic energy of the molecules.

(iii) **Drought /wind blowing over the surface.** Wind removes the saturated air layer from the surface of the liquid thereby increasing the rate of evaporation.

Example 36

Explain the occurrence of land and sea breeze. (04marks)

During day, the land is heated to a high temperature than the sea. Hot air expands and rises from land. A stream of cool air from the sea blows towards the land to replace the uprising air, hence sea breeze occurs.

At night the land cools faster because it is no longer heated by the sun. The sea retains the warmth because it is heated deeply. Warm less dense air rises from the sea surface and air from land blows to replace it leading to land breeze.

Example 18

A horizontal tube of uniform bore, closed at one end, has some air trapped by a small quantity of water. The length of the enclosed air column is 20cm at 12^0C.

Find stating any assumptions made, the length of air column when the temperature is raised to 38^0C.

[S.V.P of water at 12^0C and 38^0C are 10.5mmHg and 49.5mmHg respectively.

Atmospheric pressure = 75cmHG] (05marks)

T_1 = 273 + 12 = 285K,
T_2 = 273 + 38 = 311K;
P_1 = 750 – 10.5 = 739.5mmHg,
P_2 = 750-49.5 = 700.5mmHg

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{739.5\ x\ 20A}{285} = \frac{700.5\ x\ hA}{311}$$

h = 23.04cm

Assumption: the tube does not expand when the temperature increases.

Example 19

When hydrogen gas is collected over water, the pressure in the tube at 15^0C and 75^0C are 65.5cm and 105.6cm of mercury respectively. If the saturated vapour pressure at 15^0C is 1.42cm of mercury, find its value at 75^0C (04marks)

P_1= 65.5 – 1.42 = 62.08 and T_1 = 273 + 15 = 288K
P_2 = 105.6 – P, and T_2 = 273 + 75 = 348K
From $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

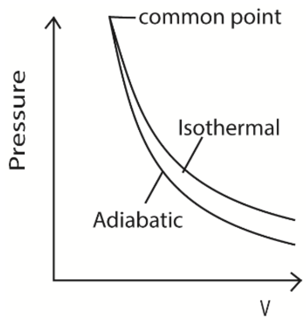
$P_2 = \left(\frac{P_1}{T_1}\right) T_2$ =105.6 - $\frac{64.08}{288}\ x\ 348$ = 28.12cmHg

Isothermal and adiabatic changes

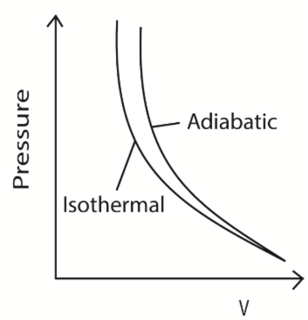
Isothermal expansion takes place at constant temperature.

Adiabatic expansion takes place at constant heat.

Sketch graphs of pressure versus volume for fixed mass of a gas undergoing isothermal and adiabatic changes.



Or



Condition necessary for realization of an isothermal change

(i) The gas must be held in thin-walled and highly conducting vessel

(ii) The process must take place slowly so that heat passes into the gas to maintain constant temperature.

(iii) The gas vessel must be surrounded by a constant temperature bath.

Conditions for adiabatic change

(i) The gas must be held in a thick walled and poorly conducting vessel

(ii) The process must be carried out rapidly to minimize heat linkage through the walls.

Relationship between volume, pressure and temperature

(i) For reversible isothermal change
PV = nRT where n is the number of mole of gas, R = gas constant

(ii) For reversible adiabatic change
Pv^{γ} = constant
$TV^{\gamma-1}$ = constant

Example 20

Ten moles of a gas, initially at 27^0C are heated at constant pressure of 1.01 x 10^5Pa and volume increased from $0.25m^3$ to $0.375m^3$. Calculate the increase in internal energy.

[Assume $C_p = 28.5Jmol^{-1}K^{-1}$] (06marks)

$T_1 = 27^0C = 300K$

Using $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$\frac{0.250}{300} = \frac{0.375}{T_2}$; $T_2 = 450K$

$\Delta T = 450 - 300 = 150K$

ΔQ = ΔU + Δw

$nC_p\Delta T = nC_v\Delta T + nR\Delta T$

$nC_v\Delta T = nC_p\Delta T - nR\Delta T$

= 10 x 28.5 x 150 – 10 x 8.31 x 150

= 3.03×10^4J

Example 21

An ideal gas at 18^0C is compressed adiabatically until the volume is halved. Calculate the final temperature of the gas. [Assume specific heat capacities of the gas at constant pressure and volume are $2100Jkg^{-1}K^{-1}$ and $1500Jkg^{-1}K^{-1}$] respectively. (04marks)

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\gamma = \frac{2100}{1500} = 1.40$

⇨ $291(V)^{1.4-1} = T_2\left(\frac{V}{2}\right)^{1.4-1}$

$T_2 = 384K$

Example 22

State the first law of thermodynamics and use it to distinguish between Isothermal and adiabatic changes in a gas. (05marks)

$\Delta Q = \Delta U + \Delta W = nC_v\Delta T + \Delta W$

During isothermal expansion, ΔT = 0. Therefore all the energy supplied is equal to the work done by the gas during expansion.

In adiabatic expansion, no heat enters or leaves the gas. Therefore ΔQ = 0 and ΔU = -ΔW.

In adiabatic expansion, work s done at the expense of its internal energy. Therefore the gas cools.

Example 23

(a) (i) What is meant by isothermal process and adiabatic process? (02marks)

Isothermal process is the expansion or compression of a gas at constant temperature.

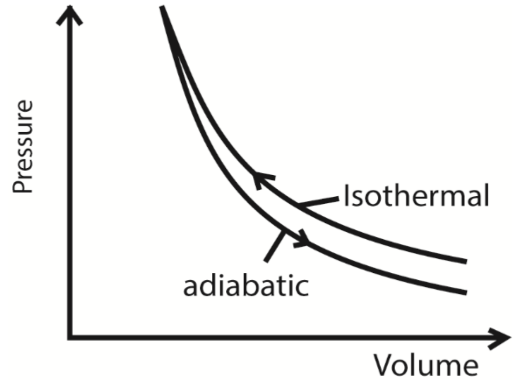
Adiabatic process is the expansion or compression of a gas where there is no heat loss or gain into the gas.

(ii) Explain why adiabatic expansion of a gas causes cooling (03marks)

During an adiabatic expansion of a gas, no heat is supplied to the gas. Molecules strike the receding piston and bounce off with reduced velocities and hence lower kinetic energy. Since kinetic energy is proportional to temperature, the gas cools during the expansion

(b) A gas at a temperature of 17^0C and pressure 1.0 x 10^5Pa compressed isothermally to half its original volume. It is then allowed to expand adiabatically to its original volume

(i) Sketch a P-V curve the above process (02marks)



(ii) If the specific heat capacity at constant pressure is 2100Jmol^{-1}K^{-1} and at constant volume is 1500Jmol^{-1}K^{-1}, find the final temperature of the gas (04marks)

$\gamma = \frac{2100}{1500} = 1.4$

$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$

$290\left(\frac{V}{2}\right)^{0.4} = T_3V^{0.4}$

$T_3 = 219.8K$

Revision exercise (Qns. as set by Global examinations' bodies)

1. (a) Define the following;

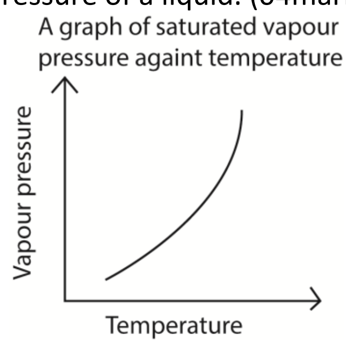
(i) Saturted vapour. (01mark)

A saturated vapour is one that is a dynamic equilibrium with its own liquid

(ii) Partial pressure of a gas. (01 marks)

Partial pressure is the pressure that would be exerted by a gas if it alone occupied the volume of the mixture.

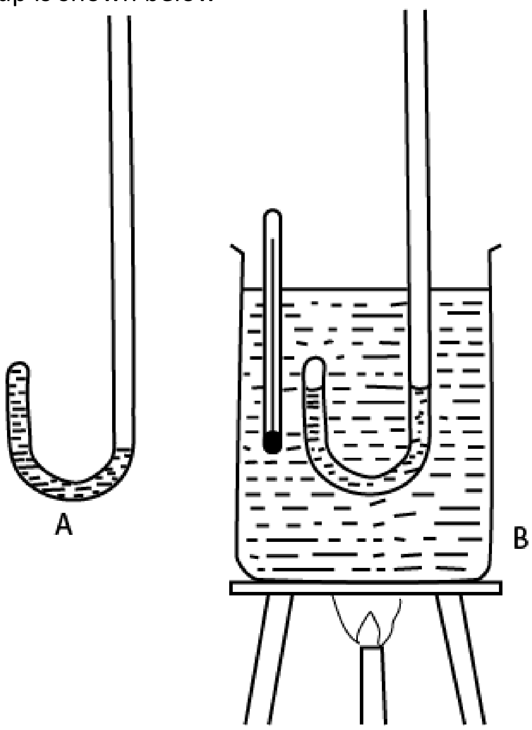
(b) (i) Explain the effect of increase in temperature on the saturated vapour pressure of a liquid. (04marks)



The saturated vapor pressure of any substance depends on its temperature. At higher temperatures, **more molecules have sufficient kinetic energy to break from the liquid surface into the vapor phase**. Under these conditions, equilibrium is reached at a higher pressure.

(ii) Describe an experiment to determine the saturated vapour pressure and boiling point of water

The setup is shown below



- Atmospheric pressure, H, is deterined using a mercury barometer
- Water is trapped in a J tube as shown in A
- The J-tube in and its content is transferred into a beaker of water B and a thermometer is inserted as shown in B.
- Water in the beaker is heated until it boils and the boiling point is obtained from the constant reading of the thermometer.
- It also noted that water in the closed and open tube of the J-tube are at the same level indicating that water boils when its saturated vapor pressure is equal to the atmospheric pressure H.

(c) (i) Define an idea gas (01ark)

An Ideal gas s one that obeys Boyle's law under all conditions

(ii) What assumptions of the kinetic theory of an ideal gas need to be modified to account for the behaviour of a real gas.

- **Negligible is Volume of Gas Molecules**: The kinetic theory assumes that the volume of individual gas molecules is negligible compared to the volume of the container. In reality, gas molecules do occupy space, and their finite size becomes significant at high pressures, where the volume of the container is reduced.

- **No Intermolecular Forces**: The theory assumes that there are no attractive or repulsive forces between gas molecules. However, real gases experience intermolecular forces (Van der Waals forces), which affect their behavior. Attractive forces become significant at low temperatures and high pressure, leading to deviations from ideal gas behavior.
- **Perfectly Elastic Collisions**: The theory assumes that collisions between gas molecules and with the walls of the container are perfectly elastic, meaning there is no loss of kinetic energy. In reality, some energy is lost in collisions, although it is often small enough to be negligible in many situations.
- **Random Motion**: While gas molecules do move randomly, the theory assumes complete randomness without considering the influence of intermolecular forces. In real gases, these forces can lead to non-random behavior, particularly under certain conditions.

To account for these deviations, the Van der Waals equation is often used as an improvement over the ideal gas law:

$$\left(P + \frac{a}{V_m^2}\right)(V_m - b) = RT$$

Where

- P is the pressure of the gas.
- V_m is the molar volume of the gas.
- T is the temperature of the gas.
- R is the universal gas constant.
- a and b are empirical constants specific to each gas, accounting for intermolecular forces and the finite volume of gas molecules, respectively.

(d) A sealed flask of volue $80cm^3$ contains argon at a pressure of 10kPa and a temperature of 27^0C. Calculate;

(i) number of molecules of argon in the flask (03marks)

From PV = nRT

n = $\frac{10\ x\ 10^3\ x\ 80\ x\ 10^{-6}}{8.31\ x\ 300}$ = 3.2 x 10^{-4} moles

Number of molecules = 3.2 x 10^{-4} x 6.02 x 10^{23} = 1.9264 x 10^{20}.

(ii) root mean square speed of the molecules in the flask (Molar mass of argon is 0.018kg) (03 mark)

$$P = \frac{1}{3}\rho \overline{c^2}$$

Mass of Argon = 3.2 x 10^{-4} x 0.018 =5.76 x10^{-6}kg

$\rho = \frac{M}{V} = \frac{5.76x10^{-6}}{80x10^{-6}} = 7.2\ x\ 10^{-2}$kgm^{-3}

$\sqrt{c^2} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3\ x\ 1\ x\ 10^4}{7.2x10^{-2}}}$ = 645.5 ms^{-1}

2. (a) What is meant by the following:

(i) Super-heated water? (01 mark)

Superheated water is **liquid water under pressure at temperatures between the usual boiling point,** 100 **°C and the critical temperature,** 374 °C.

(ii) Super cooled vapour? (01 mark)

Supercooled vapor refers to a gaseous state that remains at a temperature below its usual condensation point but doesn't condense into a liquid.

(b) Explain how:

(i) a gas in a vessel exerts pressure. (03marks)

A gas in a vessel exerts pressure due to the constant motion and collisions of its molecules with the walls of the container. Each molecule that collides with of vessel exerts a small force on the wall; and the cumulative effect of numerous collisions over a given area generates a continuous force on the wall of the vessel. This force per unit area constitutes pressure of a gas.

(ii) the atmosphere surrounding the earth prevents it from becoming unbearably cold. (03marks)

- atmosphere contain greenhouse gases and clouds that trap heat from the sun keeping the earth warm
- atmospheric gases act as insulators reducing temperature fluctuations
- circulation of atmospheric gases redistributes heat from the equator to cold paces

(c) A container of volume 0.2m^3 contains hydrogen gas of molar mass 2gmol^{-1} at a pressure of 1.5 x 10^4 Pa and a temperature of 27^{0}C.

Calculate the:

(i) number of hydrogen molecules in the container. (03 marks)

From $PV = nRT$

1.5 x 10^4 x 0.2 = n x 8.31 x (273 + 17)

The number of moles of hydrogen gas, n = $\frac{1.5x10^4x0.2}{8.31x290}$ = 1.2449

Number of hydrogen molecules = 1.2449 x 6.02 x 10^{23}

= 7.4943 x 10^{23} molecules

(ii) mean square speed of the molecules. (03 marks)

$P = \frac{1}{3}\rho \overline{c^2}$

Mass of hydrogen = 1.2449 x2 =2.4898 x 10^{-3}kg

ρ = $\frac{M}{V} = \frac{2.4898 \text{ x } 10^{-3}}{0.2} = 1.2449x\ 10^{-2}$kgm^{-3}

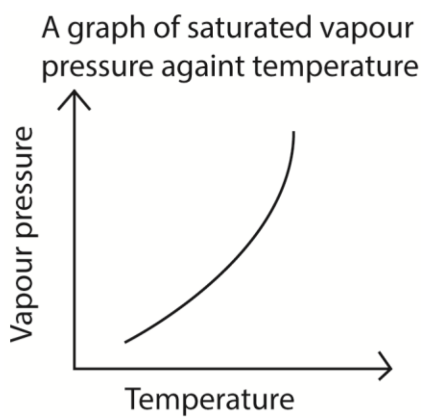
$\sqrt{\overline{c^2}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3\ x\ 1.5\ x\ 10^4}{1.2449x10^{-2}}}$ = 1,901 ms^{-1}

(iii) root mean square speed of oxygen molecules at the same temperature. (Molar mass of oxygen – 32mol^{-1}) (02 marks)

$\frac{\overline{c_O^2}}{\overline{c_H^2}} = \frac{m_H}{m_O}$

Hence root mean square speed of oxygen =$\frac{2}{32}x\ 1901$ = 119ms^{-1}

(d) Sketch a graph of saturated vapour pressure of a liquid against temperature and explain the shape of the curve. (04 marks)



Saturated vapour pressure increases with temperature due to the increase in kinetic energy and the probability of molecules overcoming the intermolecular forces holding them in the liquid phase, allowing them to escape into the vapour phase

3. (a) (i) What is meant by **isothermal** and **adiabatic** processes in a gas. (03marks)

Isothermal expansion is an expansion of gas that takes place at constant temperature.
Adiabatic expansion is an expansion of gas that takes place at constant heat.

(ii) State the conditions necessary to achieve the processes in (a)(i) (04 marks)

Isothermal process occurs at constant temperature and therefore the gas must be enclosed in thin walled container of good thermal conductivity placed in a large heat reservoir and occurs slowly enough to allow heat exchange with the surrounding.

Adiabatic process requires no heat input or out and therefore should occur rapidly in well insulated container like a thermos flask and gas should be ideal.

(iii) Explain why air coming out of a valve of a ball feels cold. (02marks)
This is due rapid (adiabatic) expansion accompanied with a decrease in temperature which causes a cold sensation.

(b) A mass of air initially occupying a volume of 2000cm^3 at a pressure of 76mmHG and a temperature of 20^{0}C expands adiabatically and reversibly to twice its volume. It is then compressed isothermally and reversibly to a volume of 3000cm^3.
(i) Find the final temperature and pressure of the gas. (06marks)

Under adiabatic expansion

Initial temperature = 273 +20 = 293

Finial temperature under adiabatic condition

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

⇨ $293(2000)^{1.4-1} = T(4000)^{1.4-1}$

T = $\frac{293}{2^{0.4}}$ = 222.1K

Final pressures under adiabatic condition

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$76\,x(2000)^{1.4} = P(4000)^{1.4}$

$P = \frac{76\,x(2000)^{1.4}}{(4000)^{1.4}}$ = 28.8mmHG

Under isothermal conditions

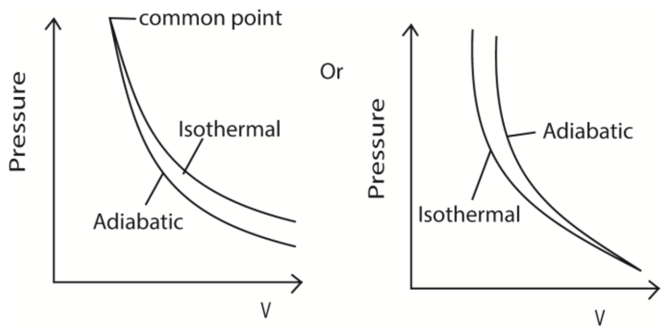
PV = constant

⇨ 28.8 x 4000 = P x 3000

P = $\frac{28.8\,x\,4000}{3000}$ = 38.4mmHg

Hence final temperature and pressure are 222.1K and 38.4mmHg respectively

(ii) *Indicate the two processes on a P-V diagram. (02 Marks)*
(The ration of the specific heat capacities of air = 1.4)



(c) Show that the work done, W, by a gas in expanding from volume V_1 to V_2 at constant pressure, P, is $W = P(V_2 - V_1)$. (04 marks)

If the piston is moved through a small distance dx, so that the pressure P is constant then

dw = Fdx

but F = PA;

=> dw = PAdx; also, Adx = dv

$\therefore dw = Pdv$

=> W = $\int_{v_1}^{v_2} Pdv$

= $[PV]_{v_1}^{v_2}$

= $P(v_2 - v_1)$

4. (a) Define the following:
   (i) Molar heat capacity of a gas at constant pressure. (01mark)
   The specific heat capacity of a gas at constant pressure is the heat required to warm unit mass of it by one degree, when its pressure is kept constant.
   (ii) Molar heat capacity of a gas at constant volume. (01mark)
   Molar heat capacity of a gas at constant volume is the amount of heat required to raise 1 mole of the gas through1K at constant volume.

(b) Derive the expression Cp –Cv = R, where Cp is the molar heat capacity of a gas at constant pressure and Cv is the molar heat capacity of a gas at constant volume and R is the gas constant. (05marks)

From dQ = dU + dW…………… (i)
But $dQ = C_pdT$, $dU = C_vdT$ and $dW = PdV = RdT$
Substituting in (i)
$C_pdT = C_vdT + RdT$
$\therefore C_p - C_v = R$

(c) (i) Differentiate between **adiabatic** and **isothermal** expansions. (02marks)

Isothermal expansion occurs at constant temperature.

Adiabatic expansion occurs at no heat input or output in the system.

(ii) State **two** examples of adiabatic changes. (01marks)

Expansion of a gas in an insulated cylinder.

release of air from a pneumatic tire.

rapidly pumping air into a bicycle tire.

(d) A fixed mass of an ideal gas of volume 400cm^3 at 15^{0}C expands adiabatically and its temperature falls to 0^{0}C. It is then compressed isothermally until the pressure returns to its original value. If the molar heat capacity at constant pressure is 28.6 J mol^{-1}K^{-1}, calculate the final volume after isothermal compression. (05marks)

Solution

Under adiabatic expansion

Initial temperature = 273 +15 = 288K

Final temperature = 273 + 0 = 273K

Cp = Cv + R = 28.6 + 8.31 = 36.91J

$\gamma = \frac{c_p}{c_v} = \frac{36.91}{28.6}$ = 1.29

Finial volume (V) under adiabatic condition

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

⇨ $288(4000)^{1.29-1}$ = $273(v)^{1.29-1}$

V = $\sqrt[0.29]{\frac{288(4000)^{0.29}}{273}}$ = 4810.2 cm^3

Final pressures(P$_2$) under adiabatic condition

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

P x (4000)$^{1.29}$ = P$_2$ x (4810.2)$^{1.29}$

P$_2$ = $\frac{P\ x\ (4000)^{1.29}}{4810.2^{1.29}}$ = 0.7883P

Final volume under isothermal conditions

PV = constant

⇨ 0.7883P x 4810.2 = P x v

V = 3,792cm^3

Hence final volume = 3.792cm^3

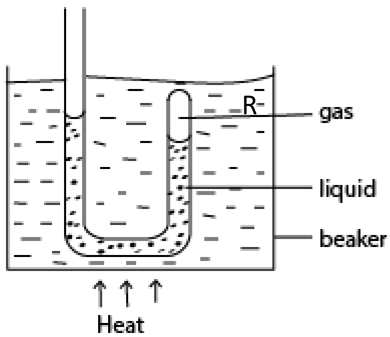
(e) (i) What is **saturated vapour pressure** of a liquid? (01mark)

A saturated vapour is one that is a dynamic equilibrium with its own liquid

(ii) Describe an experiment to show that a liquid boils when its saturated vapour pressure equals to the atmospheric pressure. (04marks)

The setup is shown below

Experiment to show that a liquid boils off when its saturated vapour pressure equals the external pressure



- Air is trapped in the closed limb of the tube by water column.
- The tube is heated in water bath.
- When the water bath begins to boil, the water in the tube comes to the same level in each limb.
- This shows that the vapor pressure in closed limb is equal to external pressure

Hence water boils when its saturated vapour pressure is equal to atmospheric pressure.

5. (a)(i) What is meant by a reversible process? (02marks)

A reversible process is a process that can proceed in a reverse direction by very small change in conditions making it take place through exactly same steps.

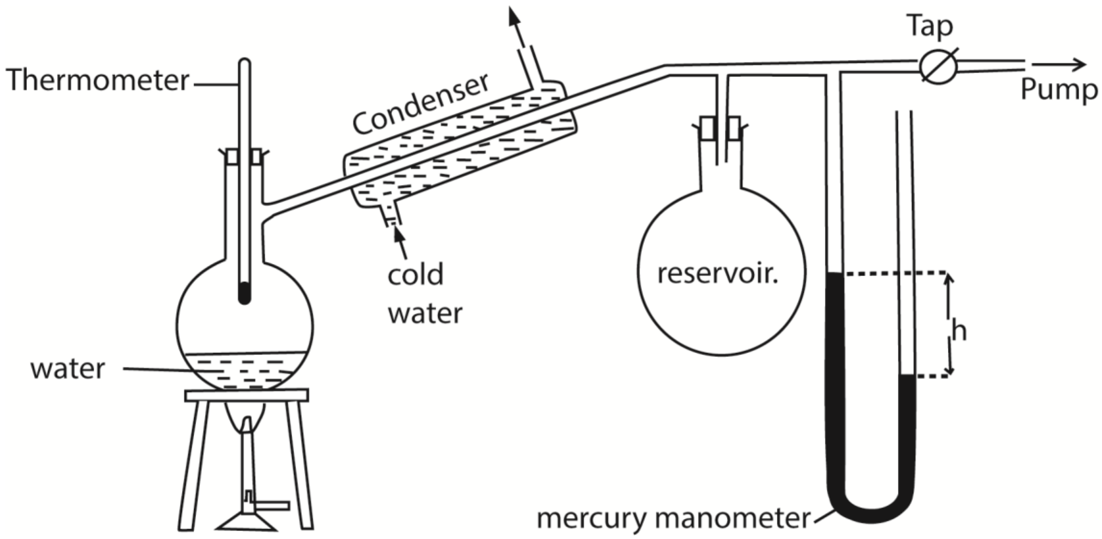
(ii) Distinguish between a saturated vapour and unsaturated vapour. (02marks)

A saturated vapour is one that is a dynamic equilibrium with its own liquid while unsaturated vapour is not in

(iii) Explain why evaporation causes cooling (03marks)

When a liquid vaporizes, it absorbs latent heat of vaporization from the body from which evaporation occurs. Hence the body cools.

(b) Describe an experiment to determine the temperature dependence of saturated vapour pressure of water. (07marks)



- The pressure of the air in R is shown by the mercury manometer; if its height is h, the pressure in mm mercury is P = H-h, where H is the barometer height.
- The tap is opened and the pressure above water varied using the pump to a suitable value.
- The tap is closed and water in the flask in heated until it boil.
- The temperature θ and difference in mercury levels, h, are noted and recorded.
- The saturated vapour pressure, P = $(H \pm h)$ is calculated
- The procedure is repeated other values of θ and h
- A graph of P versus θ is plotted and it shows that saturated vapour pressure, P, increases with temperature, θ.

(c) (i) State Dalton's law of partial pressures. (07marks)

Dalton's law states that the total pressure of a mixture of gases that to not react chemically is equal to the sum of the partial pressure of the components of a gas.

(ii) A sealed container has liquid water, water vapour and air all at 27^0C. The total pressure inside the container is 69cmHg. When the temperature is raised to 85^0C, the total pressure changes to 96cmHg. If the saturated vapour pressure of water at 27^0C is 5cmHg and water vapour remains saturated, calculate the saturated vapour pressure of water at 85^0C. (05marks)

T_1 = 27 + 273 = 300K

T_2 = 85 + 273 = 358K

Partial pressure at T_1; P_1 = 69-45 = 64cmHg

Partial pressure at T_2, P_2 = (96 –P) cmHg

Using $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ but $V_1 = V_2 = V$

$\frac{64V}{300} = \frac{(96-P)V}{358}$

P = 19.63cmHg

6. (a)(i) What is meant by boiling point? (01marks)

The boiling point of a liquid is the temperature at which its saturated vapour pressure equals the external pressure.

(ii) Explain why boiling point of a liquid increases with increase in the external pressure. (05marks)

When the liquid boils its saturated vapour pressure = external pressure and saturated vapour pressure increases with increasing temperature. When external pressure is raised, a liquid will boil at higher saturated pressure which occurs at high temperature.

(b) Explain how the pressure of a fixed mass of a gas can be increased at

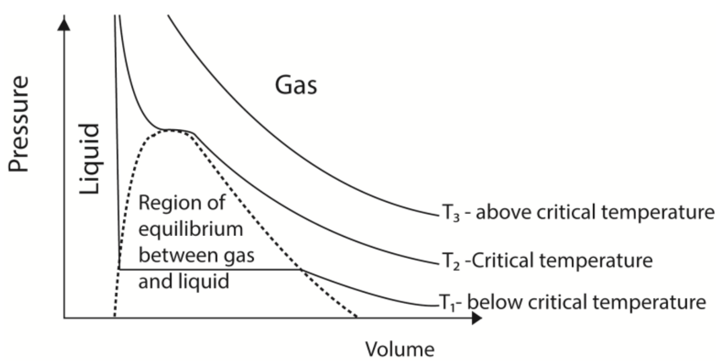
(i) Constant temperature (03marks)

By reducing the volume occupied by a gas, the molecules take less time to move between the walls of the container as the distance is reduced. The number of collisions per unit area increases hence pressure increases at constant temperature.

(ii) Constant volume (03marks)

By heating the gas, the molecules gain more kinetic energy. The molecules will bombard the walls many times per unit time per unit area. The total rate of change of momentum will increase hence pressure will increase

(c) Sketch a pressure versus volume curve for a real gas undergoing compression (03 marks)



- Above the critical temperature a gas obeys Boyle's law.
- Below the critical temperature a gas exist as unsaturated vapour at low pressure when the pressure is increase it condenses until all the gas is turned into a liquid.

(d) The cylinder of an exhaust pump has a volume of $5cm^3$. If it is connected through a valve to a flask of volume 225 cm^3 containing air at a pressure of 75cmHg, calculate the pressure of air in the flask after two strokes of the pump, assuming that the temperature of the air remain constant.

First stroke

$P_1V_1 = P_2V_2$ but $V_2 = 225 + 25 = 250cm^2$

$75 \times 225 = P_2 \times 250$

$P_2 = 674cmHg$

Second stroke

$P_2V_2 = P_3V_3$

$P_3 = \frac{67.5 \; x \; 225}{250}$= 60.8cmHG

Alternatively

$P_l = \left(\frac{V_2}{V_1 + V_2}\right)^2 P$, n = number of strokes

$= \left(\frac{225}{225+25}\right)^2 x \; 75 = 60.75cmHg$

7. (a) (i) State Dalton's law of partial pressures. (01marks)

Dalton's law of partial pressures states that the total pressure of a mixture of gases that do not react chemically is equal to the sum of partial pressures

(ii) Using the expression p = $\frac{1}{3}\rho \bar{c^2}$, where ρ is the pressure of a gas of density ρ and mean square speed c^2, derive Daltons law of partial pressures for two gases. (05marks)

P = $\frac{1}{3} N \frac{m}{V} \bar{c^2}$ = $\frac{2}{3} N \left(\frac{1}{2} m\bar{c^2}\right)$

For gas 1, P_1V_1 =$\frac{2}{3} N_1 \left(\frac{1}{2} m_1 \bar{c_1^2}\right)$

=> $N_1 = \frac{3}{2} P_1 V_1 . \frac{1}{K_1}$

Similarly for gas 2

$N_2 = \frac{3}{2} P_2 V_2 . \frac{1}{K_2}$

For a mixture of gases, N = $\frac{3}{2} PV . \frac{1}{K}$; but N = $N_1 + N_2$

$\frac{3}{2} PV . \frac{1}{K} = \frac{3}{2} P_1 V_1 . \frac{1}{K_1} + \frac{3}{2} P_2 V_2 . \frac{1}{K_2}$

Since temperature is constant, $K_1 = K_2 = K$

- $PV = P_1V_1 + P_2V_2$
- But $V = V_1 = V_2$
- $\therefore P = P_1 + P_2$

(b) (i) What is meant by isothermal process and adiabatic process. (02marks)

Isothermal process is the expansion or compression of a gas at constant temperature.

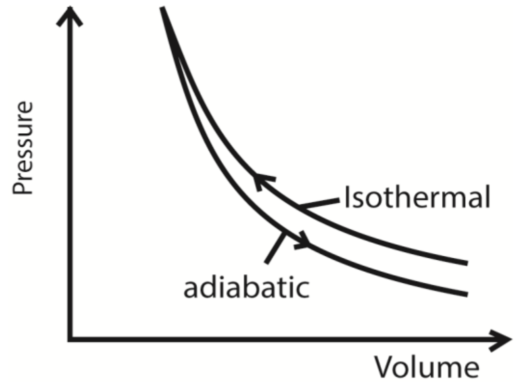
Adiabatic process is the expansion or compression of a gas where there is no heat loss or gain into the gas.

(ii) Explain why adiabatic expansion of a gas causes cooling (03marks)

During an adiabatic expansion of a gas, no heat is supplied to the gas. Molecules strike the receding piston and bounce off with reduced velocities and hence lower kinetic energy. Since kinetic energy is proportional to temperature, the gas cools during the expansion

(c) A gas at a temperature of 17^0C and pressure 1.0 x 10^5Pa compressed isothermally to half its original volume. It is then allowed to expand adiabatically to its original volume

(i) Sketch a P-V curve the above process (02marks)



(ii) If the specific heat capacity at constant pressure is 2100Jmol^{-1}K^{-1} and at constant volume is 1500Jmol^{-1}K^{-1}, find the final temperature of the gas (04marks)

$\gamma = \frac{2100}{1500} = 1.4$

$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$

$290\left(\frac{V}{2}\right)^{0.4} = T_3V^{0.4}$

$T_3 = 219.8K$

(d) (i) What is meant by saturated vapour? (01mark)

A saturated vapour is a vapour in dynamic equilibrium with its own liquid.

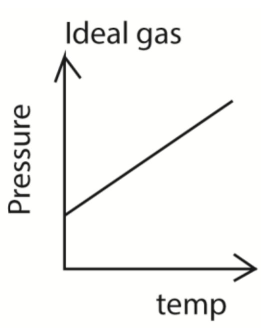
(ii) Explain briefly the effect of altitude on the boiling point of a liquid (02marks)

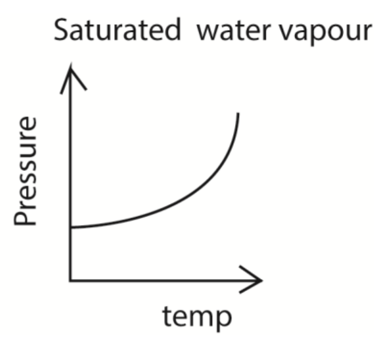
A liquid boils when its saturated vapour pressure is equal to the external vapour pressure. Atmospheric pressure decreases with increasing altitude, hence the boiling point of a liquid reduces as the altitude increases.

8. (a) (i) State two differences between saturated and unsaturated vapours. (02marks)

- A saturated vapour pressure is one which is in dynamic equilibrium with its own liquid and an unsaturated vapour is one that is not in dynamic equilibrium with its own liquid.
- A saturate vapour pressure does not obey gas laws whereas unsaturated vapour approximately obey gas laws.

(ii) Sketch graphs of pressure against temperature for an ideal gas and for saturated water vapour originally at 0^{0}C (03marks)





(b) The specific heat capacity of oxygen at constant volume is 719Jkg^{-1}K^{-1} and its density at standard temperature and pressure is 1.429kgm^{-3}. Calculate the specific heat capacity of oxygen at constant pressure (04marks)

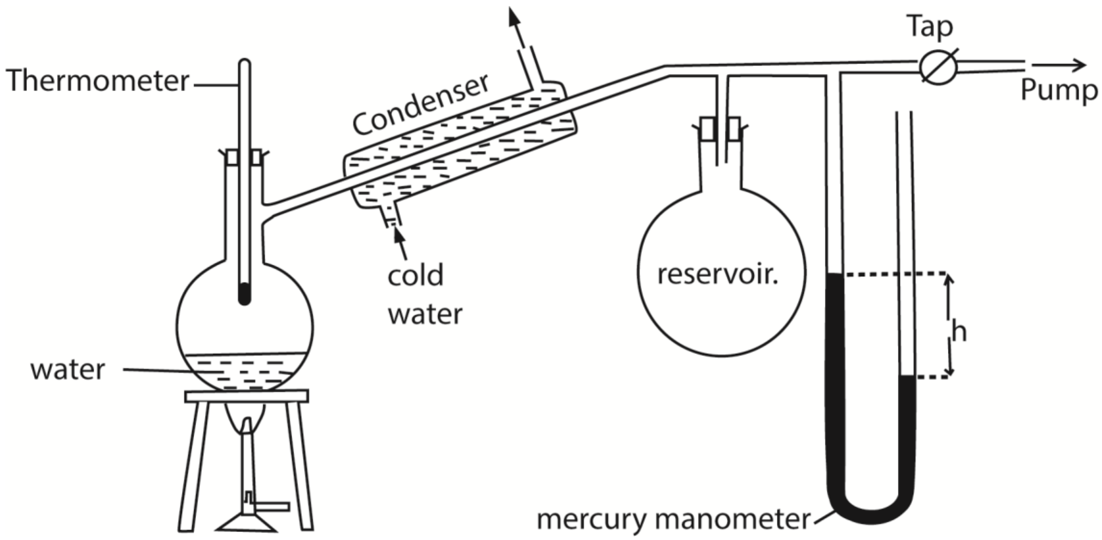
PV =mrT

r = $\frac{P}{T}\left(\frac{V}{m}\right) = \frac{1.01 \times 10^5}{273 \times 1.429}$ = 258.9Jkg^{-1}K^{-1}

$c_p - c_v = r$

c_p = 719 + 258.9 =977.9Jkg^{-1}K^{-1}

(c) (i) With the aid of a labelled diagram, describe an experiment to determine standard saturated vapour pressure of water. (05marks)



- The pressure of the air in R is shown by the mercury manometer; if its height is h, the pressure in mm mercury is P = H-h, where H is the barometer height.
- The tap is opened and the pressure above water varied using the pump to a suitable value.
- The tap is closed and water in the flask in heated until it boil.
- The temperature θ and difference in mercury levels, h, are noted and recorded.
- The saturated vapour pressure, P = $(H \pm h)$ is calculated
- The procedure is repeated other values of θ and h
- A graph of P versus θ is plotted and the saturated vapour pressure at a particular temperature is obtained.

(ii) State how the experiment set up in (c) (i) may be modified to determine a saturated vapour pressure of above atmospheric pressure (01marks)

By replacing the vacuum pump with a bicycle pump

(d)(i) Define ideal gas (01mark)

An Ideal gas s one that obeys Boyle’s law under all conditions

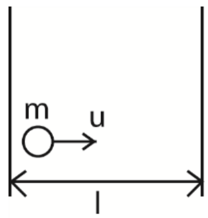
(ii) State and explain the conditions under which real gases behave as ideal gases. (04marks)

- At higher temperature, the intermolecular spacing increases and intermolecular forces become negligible
- At very low pressure, the gas occupies negligible volume of the container.

9. (a) The pressure, P, of an ideal gas is given by P= $\frac{1}{3}\rho\overline{c^2}$, where ρ is the density of the gas and $\overline{c^2}$ its mean square speed.

(i) Show clearly the steps taken to derive this expression (06marks)

Consider a molecule of mass, m, moving in a cube of length, l and velocity, u.



Change in momentum = mu - ⁻mu = 2mu

Rate of change of momentum = $\frac{2mu}{t}$

Time, t, between collision = $\frac{2l}{u}$

$F_1 = 2mu \div \frac{2l}{u} = \frac{mu^2}{l}$

For N molecules, force on the wall,

F = $\frac{mu_1^2}{l} + \frac{mu_2^2}{l} + \frac{mu_3^2}{l} + \cdots + \frac{mu_N^2}{l}$

Pressure, P = $\frac{F}{A} = \frac{m}{l^3}(u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2)$ since A = l^2

$\overline{u^2} = \frac{u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2}{N}$

$N\overline{u^2} = u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2$

$\therefore P = \frac{Nm\overline{u^2}}{l^3} = \rho\overline{u^2};\ since\ \rho = \frac{Nm}{l^3}$

$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$ and $\overline{u^2} = \overline{v^2} + \overline{w^2}$

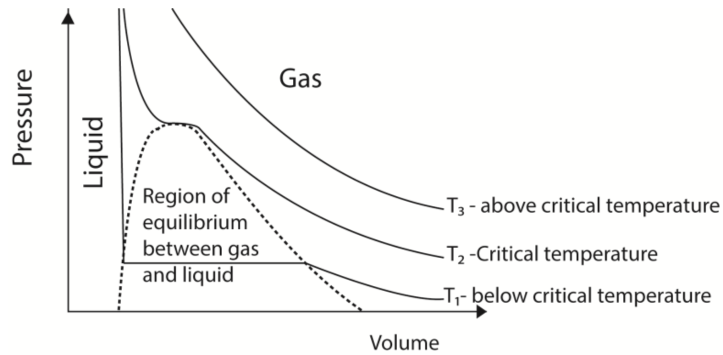
$\therefore \overline{c^2} = 3\overline{u^2}$ => $\overline{u^2} = \frac{1}{3}\overline{c^2}$

$\therefore P = \frac{1}{3}\rho\overline{c^2}$

(ii) State the assumptions made in deriving this expression (02marks)

- Intermolecular forces of attraction are negligible
- The volume of the gas molecules are negligible compared to that of the container
- Molecules make perfectly elastic collision
- The duration of collision is negligible compared to the time between collisions

(b) Sketch the pressure versus volume curve for a real gas for temperatures above and below the critical temperature. (03marks)



- Above the critical temperature a gas obeys Boyle's law.
- Below the critical temperature a gas exist as unsaturated vapour at low pressure when the pressure is increase it condenses until all the gas is turned into a liquid.

(c) For 1 mole of a real gas, the equation of state is $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

Explain the significance of the terms $\frac{a}{V^2}$ and b. (02marks)

$\frac{a}{V^2}$ - corrects deficit in pressure due to intermolecular attractions of gas molecules

b – accounts for the finite volume of molecules themselves.

(d) A balloon of volume 5.5 x $10^{-2}m^3$ is filled with helium to a pressure of 1.10 x 10^5Nm^{-2} at a temperature of 20^0C. Calculate the

(i) the number of helium atoms in the balloon (03marks)

From PV =nRT

n = $\frac{1.10 \: x \: 10^5 \: x \: 5.5 \: x \: 10^{-2}}{8.31 \: x \: 293}$ = 2.48 moles

Number of atoms = 2.48 x 6.02 x 10^{23} = 1.49 x 10^{24}.

(ii) net force acting on the square meter of material of the balloon if the atmospheric temperature is 1.01 x 10^5Nm^{-2} (04marks)

F = PA

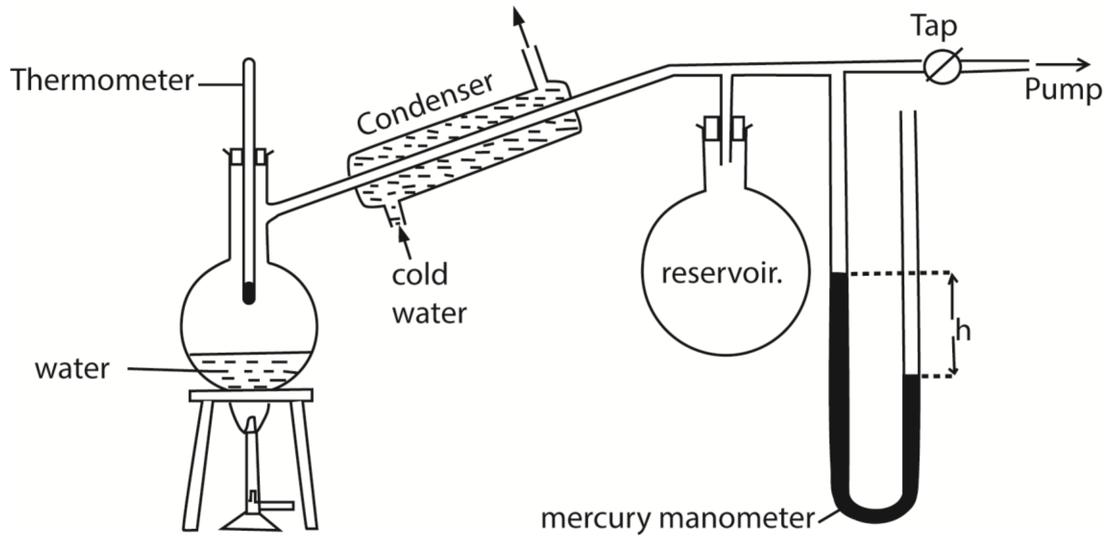
Net force = internal force in the bulb – external force on the bulb

= (1.10 x 10^5 – 1.01 x 10^5) x 1.0 = 9.0 x 10^3N

10. (a) (i) Define saturated vapour pressure. (01mark)

Saturated vapour pressure of a liquid is the pressure exerted by vapour in dynamic equilibrium with its liquid

(ii) Describe with the aid of a diagram, how saturated vapour pressure of a liquid can be determined at a given temperature. (06marks)



- The pressure of the air in R is shown by the mercury manometer; if its height is h, the pressure in mm mercury is P = H-h, where H is the barometer height.
- The tap is opened and the pressure above water varied using the pump to a suitable value.
- The tap is closed and water in the flask in heated until it boil.
- The temperature θ and difference in mercury levels, h, are noted and recorded.
- The saturated vapour pressure, P = $(H \pm h)$ is calculated
- The procedure is repeated other values of θ and h
- A graph of P versus θ is plotted and the saturated vapour pressure at a particular temperature is obtained.

(b) Use the kinetic theory to explain the following observations

(i) Saturated vapour pressure of a liquid increases with temperature. (03marks)

If a liquid is in dynamic equilibrium with its vapour, an increase in temperature increases mean kinetic energy of the molecules and hence the rate at which molecules escape from the liquid.

The density of the vapour increases implying increase in the rate of condensation until dynamic equilibrium is restored. There are now more molecules in the vapour phase than previously that are moving faster and hence higher pressure.

(ii) Saturated vapour pressure is not affected by decrease in volume at constant pressure. (03marks)

A decrease in volume leads to a momentary increase in vapour density. Consequently, the rate of condensation increases while the rate of evaporation rate is constant. When the vapour density reduces, the condensation rate also reduces. So the dynamic equilibrium is restored to the initial value.

(c) When hydrogen gas is collected over water, the pressure in the tube at 15^0C and 75^0C are 65.5cm and 105.6cm of mercury respectively. If the saturated vapour pressure at 15^0C is 1.42cm of mercury, find its value at 75^0C (04marks)

P_1= 65.5 – 1.42 = 62.08 and T_1 = 273 + 15 = 288K

P_2 = 105.6 – P, and T_2 = 273 + 75 = 348K

From $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$P_2 = \left(\frac{P_1}{T_1}\right) T_2$ =105.6 - $\frac{64.08}{288}\ x\ 348$ = 28.12cmHg

(d) Explain why the molar heat capacity of an ideal gas at constant pressure differs from the molar heat capacity at constant volume (03marks)

At constant volume, all the heat supplied goes to raising temperature (increasing internal energy of the gas) while at constant pressure, heat is used to raise the temperature of the gas (internal energy) and do external work in expansion to keep the pressure constant. Therefore the molar heat capacity at constant pressure is more than the molar heat capacity at constant volume.

11. (a) (i) Define the term specific heat capacity and internal energy and state their units. (03marks)

Specific heat capacity is the amount of heat required to raise the temperature of 1kg mass of a substance by 1K of 1^0C.

Internal energy is the total of the kinetic energy of atoms and molecules and the parallel potential energy due to mutual interactions of these atoms. Units: joules.

(ii) Why is the distinction between specific heat capacity at constant pressure and that at constant volume important for gases, but less important for solids and liquids? (04marks)

The volume of solids and liquids change very little when heated at constant pressure compared with volume changes for gas for the same temperature changes. Thus solid and liquids do very little work against atmospheric pressure. This implies that there is very little difference in energy when they expand and when they are allowed to expand.

(b) Explain why the temperature of a liquid does not change when the liquid is boiling? (02marks)

When the liquid boils, there is change in a state to vapour and all the heat supplied is used to do work by breaking the molecular bonds of the liquid and overcoming external pressure during the change into a gas. The temperature will not change until all the bonds broken.

(c) One kilogram of water is converted to steam at a temperature of 100^0C and a pressure of 1.0×10^5Pa. If the density of steam is 0.58kgm^{-3} and specific heat of vaporization of water is 2.3×10^6Jkg^{-1}, calculate the

(i) external work done (04marks)

Volume of 1kgof steam = $\frac{m}{\rho} = \frac{1}{0.58}$ = 1.724m^3

Volume of 1kgof steam = $\frac{m}{\rho} = \frac{1}{1000}$ = 0.001m^3

Change in volume = 1.724 – 0.001 = 1.723m^3

Work = pdV = 1.0 x 10^5 x 1.723 = 1.723 x 10^{5}J

(ii) internal energy (03marks)

Δu = ΔQ – Δw

= 2.3 x 10^6 – 1.723 x 10^5

= 2.13 x 10^{6}J

(d) Explain why the specific latent heat of fusion and specific latent heat of vaporization of a substance at the same pressure are different. (04marks)

Change from solid to liquid, intermolecular bonds are weakened and there is a small increase in volume. This implies there negligible change in volume and thus little work done against atmospheric pressure.

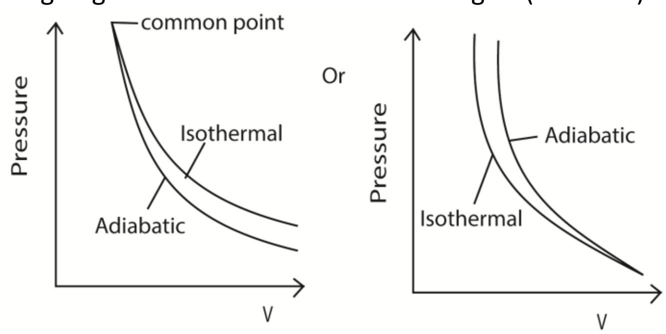
During vaporization, a lot of heat is required to break molecular bonds in a liquid and to enable expansion to larger volume of a gas against atmospheric pressure.

12. (a) (i) Explain the difference between isothermal and adiabatic expansion of a gas. (02marks)

Isothermal expansion takes place at constant temperature.

Adiabatic expansion takes place at constant heat.

(ii) Using same axes and point, sketch graphs of pressure versus volume for fixed mass of a gas undergoing isothermal and adiabatic changes. (03marks)



(b) Show that work, W, done by a gas which expands reversibly from V_0 to V_1 is given by $W = \int_{V_0}^{V_1} \rho dv$ (04marks)

If the piston is moved through a small distance dx, so that the pressure P is constant then

dw = Fdx

but F = PA; => dw = PAdx; also, Adx = dv

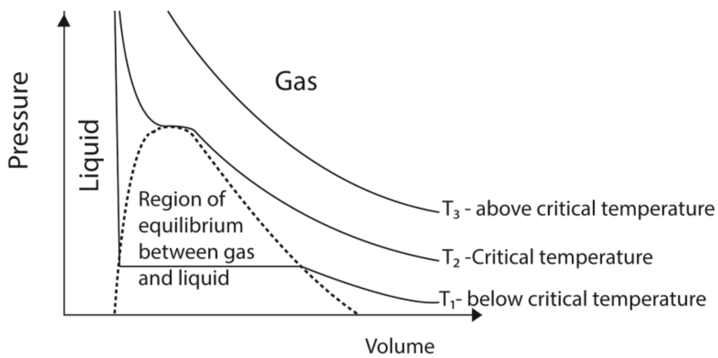
$\therefore dw = Pdv$

=> W = $\int_{v_0}^{v_1} P dv$

(c) (i) State two differences between real and ideal gases (02marks)

Real gas	Ideal gas
Intermolecular force are appreciable	Intermolecular forces are negligible
Volume of molecules compared to the volume of the container is not negligible	Volume of molecules compared to the volume of container is negligible
Obey Boyle's law at high temperature and very low pressure	Obey Boyle's law at all temperatures and pressures.

(ii) Draw a labelled diagram showing P-V isothermals for a real gas above and below the critical temperature. (03marks)



- Above the critical temperature a gas obeys Boyle's law.
- Below the critical temperature a gas exist as unsaturated vapour at low pressure when the pressure is increase it condenses until all the gas is turned into a liquid.

(d) Ten moles of a gas, initially at 27^{0}C are heated at constant pressure of 1.01 x 10^{5}Pa and volume increased from 0.25m^3 to 0.375m^3. Calculate the increase in internal energy. [Assume C_p = 28.5Jmol^{-1}K^{-1}] (06marks)

T_1 = 270C = 300K

Using $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$\frac{0.250}{300} = \frac{0.375}{T_2}$; T_2 = 450K

$\Delta T = 450 - 300 = 150K$

ΔQ = ΔU + Δw

$nC_p\Delta T = nC_v\Delta T + nR\Delta T$

$nC_v\Delta T = nC_p\Delta T - nR\Delta T$

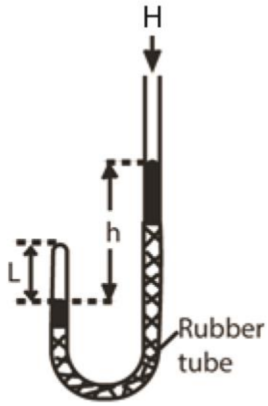
= 10 x 28.5 x 150 – 10 x 8.31 x 150

= 3.03 x 10^{4}J

13. (a) (i) state Boyles law. (01mark)

Boyle's law states that the pressure of a fixed mass of a gas at constant temperature is inversely proportional to the volume of the gas.

(ii) Describe an experiment that can be used to verify Boyles' law. (06marks)



- air in the closed limb of a U-tube barometer as shown above
- mercury is poured to a height, h, and the length of the air column, L is noted.
- the length h is varied to obtain different sets of values of h and L
- Pressure of the gas is calculated from $P = (H + h)\rho g$ where H= height of barometer corresponding to atmospheric pressure, ρ = density of mercury, g = acceleration due to gravity. Note that h can be positive or negative.
- If A is the cross section area, $V = AL$
- Values of h, L, P, V and 1/V are tabulated
- A plot of P against 1/V gives a straight line through the origin which verifies Boyle's law.

(b) Explain the following observations using the kinetic theory.

(i) A gas fills any container in which is it placed and exerts pressure on its walls. (03marks)

A gas contains molecules with negligible intermolecular forces that are free to move in all directions. As they move, they collide with each other and with the walls of the container. The unrestricted movements make them to fill the available space and collisions with the walls contribute to the pressure exerted on the wall.

(ii) The pressure of a fixed mass of a gas rises when temperature is increased at constant volume. (02 marks)

When the temperature of a gas increases, the kinetic energy of the gas molecules increases. This increases the frequency and force of collision against the wall leading to increase in pressure.

(c) (i) What is meant by a reversible process. (01marks)

A reversible process in one that takes place in reverse direction through the same values of pressure, volume and temperature in small changes or steps.

(ii) State the conditions necessary for isothermal and adiabatic processes to occur, (04marks)

Isothermal process occurs at constant temperature and therefore the gas must be enclosed in thin walled container of good thermal conductivity placed in a large heat reservoir and occurs slowly enough to allow heat exchange with the surrounding.

Adiabatic process requires no heat input or out and therefore should occur rapidly in well Insulated container like a thermos flask and gas should be ideal.

(d) A mass of an ideal gas of volume $200cm^3$ at 144K expands adiabatically to a temperature of 137K. Calculate its new volume. (Take γ =1.40)

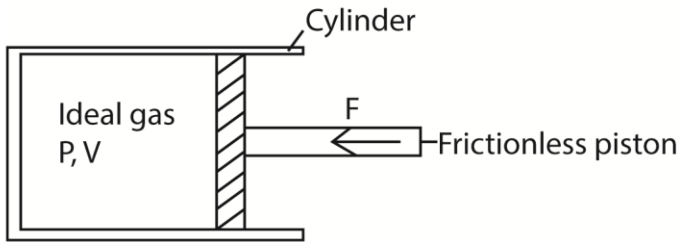
$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\gamma = \frac{2100}{1500} = 1.40$

⇨ $144(200)^{1.4-1} = 137(V)^{1.4-1}$

V = $226.5cm^3$

14. (a)



A fixed mass of an ideal gas is confined in a cylinder by frictionless piston of cross section area A. the piston is in equilibrium under the action of force, F as shown in the figure above. Show that the work done, W, by the gas when it expands from V_1 to V_2 is given by

$W = \int_{V_1}^{V_2} PdV$ (03marks)

Suppose the gas expands by dv so that the piston moves out through a small distance dx.

Work done by the gas, dW = Fdx

= PAdx

= Pdv

Total work done during expansion from v_1 to v_2 is given by

$$W = \int_{v_1}^{v_2} Pdv$$

(b) State the first law of thermodynamics and use it to distinguish between Isothermal and adiabatic changes in a gas. (05marks)

$\Delta Q = \Delta U + \Delta W = nC_v\Delta T + \Delta W$

During isothermal expansion, ΔT = 0. Therefore all the energy supplied is equal to the work done by the gas during expansion.

In adiabatic expansion, no heat enters or leaves the gas. Therefore ΔQ = 0 and ΔU = -ΔW.

In adiabatic expansion, work s done at the expense of its internal energy. Therefore the gas cools.

(c) The temperature of 1mole of helium gas at a pressure of 1.0×10^5Pa increases from 20^0C to 100^0C when the gas is compressed adiabatically.

Find the final pressure of the gas. (Take γ =1.67) (04 marks)

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

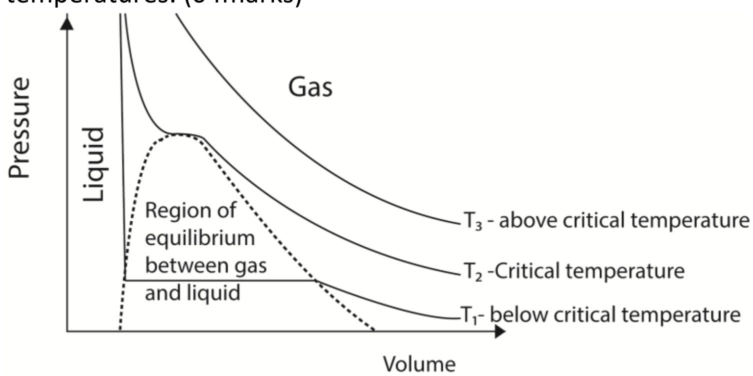
$but\ V = \frac{nRT}{P} => \frac{P_1T_1^{\gamma}}{P_1} = \frac{P_2T_2^{\gamma}}{P_2}$

$=> \frac{T_1^{\gamma}}{P_1^{\gamma-1}} = \frac{T_2^{\gamma}}{P_2^{\gamma-1}}$

$$\frac{(293)^{1.67}}{(1.0\ x\ 10^5)^{0.67}} = \frac{(373)^{1.67}}{(P)^{0.67}}$$

$$P = 1.87\ x\ 10^5 Pa$$

(d) With the aid of a P-V diagram, explain what happens when a real gas is compressed at different temperatures. (04marks)



- Above the critical temperature a gas obeys Boyle's law.
- Below the critical temperature a gas exist as unsaturated vapour at low pressure when the pressure is increase it condenses until all the gas is turned into a liquid.

(e) The root-mean square speed of the molecules of a gas is 44.72ms^{-1}. Find the temperature of the gas if its density is 9.0 x 10^{-2}kgm^{-3} and the volume is 42.0m^3. (04marks)

P= $\frac{1}{3}\rho\overline{c^2}$

⇨ $\frac{RT}{V} = \frac{1}{3}\rho\overline{c^2}$

T = $\frac{1}{3}\rho\overline{c^2} x\ \frac{V}{R}$ = $\frac{1}{3}\ x\ 9.0\ x\ 10^{-2} x\ (44.72)^2\ x\ \frac{42}{8.31}$ = 303.2K

15. (a) Define saturated vapour pressure (S.V.P) (01mark)

Saturated vapour pressure in the one in dynamic equilibrium with its own liquid

(b) Use the kinetic theory of matter to explain the following observations

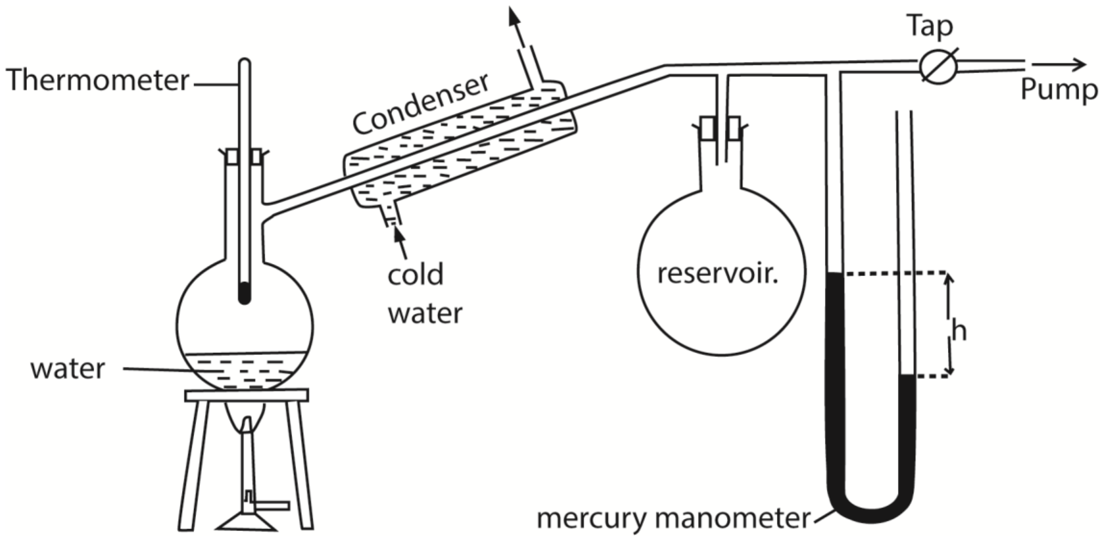
(i) saturated vapour pressure of a liquid increases with temperature. (03marks)

Increase in temperature increases the kinetic energy of the liquid molecules and the rate of evaporation increases. Therefore the pressure of the vapour rises. As the rate at which the molecules bombard the liquid surface increases, dynamic equilibrium is restored at a higher saturated vapour pressure.

(ii) saturated vapour pressure is not affected by decrease in volume at constant temperature. (03marks)

A decrease in volume leads to momentary increase in the density of the vapour. The rate of condensation increases than the rate of evaporation. As the density of the vapour falls the rate of condensation also falls. Dynamic equilibrium is re-established to original values of density and pressure of vapour. Therefore no increase in saturated vapour pressure occurs.

(c) Describe how saturated vapour pressure of a liquid at various temperatures can be determined. (07marks)



- The pressure of the air in R is shown by the mercury manometer; if its height is h, the pressure in mm mercury is P = H-h, where H is the barometer height.
- The tap is opened and the pressure above water varied using the pump to a suitable value.
- The tap is closed and water in the flask in heated until it boil.
- The temperature θ and difference in mercury levels, h, are noted and recorded.
- The saturated vapour pressure, P = $(H \pm h)$ is calculated
- The procedure is repeated other values of θ and h
- A graph of P versus θ is plotted and the saturated vapour pressure at a particular temperature is obtained.

(d) (i) State Dalton's law of partial pressures (01mark)

Daltons law of partial pressures states that the pressure of a mixture of gases that do not react chemically is the sum of partial pressures of its components.

(ii) A horizontal tube of uniform bore, closed at one end, has some air trapped by a small quantity of water. The length of the enclosed air column is 20cm at 12^0C.

Find stating any assumptions made, the length of air column when the temperature is raised to 38^0C.

[S.V.P of water at 12^0C and 38^0C are 10.5mmHg and 49.5mmHg respectively. Atmospheric pressure = 75cmHG] (05marks)

T_1 = 273 + 12 = 285K, T_2 = 273 + 38 = 311K; P_1 = 750 – 10.5 = 739.5mmHg, P_2 = 750-49.5 = 700.5mmHg

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{739.5\ x\ 20A}{285} = \frac{700.5\ x\ hA}{311}$$

h = 23.04cm

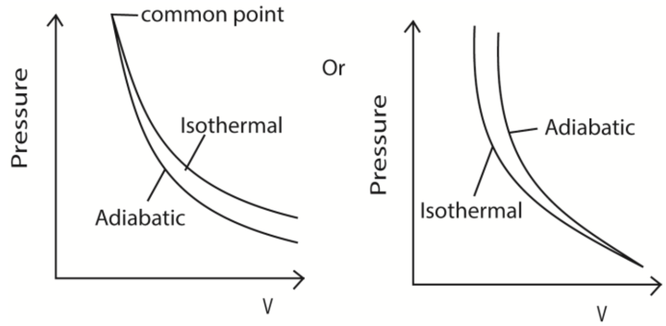
Assumption: the tube does not expand when the temperature increases.

16. (a)(i) What is meant by isothermal and adiabatic changes? (02marks)

Isothermal expansion takes place at constant temperature.

Adiabatic expansion takes place at constant heat.

(ii) Using same axes and point, sketch graphs of pressure versus volume for fixed mass of a gas undergoing isothermal and adiabatic changes. (03marks)



(b) An ideal gas is trapped in a cylinder by a movable piston. Initially it occupies a volume of 8 x $10^{-3}m^3$ and exerts a pressure of 108kPa. The gas undergoes an isothermal expansion until its volume is 27 x $10^{-3}m^3$. It is then compressed adiabatically to the original volume of the gas.

(i) Calculate the final pressure of the gas (06marks)

Under isothermal, $P_1V_1 = P_2V_2$

$108 \times 10^3 \times 8 \times 10^{-3} = P_2 \times 27 \times 10^{-3}$

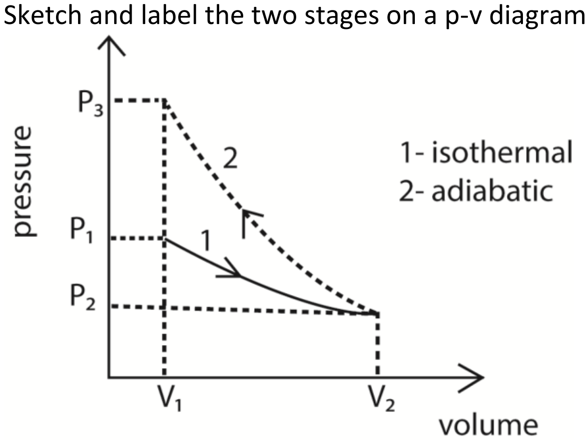
$P_2 = 3.2 \times 10^3$Pa

Under adiabatic, $P_3V_3^\gamma = P_2V_2^\gamma$

$P_3(8\ x\ 10^{-3})^{\frac{5}{3}} = 32\ x\ 10^3\ x\ (27\ x\ 10^3)^{\frac{5}{3}}$

$P_3 = 243\ x\ 10^3$ Pa

(i) Sketch and label the two stages on a p-v diagram. (02marks)



[The ratio of the principal molar heat capacities of the gas = 5:3]

(c) (i) Define molar heat capacities at constant pressure. (01mark)

The specific heat capacity of a gas at constant pressure is the heat required to warm unit mass of it by one degree, when its pressure is kept constant.

(ii) Derive the expression $C_p - C_v = R$, for 1mole of a gas (05marks)

From dQ = dU + dW……………. (i)
But dQ = C_pdT, dU = C_vdT and dW= PdV = RdT
Substituting in (i)

$C_pdT = C_vdT + RdT$
$\therefore C_p - C_v = R$

(iii) In which ways does a real gas differ from an ideal gas? (02marks)

Real gas	Ideal gas
Intermolecular force are appreciable	Intermolecular forces are negligible
Volume of molecules compared to the volume of the container is not negligible	Volume of molecules compared to the volume of container is negligible
Obey Boyle's law at high temperature and very low pressure	Obey Boyle's law at all temperatures and pressures.

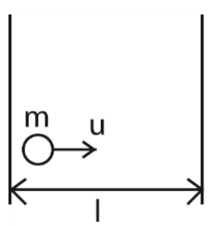
17. (a) Derive the expression P = $\frac{1}{3}\rho\overline{c^2}$ for the pressure, P, of an ideal gas of density ρ and mean square speed, $\overline{c^2}$. State any assumptions made (07marks)

Assumptions

- The intermolecular forces are negligible
- The volume of the gas is negligible compared the volume of the container
- Collision are perfectly elastic
- The duration of collision is negligible

Derivation

Consider a molecule of mass, m, moving in a cube of length, l and velocity, u.



Change in momentum = mu - ⁻mu = 2mu

Rate of change of momentum = $\frac{2mu}{t}$

Time, t, between collision = $\frac{2l}{u}$

$F_1 = 2mu \div \frac{2l}{u} = \frac{mu^2}{l}$

For N molecules, force on the wall,

F = $\frac{mu_1^2}{l} + \frac{mu_2^2}{l} + \frac{mu_3^2}{l} + \cdots + \frac{mu_N^2}{l}$

Pressure, P = $\frac{F}{A} = \frac{m}{l^3}(u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2)$ since A = l^2

$$\overline{u^2} = \frac{u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2}{N}$$

$$N\overline{u^2} = u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2$$

$$\therefore P = \frac{Nm\overline{u^2}}{l^3} = \rho\overline{u^2};\ since\ \rho = \frac{Nm}{l^3}$$

$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$ and $\overline{u^2} = \overline{v^2} + \overline{w^2}$

$\therefore \overline{c^2} = 3\overline{u^2}$ => $\overline{u^2} = \frac{1}{3}\overline{c^2}$

$$\therefore P = \frac{1}{3}\rho\overline{c^2}$$

(b) A gas is confined in a container of volume $0.1m^3$ at a pressure of $1.0 \times 10^5 Nm^{-2}$ and temperature of 300K. If the gas is assumed to be ideal, calculate the density of the gas.
(The relative molecular mass of the gas is 32) (05marks)

PV = nRT

n = $\frac{1.0 \times 10^5 \times 0.1}{8.31 \times 300}$ = 4

mass of gas, m =nM =4 x 32 x 10^{-3} =0.128kg

Density = $\frac{m}{V} = \frac{0.128}{0.1}$ = $1.28kgm^{-3}$

(c) What is meant by

(i) isothermal change

Isothermal expansion takes place at constant temperature.

(ii) adiabatic change (02marks)

Adiabatic expansion takes place at constant heat.

(d) A gas at a pressure of 1.0×10^6Pa is compressed adiabatically to half its volume and then allowed to expand isothermally to its original volume. Calculate the final pressure of the gas.
[Assume the ratio of the principal specific heat capacities $^{C_p}/_{C_v}, \gamma = 1.4$] (06marks)

For adiabatic change

PV^γ = constant

⇨ $1.0 \times 10^6 \times V^{1.4} = P \times \left(\frac{V}{2}\right)^{1.4}$

P =2.64×10^6Pa

For isothermal change

PV = constant

$2.64 \times 10^6 \times \frac{V}{2}$ = PV

P = 1.32×10^6Pa

18. (a) (i)What is meant by kinetic theory of gases? (03marks)

Gases are composed of molecules which are in continuous random motion. The molecules collide elastically with one another and also with the walls of the container. When heat energy is supplied, their kinetic energy increases.

(ii) Define an ideal gas (01mark)

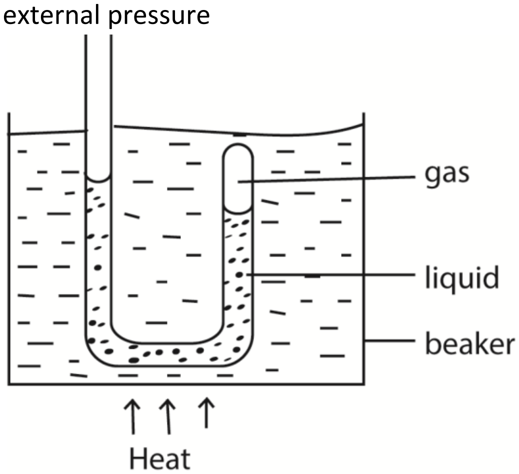
An ideal gas is one which exactly obeys Boyles law at all conditions

(iii) State and explain conditions under which real gases behave like ideal gases. (04mark)

At very high temperature, the intermolecular forces of attraction become negligible.
At low pressure, the volume of gas molecules become negligible compared to the volume of the container.

(b) (i) Describe an experiment to show that a liquid boils only when its saturated vapor pressure is equal to external pressure (05marks)

Experiment to show that a liquid boils off when its saturated vapour pressure equals the external pressure



- Air is trapped in the closed limb of the tube by water column.
- The tube is heated in water bath.
- When the water bath begins to boil, the water in the tube comes to the same level in each limb.
- This shows that the vapor pressure in closed limb is equal to external pressure.

(ii) Explain how cooking at a pressure of 76cm of mercury and temperature of 100^0C may be achieved on top of high mountains. (03marks)

Cooking pans are fitted with lids that possess safety valves that open when the pressure exceeds 76cmmmHg. During cooking the vapour pressure inside the cooking pan increase and the temperature increases to 100^0C. The safety valves prevents pressure to exceed 76cmHg and therefore boiling occurs at 100^0C.

(c) (i) Define root-mean-square speed of molecules of a gas. (01mark)

Root mean square speed of the gas is the average of the square speeds of individual molecules of a gas.

(ii) The mass of hydrogen and oxygen atoms are 1.66 x 10^{-27}kg and 2.66 x 10^{-26}kg respectively. What is the ratio of the root mean square speed of hydrogen to that of oxygen molecules at the same temperature? (03marks)

From $\frac{1}{3}Nm\overline{c^2}$ = RT

$$\frac{1}{3}N\ x\ 1.66\ x\ 10^{-27} x\ \overline{c_H^2} = \frac{1}{3}N\ x\ 2.66\ x\ 10^{-26}\ x\ \overline{c_O^2}$$

$$\frac{\overline{c_H^2}}{\overline{c_O^2}} = \frac{2.66\ x\ 10^{-26}}{1.66\ x\ 10^{-27}} = 16$$

19. (a) State the assumption made in the derivation of the expression P = $\frac{1}{3}\rho\overline{c^2}$ for pressure of an ideal gas (02marks)

- The intermolecular forces are negligible
- The volume of the gas is negligible compared the volume of the container
- Collision are perfectly elastic
- The duration of collision is negligible

(b) Use the expression in (a) above to deduce Dalton's law of partial pressures. (03marks)

P = $\frac{1}{3}N\frac{m}{V}\overline{c^2} = \frac{2}{3}N\left(\frac{1}{2}m\overline{c^2}\right)$

For gas 1, $P_1V_1 = \frac{2}{3}N_1\left(\frac{1}{2}m_1\overline{c_1^2}\right)$

=> $N_1 = \frac{3}{2}P_1V_1.\frac{1}{K_1}$

Similarly for gas 2

$N_2 = \frac{3}{2}P_2V_2.\frac{1}{K_2}$

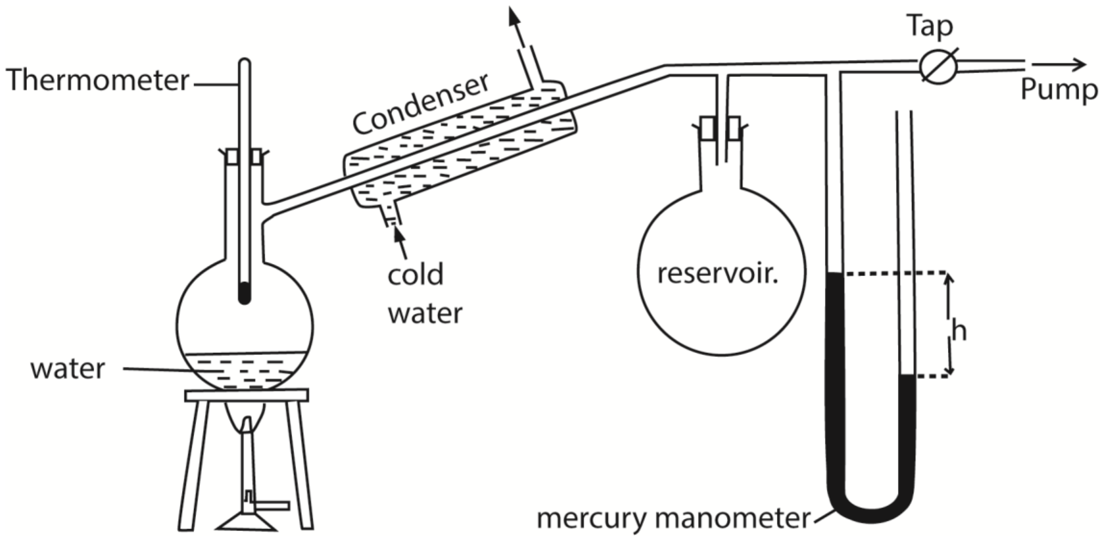
For a mixture of gases, N = $\frac{3}{2}PV.\frac{1}{K}$; but N = $N_1 + N_2$

$\frac{3}{2}PV.\frac{1}{K} = \frac{3}{2}P_1V_1.\frac{1}{K_1} + \frac{3}{2}P_2V_2.\frac{1}{K_2}$

Since temperature is constant, $K_1 = K_2 = K$

- $PV = P_1V_1 + P_2V_2$
- But $V = V_1 = V_2$
- $\therefore P = P_1 + P_2$

(c) Describe an experiment to determine the saturation vapor pressure of a liquid. (06marks)



- The pressure of the air in R is shown by the mercury manometer; if its height is h, the pressure in mm mercury is P = H-h, where H is the barometer height.
- The tap is opened and the pressure above water varied using the pump to a suitable value.
- The tap is closed and water in the flask in heated until it boil.
- The temperature θ and difference in mercury levels, h, are noted and recorded.
- The saturated vapour pressure, P = (H$\pm h$) is calculated
- The procedure is repeated other values of θ and h
- A graph of P versus θ is plotted and the saturated vapour pressure at a particular temperature is obtained.

(d) (i) What is meant by a reversible isothermal change? (02marks)

The change taking place at constant temperature and can be taken back from the final to initial states through exactly the same values of pressure and volume at every stage.

(ii) State the conditions for achieving a reversible isothermal change. (02marks)

Use vessels with thin good conducting walls having a frictionless piston, surrounded by constant temperature bath and the process must occur slowly.

(e) An ideal gas at 27^0C and at a pressure of 1.01 x 10^5Pa is compressed reversibly and isothermally until its volume is halved. It is then expanded reversibly and adiabatically to twice its original volume. Calculate the final pressure and temperature of the gas if γ=1.4 (05marks)

For isothermal: P$\frac{V}{2} = 1.01\ x\ 10^5 V$; P = 2.02 x 10^5Pa

For adiabatic; 2.02 x 105$\left(\frac{V}{2}\right)^{1.4}$ = $P_1(2V)^{1.4}$; P_1 = 2.9 x 10^4Pa

Final pressure = 2.9 x 10^4Pa

Also,

$TV^{\gamma-1}$ = constant.

⇨ $(300.15)\left(\frac{V}{2}\right)^{0.4} = (2V)^{0.4}$; T = 172K

20. (a) (i)Explain what happens when a quantity of heat is applied to a fixed mass of a gas (02marks)

Heat supplied increase internal energy of the gas and used to overcome external pressure during the expansion.

(ii) Derive the relationship between the principal molar heat capacities C_p and C_v for an deal gas. (05marks)

From dQ = dU + dW……………. (i)

But dQ = C_pdT, dU = C_vdT and dW= PdV = RdT

Substituting in (i)

$C_p dT = C_v dT + RdT$

$\therefore C_p - C_v = R$

(b) (i) What is adiabatic process? (01mark)

An adiabatic process is one in which no heat is added or removed from the system.

(ii) A bicycle pump contains air at 290K. The piston of the pump is slowly pushed in until the volume of the air enclosed is one fifth of the total volume of the pump. The outlet is sealed off and the piston suddenly pulled out to full extension. If no air escapes, find its temperature immediately after pulling the piston. $\left(Take\ {}^{C_p}\!/_{C_v} = 1.4\right)$ (03marks)

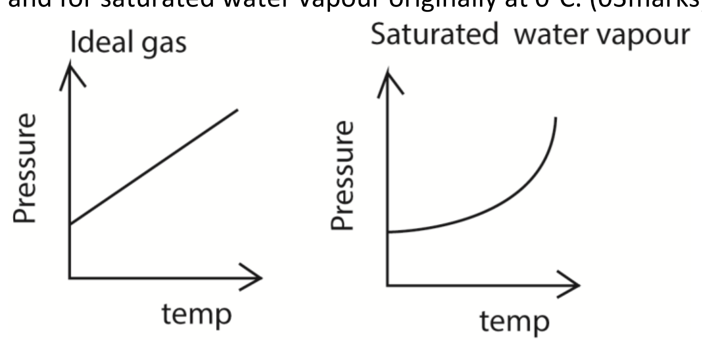
$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$

$T_2 = 290\left(\frac{5V}{V}\right)^{0.4}$ = 152K

(c) (i) Distinguish between unsaturated and saturated vapors. (02marks)

Unsaturated vapour is a vapour that is not in dynamic equilibrium with its own liquid while saturated vapour is a vapour that is in dynamic equilibrium with its own liquid.

(ii) Draw graphs to show the relationship between pressure and temperature for ideal gas and for saturated water vapour originally at 0^0C. (03marks)



(d) In an experiment, the pressure of a fixed mass of air at constant temperature is 10.4kPa. When the volume is halved, keeping the temperature constant, the pressure becomes 19.0kPa. Discuss the applicability of the above results in verifying Boyle's law. (04marks)

From PV = constant

Halving the volume ta constant temperature doubles pressure. Since pressure was not doubled, Boyles's law is not verified.

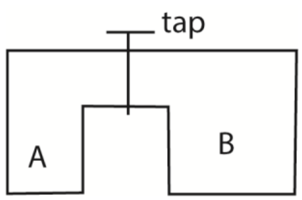
21. (a) (i) State Boyles law. (01mark)

The pressure of a fixed mass of a gas is inversely proportional to volume.

(ii) What is meant by partial pressure of a gas? (01 mark)

Partial pressure is the pressure that would be exerted by a gas if it alone occupied the volume of the mixture.

(iii)



Two cylinders A and B of volumes V and 3V respectively are separately filled with a gas. The cylinders are connected as shown above with the tap closed. The pressures of A and B are P and 4P respectively. When the tap is opened the common pressure becomes 60Pa. Assuming isothermal conditions find the value of P. (04marks)

Solution

From PV = nRT

Moles n_1 of the gas in A before mixing= $\frac{PV}{RT}$

Moles n_2 of the gas in B before mixing = $\frac{4P \; x \; 3V}{RT} = \frac{12PV}{RT}$

Moles n_3 of the gas when tap is opened = $\frac{60 \; x \; 4V}{RT}$

But moles of the gas before mixing = mole of the gas after mixing

⇨ $n_1 + n_2 = n_3$

$\frac{PV}{RT} + \frac{12PV}{RT} = \frac{60 \; x \; 4V}{RT}$

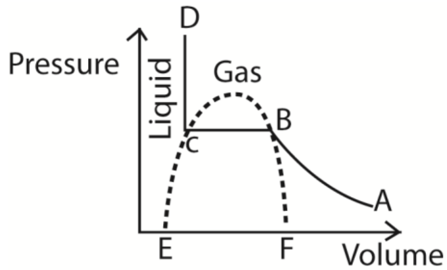
$13P = 240$

P = 18.46Pa

(b) (i) State three differences between ideal and real gases. (03marks)

Real gas	Ideal gas
Intermolecular force are appreciable	Intermolecular forces are negligible
Volume of molecules compared to the volume of the container is not negligible	Volume of molecules compared to the volume of container is negligible
Obey Boyle's law at high temperature and very low pressure	Obey Boyle's law at all temperatures and pressures.

(ii)Sketch a pressure versus volume curve for a real gas undergoing compression below its critical temperature. (01mark)



(iii) Explain the main features of the curve in (b)(ii) above (03marks)

- AB represents unsaturated vapour that approximately obey Boyle's law.
- BC represents saturated vapour, the gas turns into a liquid at constant pressure.
- CD is a liquid, small decrease in volume leads to a big increase in pressure because liquids are incompressible

(c) Two similar cylinders P and Q contain different gases at the same pressure. When gas is released from P the pressure remains constant for some time before it starts dropping. When gas is released from Q the pressure continuously drops. Explain the observation above. (05marks)

- The gas in P is in form of a saturated vapour; that is, in dynamic equilibrium with a liquid. As the gas is released, more liquid turns into a gas to restore pressure until the gas becomes unsaturated and the pressure begins to drop as the moles of the gas decrease
- The gas in Q is unsaturated, and thus pressure reduces as the moles of the gas reduces up on release.

(d) Using the expression for the kinetic pressure of an ideal gas, deduce the ideal gas equation of $\frac{1}{2}m\overline{c^2} = \frac{3}{2}K_BT$ (02marks)

Given $\frac{1}{2}m\overline{c^2} = \frac{3}{2}K_BT$

From P = $\frac{1}{3}\rho\overline{c^2}$ = $\frac{1}{3}x\ \frac{M}{V}\overline{c^2}$

PV = $\frac{1}{3}M\overline{c^2} = \left(\frac{1}{2}M\overline{c^2}\right)x\ \frac{2}{3}$ = $\frac{3}{2}K_BT\ x\ \frac{2}{3}$ = K_BT = Constant

$\therefore PV = constant$

Thank you
Dr. Bbosa Science