SENIOR FIVE TERM 3

TOPIC 1/6: REFLECTION OF LIGHT

Competency: The learner investigates the light reflecting properties of different surfaces and their applications in real life.

Optics

Optics is the study of light energy and its properties

Evidences that light is a form of energy

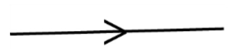
(i) When light enters our eyes, it enables us to see.

(ii) When light is absorbed by green plants, it enables then to make their food by photosynthesis

(iii) When light is incident on certain materials, the surface electrons acquire energy and escape. This process is called photoelectric emission.

Light rays

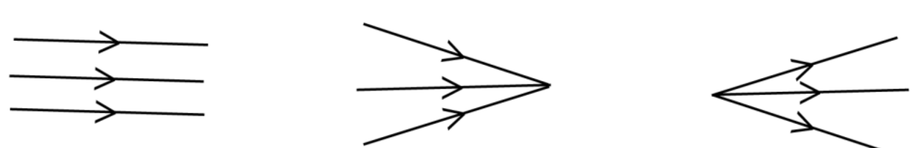
The path taken or indicating the direction along which light energy travels is known as a ray of light. A ray is indicated with an arrow.



ray of light

Beam of light

A group of light is called a beams. There are three types of beams namely



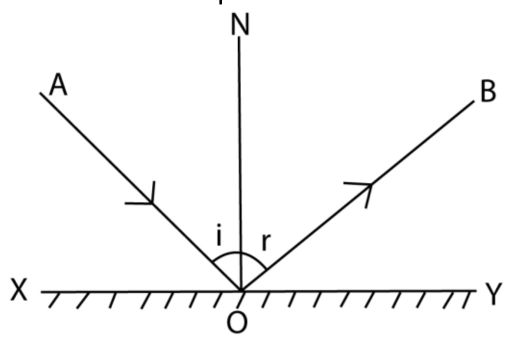
(a) Parallel beam (b) Converging beam (c) Diverging beam

Reflection of light at a plane mirror

Reflection is the bouncing back of light energy when it meets an obstacle

Laws of reflection of light

Consider a ray of light AO incident on a plane surface and then reflected along OB as shown.



O = point of Incidence.

AO = incident ray

OB = reflected ray.

ON = normal to the reflecting surface

<i = angle of incidence

<r =angle of reflection

LAW 1:

The incident ray, the reflected ray, and the normal at the point of incidence all lie in the same plane.

LAW 2:

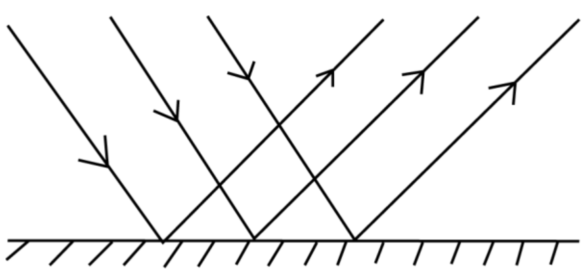
The angle of incidence is equal to the angle of reflection.

Example 1

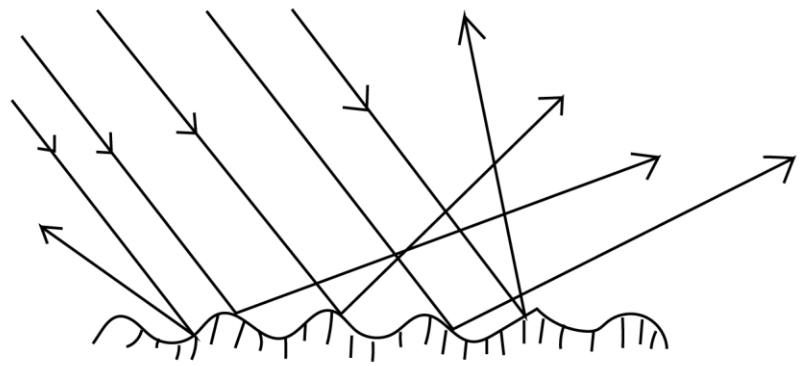
State laws of reflection

Types of reflection:

(i) **Regular reflection**: This occurs when a parallel beam of light incident on a smooth surface such as a plane mirror gets reflected as a parallel beam as shown.



(ii) **Diffuse /irregular reflection**: This occurs when a parallel beam of light incident on a rough surface such as a paper gets reflected while scattered in different directions as shown



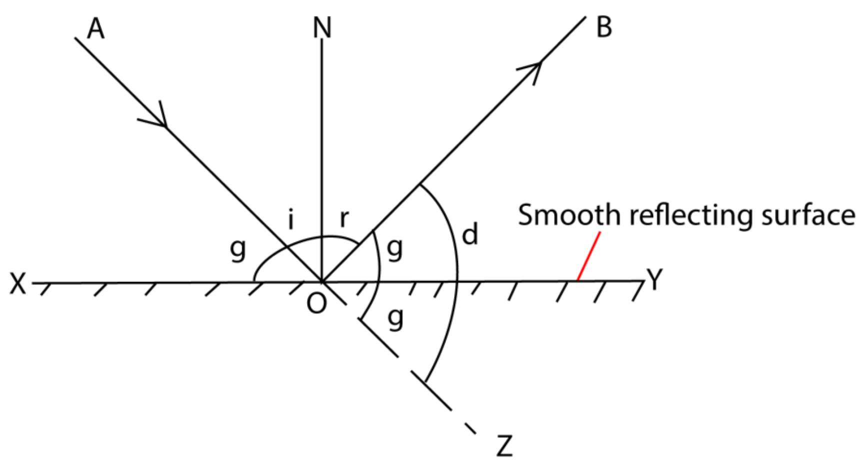
Differences between regular and irregular reflection

Regular reflection	**Irregular reflection**
Occurs on smooth surface	Occurs on a rough surface
Parallel incident beam is reflected parallel	Parallel incident beam is scattered after reflection
Reflected beam is very bright	Reflected beam is dull

Deviation of light at plane mirror surfaces

This is the property of a plane mirror to deviate incident light from its original direction to another.

Let g be the glancing angle made by the ray AO with the mirror XY as shown



d = ZOB
d = g + YOB
YOB = (90-r)
but r =i (laws of reflection)
➔ (90 – r) = (90 – i) = g
∴ d = g + g
d = 2g

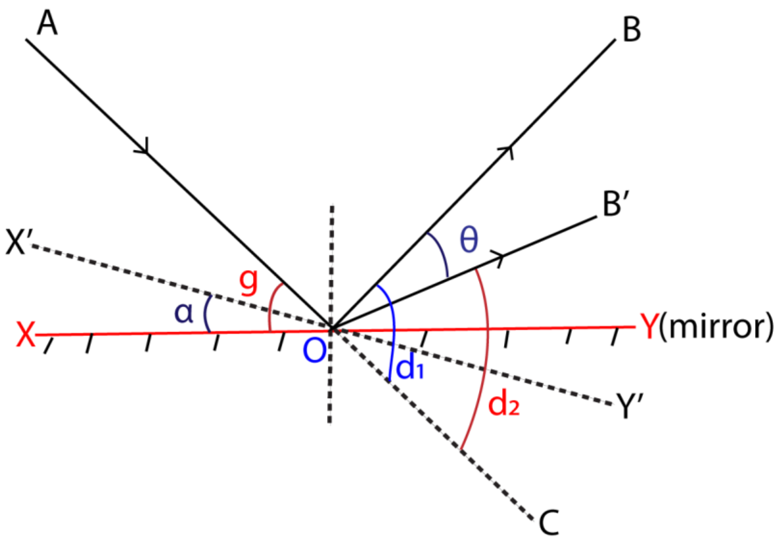
Hence, the angle of deviation of a ray by a plane surface is twice the glancing angle

Example 2

Prove that a ray incidence on a plane mirror, d = 2g. where d is the deviation and g is the glancing angle.

Deviation of reflected ray by rotated mirror

Let XY be the initial position of the mirror with ray AO making a glancing angle g. By keeping the direction of the incident ray fixed, the mirror is rotated through an angle **α** to a new position X'Y' as shown.



Case 1 (mirror in position XY)

Glancing angle = g

Deviation d_1 = 2g……… (i)

Case 2**(mirror in position X'Y')**

Glancing angle = (g – α)

Deviation d_2 = 2(g – α) …..(ii)

$\theta = d_1 - d_2$

$= 2g - 2(g - \alpha)$

$= 2g - 2g + 2\alpha$

$= 2\alpha$

Hence, if the direction of incident ray is constant, a reflected ray rotates through an angle = 2α, when the mirror is rotated through an angle α.

This is applied in optical lever in mirror galvanometer where a beam of light serves as a pointer.

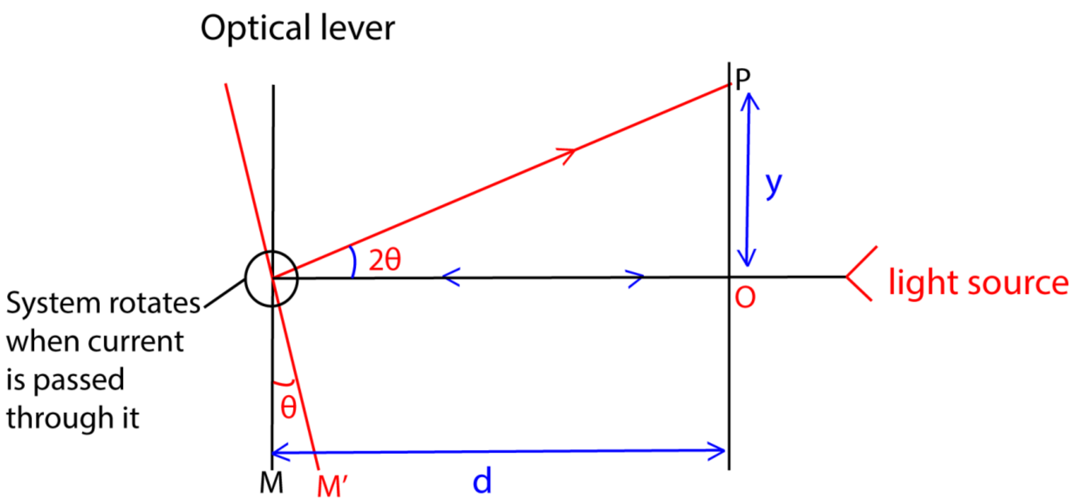
Advantages of optical lever to metallic pointer

(i) Its weight is negligible hence highly sensitive

(ii) Magnifies the angle of rotation.

(iii) Can be used to measure very small electric current and small changes in length due to expansion and contraction of solid.

Action of optical lever



(i) A beam of light from the light source is normal to the mirror and is reflected back through the same direction at O

(ii) Current through the galvanometer causes the coil to rotate the mirror through an angle θ to position M' and the beam of light is reflected through angle 2θ and deflected to position P by displacement y.

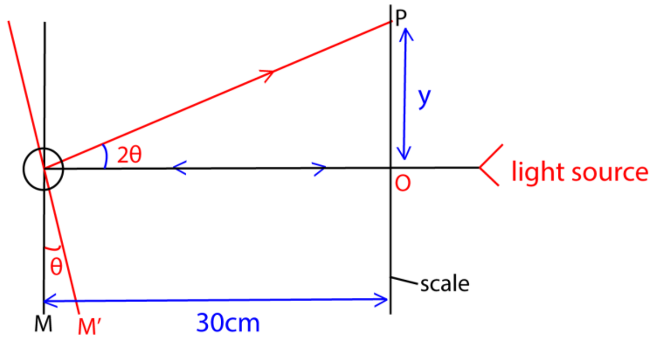
Example 3

(a) Describe briefly the action of optical lever.

(b) An optical galvanometer of sensitivity 0.05 radian per M.A is used to measure current of 0.2A. the distance of the cell from the linear scale is 30cm. Find the displacement of light spot on the scale from the initial position.

Solution

Sketch



$360^0 = 2\pi$ radian

1MA of current through the coil turns the mirror through an angle 0.05radian

It implies that, 0.02MA turns the mirror through 0.05 x 0.2 = 0.01radians

The reflected ray turns through $2\theta = 2 \times 0.01 = 0.02$ radians

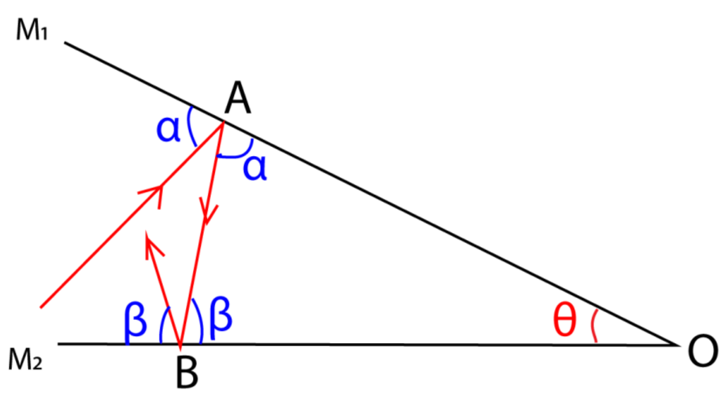
But $\tan 2\theta = y/30$

$\tan 0.02 = y/30$;

$y = 0.6$cm

Deviation by successive reflections at two inclined mirrors

Consider an incident ray of light reflected successively from two mirrors M_1 and M_2 inclined at an angle θ to each other at O as shown



Let the glancing angles at A and B be ▯ and ▯ respectively.

Deviation by M_1 = 2α (clockwise direction)

Deviation by M_2 = 2β (clockwise direction)

Total deviation = $2\alpha + 2\beta$

= $2(\alpha + \beta)$ ----------------------(i)

But, $\alpha + \beta + \theta = 180^0$ (Angle sum of a triangle)

=> $\alpha + \beta = (180^0 - \theta)$ ---------------------(ii)

Combining equation (i) and (ii) gives

Total deviation = $2(180^0 - \theta)$ (clockwise direction)

= $360^0 - 2\theta$ (clockwise direction)

= 2θ (anti-clockwise direction)

Thus, the deviation produced by two inclined mirrors is twice the angle between the mirrors when a ray under goes two successive reflections.

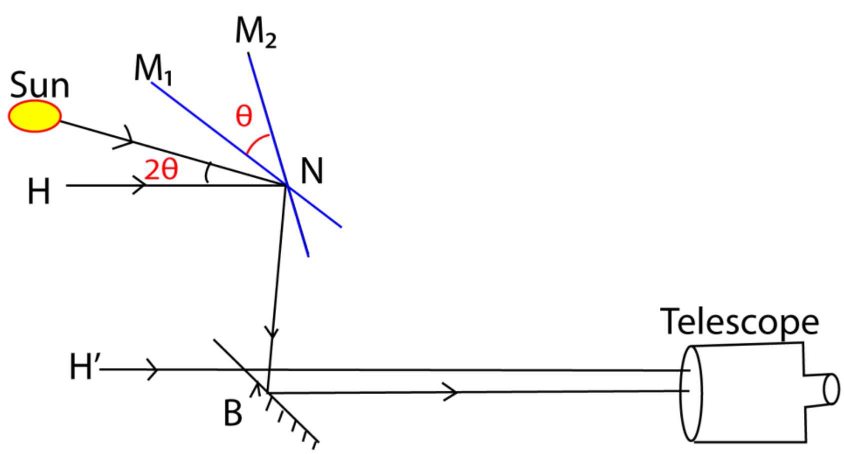
NOTE

(i) Clock wise deviation **(360^0 - 2** θ) **+** anti-clockwise deviation (**2** θ**)** **=** **360^0**
(ii) The above result finds application in the sextant, a device for measuring the angle of elevation of the sun or stars.

The sextant
A sextant is an instrument used in navigation for measuring the angle elevation of the sun or stars or the angle between two objects in space. It works on the principle of two inclined mirrors. The estimation of this angle, is known is sighting or shooting the object, or taking a sight.

Principle of a Sextant

The principle of a sextant is that, when the ray of light is reflected from two mirrors in succession in the same plane, then the angle between the incident and reflected ray is two times the angle between the mirrors.



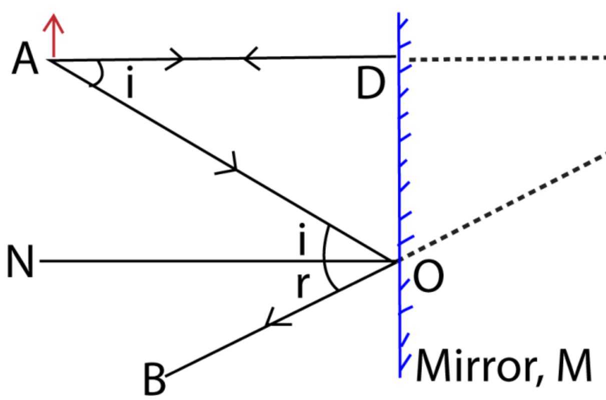
(i) A sextant consists of a fully silvered mirror M1which can be rotated about a horizontal axis and a fixed half silvered mirror B.
(ii) Mirror M1 is adjusted to become parallel to B by rotating it until the image of the horizon, H' is seen directly through the un silvered part of mirror B. Note that, if the two mirrors are parallel, the incident ray from any observed body must be parallel to the observer's line of sight through the horizon glass
(iii) H is the horizon as seen by successive reflection by successive reflection in mirror M_1 and B respectively
(iv) The mirror M_1 is rotated to position M_2 such that the image of the horizon H, sun coincides with H'
(v) The angle of rotation is measured from the scale on the instrument. The elevation of the sun is 2θ.

Example 4

Describe briefly how the elevation of the sun can be determined using the principle of the sextant or briefly describe the operation of the sextant.

Image formation in plane mirrors

Consider an object A placed in front of a mirror **M**.



A ray AD from A incident normally on the mirror at D is reflected back along DA. Thus this reflected ray appears to come from a point **I** behind the mirror. The intersection **I** of the rays AD and BO is the image position**.**

From above,

<DAO = <AON -------------------------- (alternating angles)

<AON = < NOB ------------------------- (2nd law of reflection)

<NOB = < DIO --------------------------- (corresponding angles)

Combining all the equations gives

<DAO = <AON = < NOB = < DIO

=> <DAO = < DIO

tan < DAO = tan < DIO

$$\frac{DO}{AD} = \frac{DO}{ID}$$

Thus AD = ID.

∴**The image is as far behind the mirror as the object is in front**

CHARACTERISTICS OF IMAGES FORMED BY PLANE MIRRORS

(i) It is virtual
(ii) It is erect
(iii) It is laterally inverted
(iv) It is of the same size as the object
(v) It is as far behind the mirror as the object is in front

Uses of plane mirrors

(i) Dressing mirrors
(ii) In telescope
(iii) In periscopes

Number of images formed by two plane mirror inclined at an angle, θ

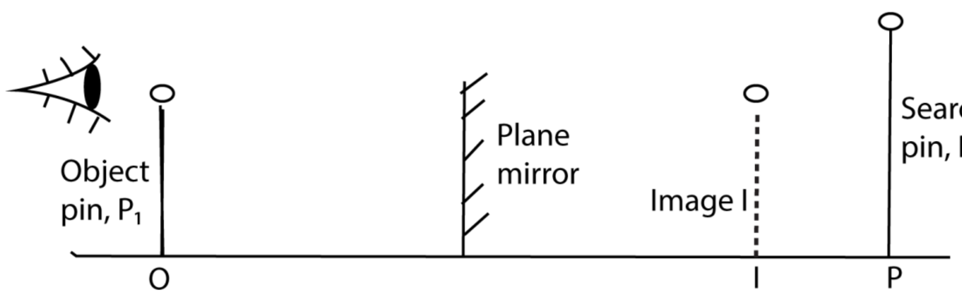
Two plane mirrors inclined at an angle θ to each other. The number of images formed is given by the formula

$n = \frac{360}{\theta} - 1$

Example 4

What is the number of images formed between two mirrors inclined at 600 to each other? (5)

Image location by no parallax



An object pin **O** is placed in front of a plane mirror to form a virtual image **I**.

A large search pin **P** is placed behind the mirror and moved to such a position that there is no parallax between pin **P2** and the image **I**.

The image position has therefore been located.

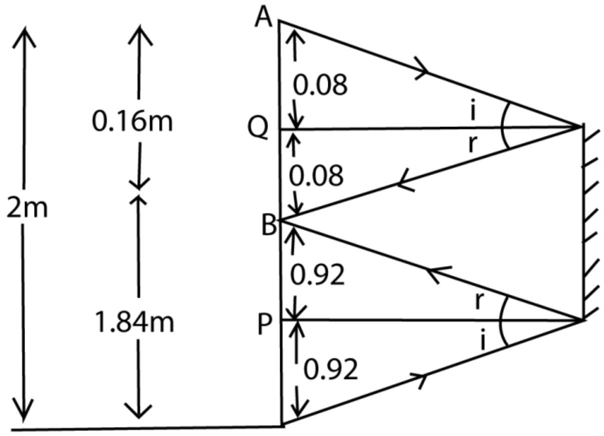
NOTE

Parallax is the perceived change in position of an object seen from two different places

No parallax method of locating an image involves an image coinciding in positionwith a search pin such that when the observer's head is moved sideways, both the image and the search pin appear to move in the same direction. This is when there is no relative motion between them.

Example 5

A man 2m tall whose eye level is 1.84m above the ground looks at his image in a Vertical mirror. What must be the minimum vertical length of the mirror so that the man can see the whole of himself completely in the mirror?



Rays from the top of the man are reflected from the top of the mirror and are incident in the man's eyes (point **B** is the man's eye level)

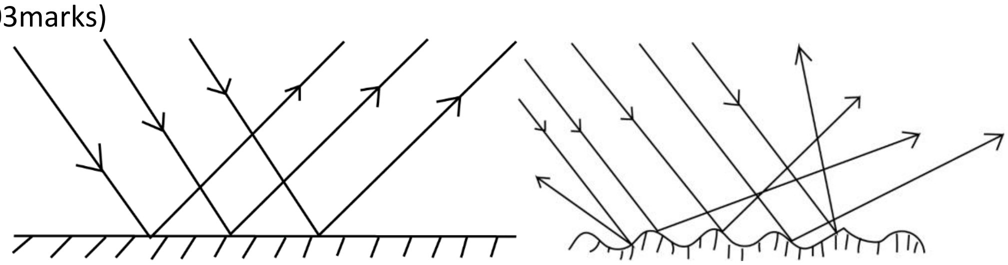
Since AQ = BQ then, BQ = $\frac{1}{2}$ x 0.16 = 0.08m
Similarly BP = PC.
Thus BP = $\frac{1}{2}\ x$ 1.84 = 0.92m
The minimum length of the mirror = BQ + BP
= 0.08 + 0.92
= 1m

Comparison of plane mirrors and reflecting prisms

(i) Un like in prisms, plane mirrors produce multiple images
(ii) The silvering in plane mirrors wears out with time while no silvering is required in prisms
(iii) Unlike in prisms, plane mirrors exercise loss of brightness when reflection occurs at its surface.

Revision exercise 1

1. (a) (i) With the ais of digrams, differentiatebetween regular and irregular refelection (03marks)



Regular reflection Irregular reflection

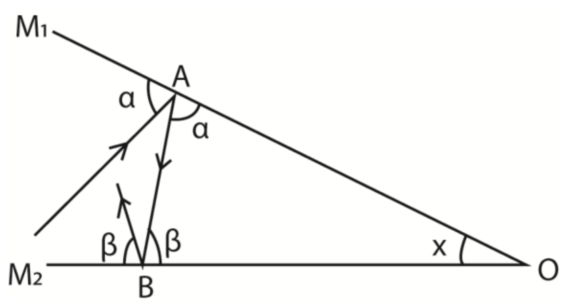
Differences between regular and irregular reflection

Regular reflection	Irregular reflection
Occurs on smooth surface	Occurs on a rough surface
Parallel incident beam is reflected parallel	Parallel incident beam is scattered after reflection
Reflected beam is very bright	Reflected beam is dull

(ii) Show that if two plane mirrors are inclined at an angle, x, then the total deviation of a light ray produced is 2x after reflection once in each mirror. (03 marks)

Deviation by successive reflections at two inclined mirrors

Consider an incident ray of light reflected successively from two mirrors M_1 and M_2 inclined at an angle x to each other at O as shown



Let the glancing angle at A and B be α and β respectively

Deviation by M_1 = 2α (clockwise direction)

Deviation by M_2 = 2β (clockwise direction)

Total deviation = 2α + 2β

= 2(α + β) ………………… (i)

But α + β + x = 180 (Angle sum of a triangle)

α + β = 180 – x …………………….. (ii)

Combining (i) and (ii)

Total deviation = 2(180 – x) (clockwise)

= 360 - 2x (clockwise)

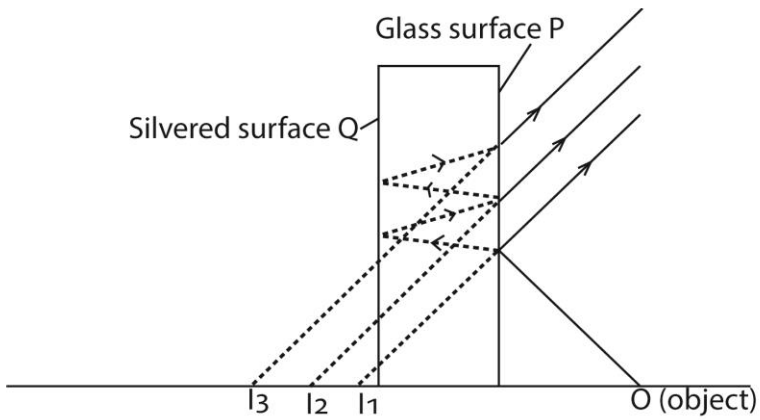
= 2x (anticlockwise)

2. (a) (i) State two differences between real and virtual images (02marks)

A real image is formed by actual intersection of rays and can be formed on a screen while virtual image is formed by apparent intersection of rays and cannot be formed on a screen.

(ii) Explain with the aid of a diagram how thick plane mirror forms multiple images (04marks)

Formation of multiple images in thick plane mirror



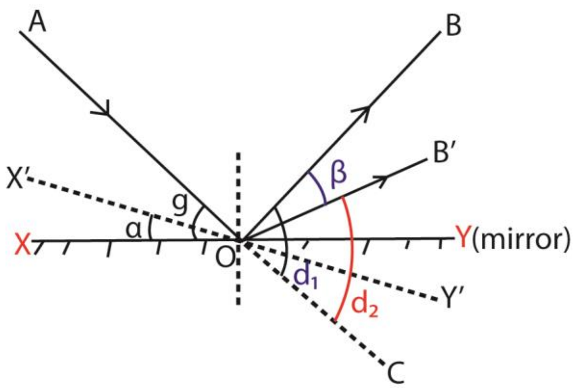
Multiple images are formed due to partial reflection and refraction at the non-silvered surface of the mirror.

- İmage I_1 is formed by reflection on the glass surface P
- The image I_2 (the brightest is formed by reflection of the most light on the silvered surface Q
- Others by partial refraction

3. (a) (i) State the laws of reflection of light. (02marks)
   - The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane
   - The angle of incidence is equal to the angle of reflection

(ii) A ray of light is incident on a plane mirror. The mirror is then turned through an angle α keeping the direction of the incident ray constant. If a reflected ray turned through angle β, find the relationship between α and β.

Let XY be the initial position of the mirror with ray AO making a glancing angle g. By keeping the direction of the incident ray fixed, the mirror is rotated through an angle **α** to a new position X'Y' as shown.



Case 1 (mirror in position XY)

Glancing angle = g

Deviation d_1 = 2g……… (i)

Case 2**(mirror in position X'Y')**

Glancing angle = (g – α)

Deviation d_2 = 2(g – α) …..(ii)

$\beta = d_1 - d_2$

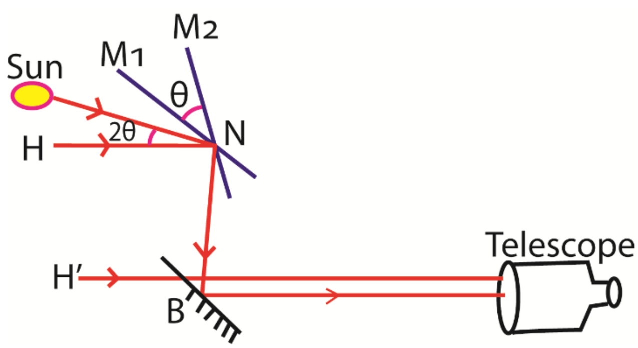
$= 2g - 2(g - \alpha)$

$= 2g - 2g + 2\alpha$

$= 2\alpha$

(b) Describe how a sextant is used to determine the angle of elevation of a star. (05marks)

Setup



(i) A sextant consists of a fully silvered mirror M1which can be rotated about a horizontal axis and a fixed half silvered mirror B.

(ii) Mirror M1 is adjusted to become parallel to B by rotating it until the image of the horizon, H' is seen directly through the unsilvered part of mirror B by successive reflection in mirror M_1 and B respectively

(iii) The mirror M_1 is rotated to position M_2 such that the image of the horizon H, and the sun coincides at H'

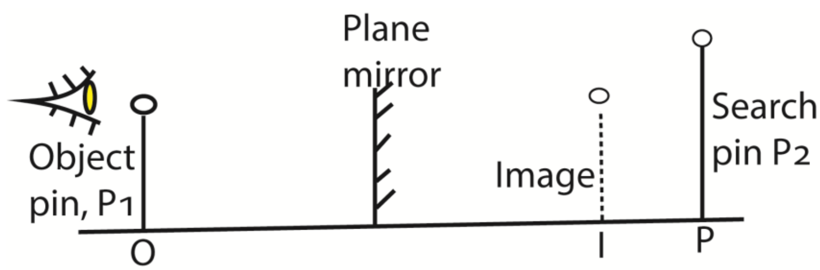
(iv) The angle of rotation is measured from the scale on the instrument. The elevation of the sun is 2θ.

4. (a) (i) Distinguish between real and virtual images. (02marks)

A real image is one formed by actual intersession of rays and can be formed on the screen while a virtual image is formed by apparent intersession of rays and cannot be formed on a screen.

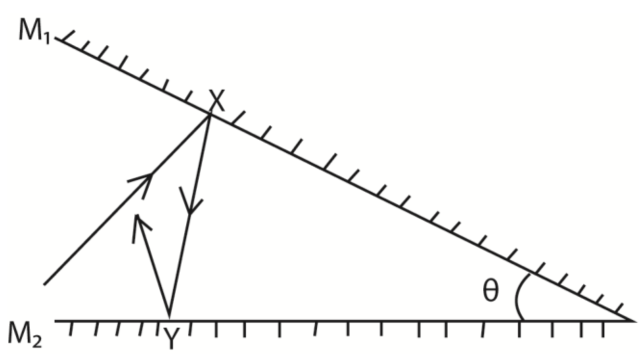
(ii) Describe how the position of an image in a plane mirror can be located (03marks)

Image location by no parallax



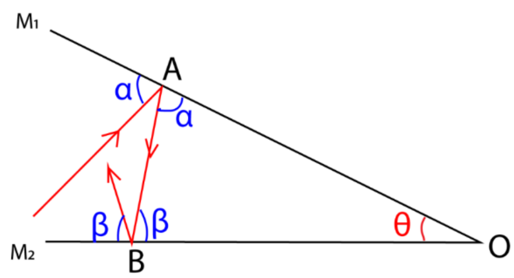
- An object pin **O** is placed in front of a plane mirror to form a virtual image **I**.
- A large search pin **P** is placed behind the mirror and moved to such a position that there is no parallax between pin **P2** and the image **I**.
- The image position has therefore been located.

(b) The diagram in the figure below shows a ray of light undergoing two successive reflections at points X and Y in two mirrors M_1 and M_2 inclined at an angle θ.



Show that the ray is deviated through an angle 2θ (03marks)

Consider an incident ray of light reflected successively from two mirrors M_1 and M_2 inclined at an angle θ to each other at O as shown



Let the glancing angles at A and B be α and β respectively.

Deviation by $M_1 = 2\alpha$ (clockwise direction)

Deviation by $M_2 = 2\beta$ (clockwise direction)

Total deviation $= 2\alpha + 2\beta$

$= 2(\alpha + \beta)$ ---------------------- (i)

But, $\alpha + \beta + \theta = 180^0$ (Angle sum of a triangle)

⇨ $\alpha + \beta = (1800 - \theta)$ ---------------------(ii)

Combining equation (i) and (ii) gives

Total deviation $= 2(180^0 - \theta)$ (clockwise direction)

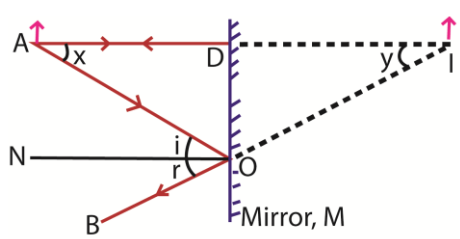
$= 360^0 - 2\theta$ (clockwise direction)

= 2θ (anti-clockwise direction)

5. (a) (i) State the laws of reflection (02marks)
   - Incident ray, refracted ray and the normal at the point of incidence all lie in the same plane
   - Angle of reflection is equal to the angle of incidence

   (ii) Show that the image formed in a plane mirror is as far behind the mirror as the object is in front. (04marks)

Consider an object A placed in front of a mirror **M**.



A ray AD from A incident normally on the mirror at D is reflected back along DA.

The Ray AO is reflected along OB and appears to come from point I behind the mirror.

The intersection **I** of the rays AD and BO is the image position.

From above angles,

x = i (alternating angles)

i = r (2^{nd} law of reflection)

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Combining all the equations gives

x = i = r = y

⇨ angle x = angle y

Tan x = t a n y

$$\therefore \quad \frac{DO}{AD} = \frac{DO}{DI}$$

Thus AD = ID.

∴ The **image is as far behind the mirror as the object is in front**

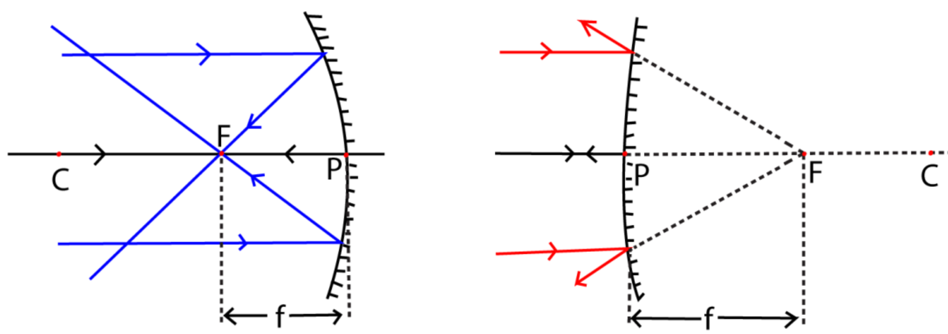
Reflection at curved mirrors

Curved mirrors are of two types namely;

(i) Concave (Converging) mirror: it is part of the sphere whose centre **C** is in front of its reflecting surface.

(ii) Convex (Diverging) mirror: it is part of the sphere whose centre **C** is behind its reflecting surface.

Consider the reflection of a parallel narrow beam of light at curved mirrors as shown.



Concave mirror

Convex

Definitions

1. **Centre of curvature C:** it is the centre of the sphere of which the mirror forms part.
2. **Radius of curvature r:** it is the radius of the sphere of which the mirror forms part.
3. **Pole of the mirror:** it is the mid-point (centre) of the mirror surface.
4. **Principal axis CP**: it is the line that passes through the centre of curvature and the pole of the mirror.
5. **Secondary axes:** These are lines parallel to the principal axis of the mirror.
6. **Paraxial rays:** These are rays drawn close and parallel to the principal axis
7. **Marginal rays:** These are parallel rays furthest from the principal axis of the mirror.
8. **(i) Principal focus "F" of a concave mirror:** it is a point on the principal axis where paraxial rays incident on the mirror and parallel to the principal axis converge after reflection by the mirror.

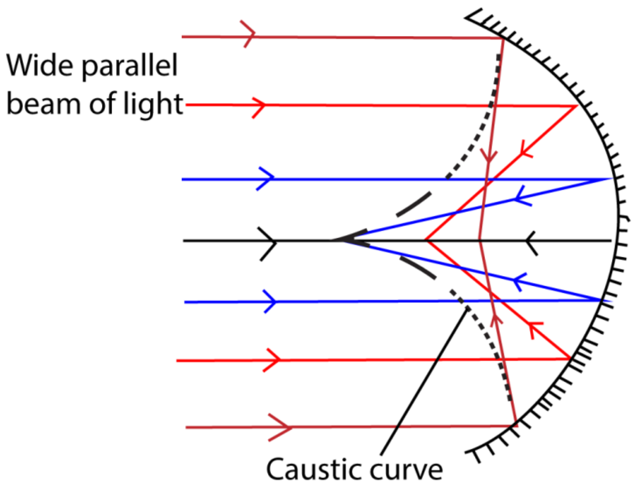
   (ii). Principal focus "F" of a convex mirror: it is a point on the principal axis where paraxial rays incident on the mirror and parallel to the principal axis appear to diverge from after reflection by the mirror
9. **(i) Focal length "f" of a concave mirror:** it is the distance from the pole of the mirror to the point where paraxial rays incident and parallel to the principal axis converge after reflection by the mirror.

   (ii) Focal length "f" of a convex mirror: it is the distance from the pole of the mirror to the point where paraxial rays incident and parallel to the principal axis appear to diverge from after reflection by the mirror.

10. Aperture of the mirror: it is the length of the mirror surface.

Reflection of a parallel wide beam of light at curved mirrors

Consider the reflection of a wide parallel beam of light incident on a concave mirror as shown.



When a wide parallel beam of light is incident on a concave mirror, the different reflected rays are converged to different points. However these reflected rays appear to touch a surface known as a **caustic surface** having a cusp (an apex) at the principal focus **F**.

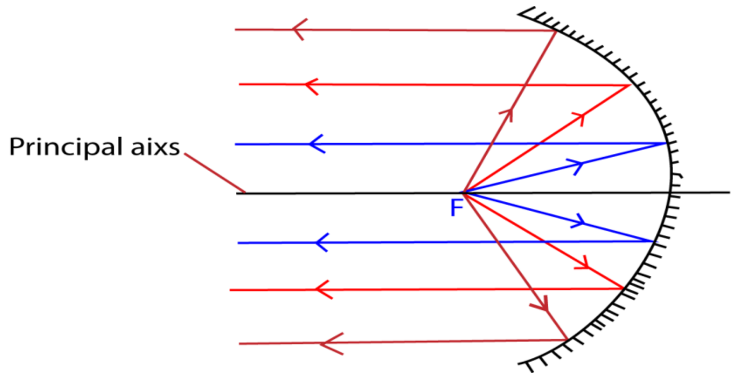
Note

(i) The marginal rays furthest from the principal axis are converged nearer to the pole of the mirror than the paraxial rays.

(ii) Similarly, if a wide parallel beam of light is incident on a convex mirror, the different reflected rays appear to have diverged from different points.

Hyperbolic mirror

This is a type of concave mirror that reflects all the parallel incident beam on to a unique point F, the principal focus of the mirror.



Note that in car headlight and torches, the reflecting surfaces are hyperbolic. When a bulb is placed at the principal focus of reflecting surface, a strong parallel beam of light is obtained. Hyperbolic mirrors are also used in reflecting telescopes.

Example 6

Explain why parabolic surface is preferred to concave surface in car headlights?

Solution

Parabolic mirror reflect a wide beam of light of high intensity from a lamp at its principal focus.

Concave mirror reflect light of low intensity of light which diminish with distance since they do not reflect parallel beam when a lamp a lamp is placed at their principal focus.

Geometrical rules for the construction of ray diagrams

The following is a set of rules for easy location of the images formed by spherical mirrors

1. Rays parallel to the principal axis are reflected through the principal focus.
2. Rays through the principal focus are reflected parallel to the principal axis.
3. Rays passing through the centre of curvature are reflected back along their own paths.
4. Rays incident to the pole are reflected back, making the same angle with the principal axis.

NOTE:

(i) The normal due to reflection at the mirror surface at any point must pass through the centre of curvature.

(ii) The image position can be located by the intersection of two reflected rays initially coming from the object.

Real and virtual images

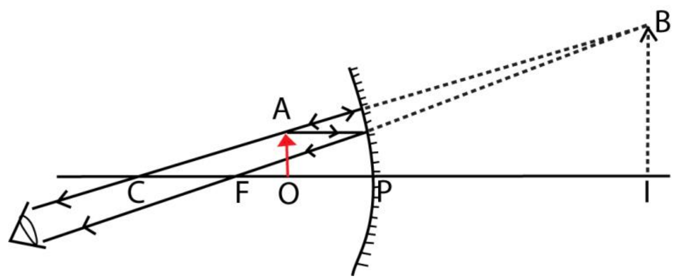
A real image: This is the image formed by the actual intersection of light rays from an object and can be received on the screen.

A virtual image: This is the image formed by the apparent intersection of light rays and cannot be received on the screen

Images formed by a concave mirror

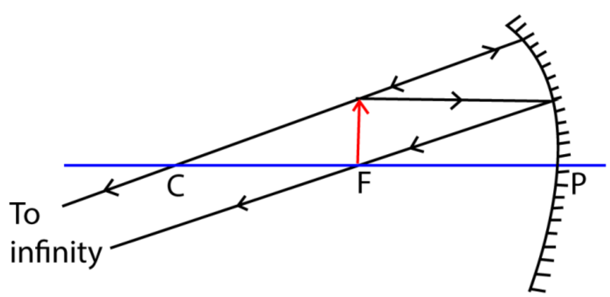
The nature of the image formed by a concave mirror is either real or virtual depending on the object distance from the mirror as shown below;

(i) **Object between F and P the image is**
- Behind the mirror
- Virtual
- Erect
- Magnified

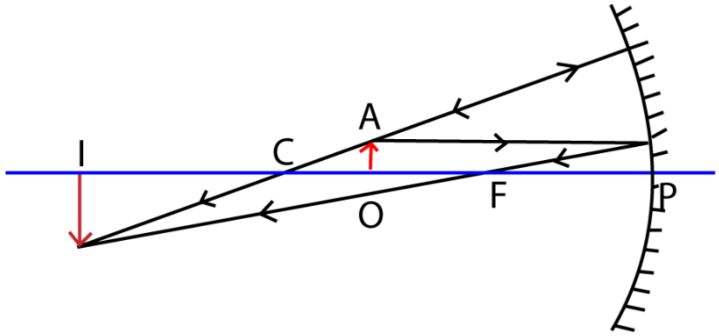


The property of a concave mirror to form erect, virtual and a magnified image when the object is nearer to the mirror than its focus makes it useful as a shaving mirror and also used by dentists for teeth examination.

(ii) **Object at F the image is**
- at infinity

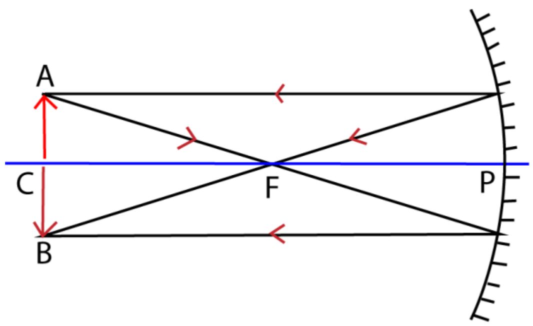


(iii) **Object between F and C the image is**
- Beyond C
- Real
- Inverted
- Magnified



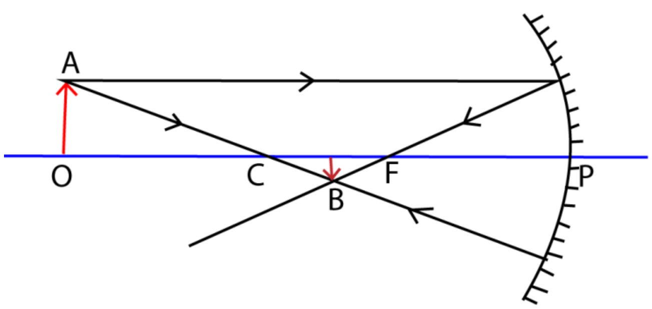
(iv) **Object at C the image is**

- At C
- Real
- Inverted
- Same size as the object



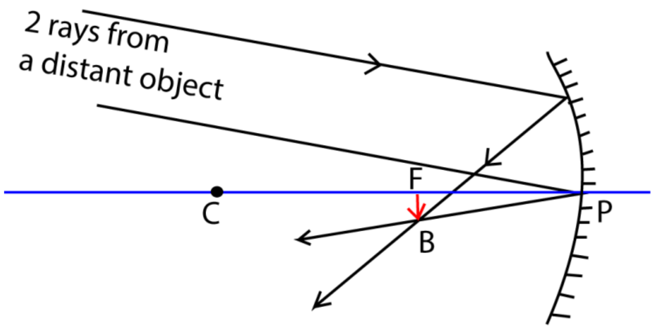
(v) **Object beyond C the image is**

- Between C and F
- Real
- Inverted
- Diminished



(vi) **Object at infinity the image is**

- At F
- Real
- Inverted
- Diminished



Note;

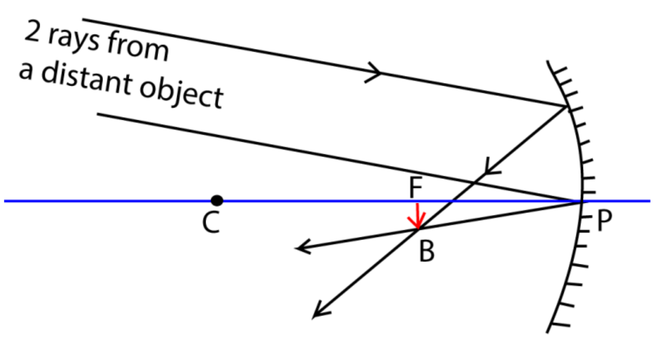
Generally the image of an object in a concave mirror is virtual only when the object is nearer to the mirror than its focus.

Uses of concave mirrors

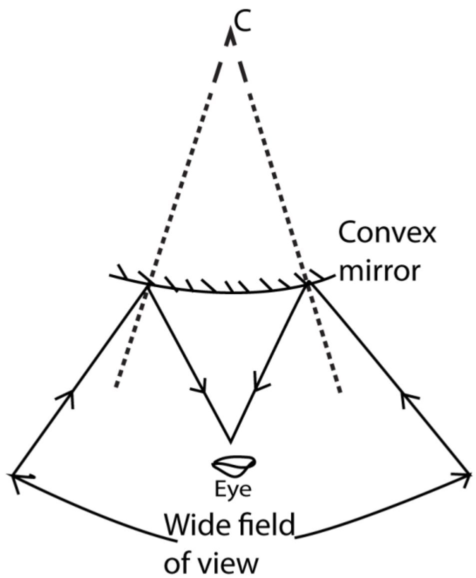
(i) They are used as shaving mirrors.
(ii) They are used by dentists for teeth examination.
(iii) They are used as solar concentrators in solar panels.
(iv) They are used in reflecting telescopes, a device for viewing distant objects
(v) They are used in projectors, a device for showing slides on a screen.

Images formed by a convex mirror

The image of an object in a convex mirror is erect, virtual, and diminished in size no matter where the object is situated as shown below



In addition to providing an erect image, convex mirrors have got a wide field of view as illustrated below.



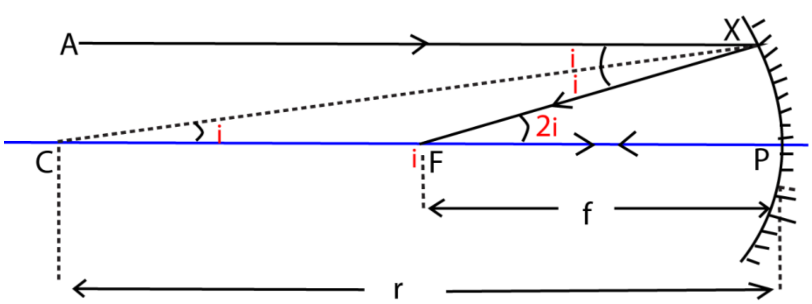
Uses of convex mirrors

(i) They are used as car driving mirrors

(ii) They are used in reflecting telescopes, a device for viewing distant objects

Relationship between the focal length f and the radius of curvature r of curved mirrors

Consider the reflection of a paraxial ray, AX, parallel to the principal axis of a concave mirror as shown.



Taking **FP** = focal length and **C** = centre of curvature, then , **CX**, is the normal to the mirror surface and, **CP**, is the radius of curvature.

$\tan 2i \approx 2i$

If, X, is close to P, then $\tan 2i \approx 2i = \frac{XP}{FP}$---------------- (for small angles)

=> $2i = \frac{XP}{FP}$ --(i)

Similarly tan i $\approx$ i = $\frac{XP}{CP}$

=> i = $\frac{XP}{CP}$ -- (ii)

Combining equations (i) and (ii) gives,

$$2\frac{XP}{CP} = \frac{XP}{FP}$$

Canceling XP throughout and simplifying for CP gives

CP = 2FP where CP = r, FP = f

$\therefore$ **r = 2f**.

Thus, the radius of curvature of a concave mirror is twice its focal length.

Note:

It can be shown that the relation between f and r holds for both concave and convex mirrors.**(This is left as an exercise for the reader)**

Mirror formula and sign convention

In order to obtain a formula which holds for both concave and convex mirrors, a Sign rule or convention must be obeyed and the following shall be adopted.

(i) Distances of real objects and images are positive.

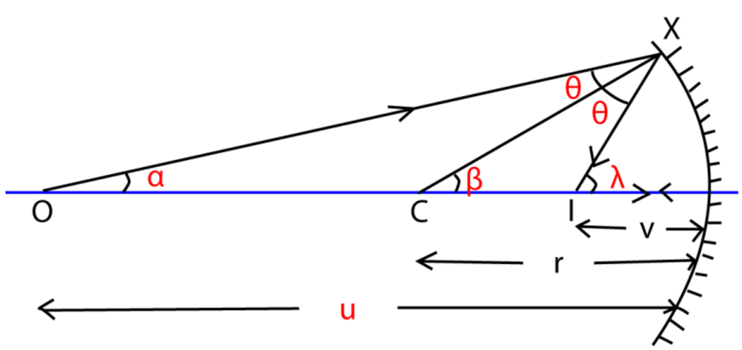
(ii) Distances of virtual objects and images are negative.

Note:

A concave mirror has a positive focal length while a convex mirror has a negative focal length.

Concave (converging) mirror formula

Consider the incidence of ray **OX** on to a concave mirror from a point object **O** placed along the principal axis that is reflected in the direction XI making an angle θ with the normal CX.



Ray OP strikes the mirror incident normally at P is reflected back along its own path. The point of intersection **I** of the two reflected rays is the image position.

From ΔOXC, $\alpha + \theta = \beta$ -------------------------------(i) From

Δ CXI, $\beta + \theta = \lambda$ ------------------------------(ii)

Equation (i) - Equation (ii) eliminates θ to give.

$\alpha - \beta = \beta - \lambda$

=> $\alpha + \lambda = 2\beta$ ---(a)

If X is very close to P, then

$\alpha \approx \tan\alpha = \frac{XP}{u}$, $\beta \approx \tan\beta = \frac{XP}{r}$, $\lambda \approx \tan\lambda = \frac{XP}{v}$

Equation (a) becomes = $\frac{XP}{u} + \frac{XP}{v} = \frac{2XP}{r}$

$= \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$

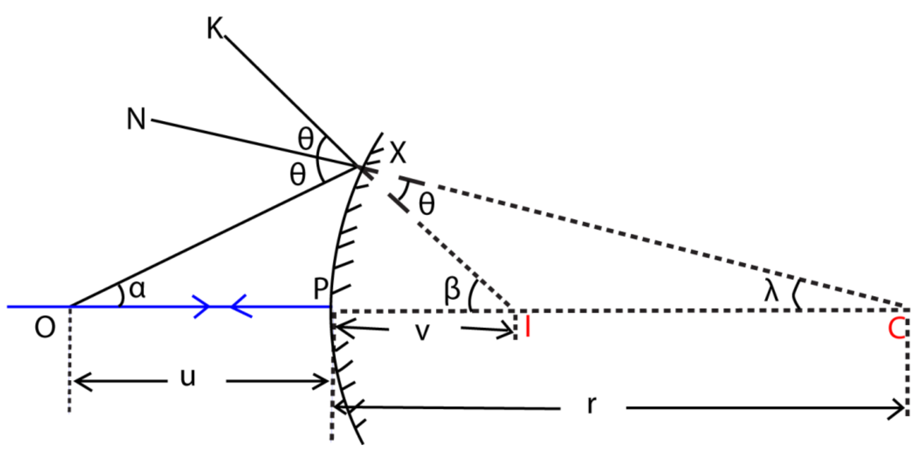
But 2 = 2f

$= \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Note:
The focal length of a concave mirror is a positive distance.

Convex (diverging) mirror formula

Consider the incidence of ray **OX** on to a convex mirror from a point object **O** placed along the principle axis and then suddenly reflected in the direction **XK** making an angle with the normal **XN**.



From ΔXIC, $\theta + \lambda = \beta$

=> $\theta = \beta - \lambda$ --------------------------------(i)

From ΔOXI, $\alpha + \beta = 2\theta$ -------------------------------(ii)

Substituting equation (i) into (ii) gives.

$\alpha + \beta = 2(\beta - \alpha)$

=> α - β = -2λ ---------------------------------------(a)

If X is very close to P, then

$\alpha \approx \tan\alpha = \frac{XP}{u}$, $\beta \approx \frac{XP}{-v}$ (I is virtual) and $\lambda \approx \tan\lambda = \frac{XP}{-r}$ $(C\ is\ virtual)$

Equation (a) becomes

$$\frac{XP}{u} - \frac{XP}{-v} = \frac{-2XP}{-r}$$

$$= \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

But 2 = 2f

$$= \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

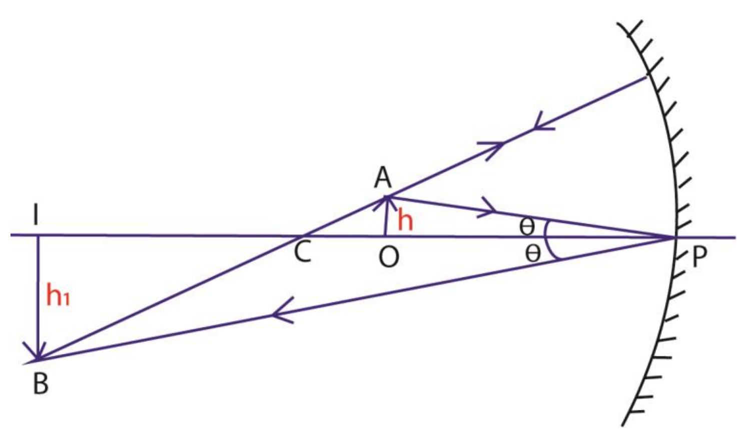
Note that the focal length of a convex mirror is a negative distance.

Formula for magnification

Linear magnification, m, = $\frac{\text{Image height}}{\text{Object height}} = \frac{\text{Image distance}}{\text{Object distance.}}$.

Proof

Consider the incidence of ray **AP** on to the pole of a concave mirror from an object of height **h** placed a distance, **u,** from the mirror and then reflected back making the same angle with the principal axis to form an image of height **h_1**, located at distance, **v**, from the mirror as shown



Ray **AP** makes an angle θ with the normal **OP**, then,

From ΔOAP, $\tan\theta = \frac{h}{u}$ ……………………….(i)

From ΔIPB, tan θ $= \frac{h_1}{v}$..(ii)

Equating equation (i) and (ii) gives

$$\frac{v}{u} = \frac{h_1}{h}$$

Thus magnification, m= $\frac{v}{u} = \frac{h_1}{h}$

NOTE:

(i) No signs need be inserted in the magnification formula.

(ii) Using the mirror formula, a connection relating magnification to the focal length of the mirror with either the object distance or the image distance can be established.

Relationship connecting m, v and f

Using the mirror formula; $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Multiplying, v, throughout we get

$$\frac{V}{f} = \frac{V}{u} + 1$$

Since $\frac{v}{u}$ = m

Then, $\frac{V}{f}$ = m + 1 or m = $\frac{V}{f}$ - 1

Examples 7

An object is placed 10cm in front of a concave mirror of focal length 15cm. Find the image position and magnification.

Solution:

For a concave mirror, focal length f = +15cm, u = 10cm.

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$v = \frac{f \, x \, u}{u-f} = \frac{15 \, x \, 10}{10-15} = -30cm$$

The negative sign implies that the image formed is **virtual** and it is formed 30cm from the mirror.

Hence, **Magnification, m** $= \frac{v}{u} = \frac{30}{10} = 3$

Examples 8

The image of an object in a convex mirror is 6cm from the mirror. If the radius of curvature of the mirror is 20cm, find the object position and the magnification.

Solution:

For a convex mirror,

$f = \frac{-r}{2} = \frac{-20}{2} = 10\text{cm}$

v = -6cm (The image in a convex mirror is always virtual)

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, gives

$$u = \frac{f\,x\,v}{v-f} = \frac{-10\,x-6}{-6-(-10)} = 15\text{cm}$$

Magnification, m $\frac{v}{u} = \frac{6}{15} = 0.4$

Examples 9

Show that an object and its image coincide in position at the centre of curvature of a concave mirror. Hence find the magnification produced in this case.

Solution

At the centre of curvature of a concave mirror, object distance **u = r** where r is the radius of curvature of the mirror.

Using the mirror formula = $\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$ and substituting r for u gives

$\frac{1}{r} + \frac{1}{v} = \frac{2}{r}$

V= r

Thus the image is also formed at the centre of curvature and therefore it coincides in position with its object.

Hence magnification m = $\frac{v}{u} = \frac{r}{r} = 1$

$\therefore$ **The object and its image are of the same size in this case.**

Examples 10

A concave mirror forms on a screen a real image of three times the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is **30cm,** determine the

(i) focal length of the mirror

(ii) shift of the object

Solution

(i) Using the relationship m = $\frac{u}{f} - 1$ for values m_1= 3 and m_2 = gives

$v_1 = (m_1 + 1)f$ ……………………………a

$v_2 = (m_2 + 1)f$ ………………………… b

For $m_2 > m_1$, it follows that $v_2 > v_1$

thus, the given shift in image = $v_2 - v_1$ = 30cm

Equation a – equation b gives
$v_2 - v_1 = (m_2 - m_1)f$

$f = \frac{v_2 - v_1}{m_2 - m_1} = \frac{30}{5-3} = 15$cm

(ii) Using the relationship $\frac{1}{m} = \frac{u}{f} - 1$ for values m_1= 3 and m_2 = gives

$u_1 = \frac{f}{m_1} + 1$ …………………(a)

$u_2 = \frac{f}{m_2} + 1$ …………………(b)

For $m_2 > m_1$, follows that $u_1 > u_2$

The required shift in object distance is equal to $u_1 - u_2 = (\frac{f}{m_1} + 1)$ **−(** $\frac{f}{m_2} + 1)$

$= (\frac{f}{m_1} - \frac{f}{m_2})$

$= \frac{15}{3} - \frac{15}{5} = 2$cm

Examples 11

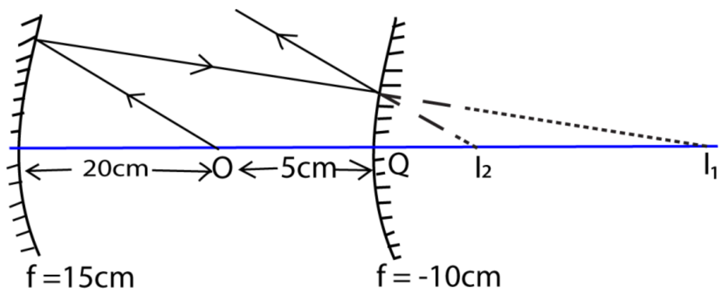
A concave mirror **P** of focal length 15cm faces a convex mirror **Q** of focal length

10cm placed 25cm from it. An object is placed between **P** and **Q** at a point 20cm from **P**.

(i) Determine the distance from **Q** of the image formed by reflection**,** first in **P** and then in **Q**

(ii) Find the magnification of the image formed in **(i)** above

Solution



Consider the action of a concave mirror

u = 20cm, and f = 15cm

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, gives, v = $\frac{fu}{u-f} = \frac{20 \, x \, 15}{20-15}$ = 60cm

∴The image distance from a concave mirror = **60cm**

Thus, the image distance behind a convex mirror = **60 – (5 + 20)cm = 35cm.**
Consider the action of a convex mirror

The image formed by a concave mirror acts as a virtual object for the convex mirror. Thus u = -35cm and f = -10cm

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, gives, v = $\frac{fu}{u-f}$ = $\frac{-10\ x -35}{-35--15}$ = -14cm
∴A final virtual image is **14cm** behind the convex mirror

Magnification, m = $\frac{h_2}{h_o} = \frac{h_2}{h_1}\ x\ \frac{h_1}{h_0}$ = m_2 x m_1

m = $\frac{v_1}{u_1}\ x\ \frac{v_2}{v_2}$ where u_1 = 20cm, v_1 = 60cm, u_2 = 35cm, v_2 = 14cm

thus m = $\frac{60}{20}\ x\ \frac{14}{35} = 1.2$

Examples 12

A small convex mirror is placed **60cm** from the pole and on the axis of a large concave mirror of radius of curvature **200cm.** The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole.

(a) (i) Draw a ray diagram to show how a convex mirror forms an image of a non- axial point of a distant object

(ii) Suggest a practical application for the arrangement of the mirrors in **a (i)** above**.**

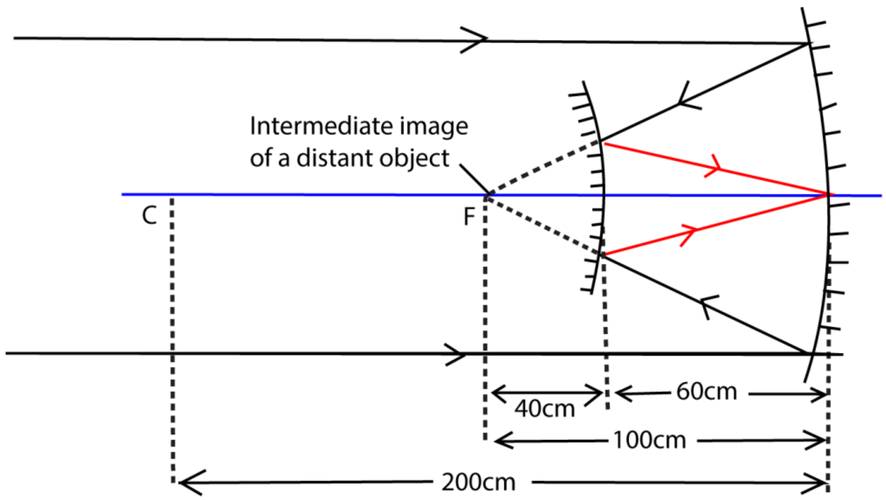
(b) Calculate the

(i) radius of curvature of the convex mirror.

(ii) height of the real image if the distant object subtends an angle of **0?5?** at the pole of the convex mirror.

Solution

(a)(i)



(ii) The mirror arrangement finds application in a reflecting telescope, a device for viewing distant objects

(b) (i) Consider the action of a concave mirror

The image of a distant object is formed at the principal focus of the concave mirror. This image acts as a virtual object for a convex mirror.

Consider the action of a convex mirror

u = -40cm and v = 60cm

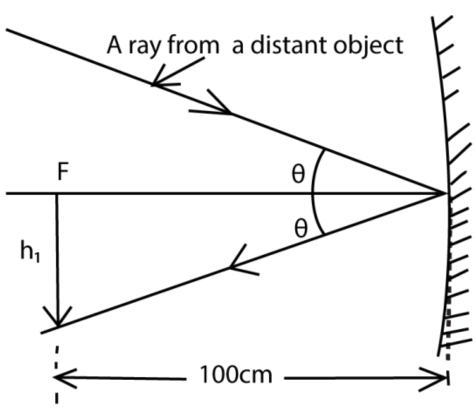
Applying the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$, gives

$$r = \frac{2uv}{(u+v)} = \frac{2\ x{-40}\ x\ 60}{-40+60} = -240\ \text{cm}$$

The required radius of curvature r = 240cm

(ii) Consider the magnification produced by a convex mirror

Let **h1 =** height of the intermediate image formed by a concave mirror as shown.



From above, tan 0.50 = $\frac{h_1}{100}$

h1 = 0.873cm

Let h2 = height of the image formed by a convex mirror

Magnification, m = $\frac{h}{h_1} = \frac{v}{u}$

h_2 = $\frac{v}{u}\ x\ h_1$ = $\frac{60}{40}\ x\ 0.873$ = 1.3 cm

Required image height = 1.3cm

Revision EXERCISE 2

1. Define the terms centre of curvature, radius of curvature, principal focus and focal length of a converging mirror.
2. Distinguish between real and virtual images.
3. Explain with the aid of a concave mirror the term **a caustic surface.**
   .
4. .Explain why a parabolic mirror is used in searchlights instead of a concave mirror
5. An object is placed a distance **u** from a concave mirror. The mirror forms an image of the object at a distance **v.** Draw a ray diagram to show the path of light when the image formed is:

   (i) real

   (ii) virtual
6. Give two instances in each case where concave mirrors and convex mirrors are useful.
7. **(i)** Explain the suitability of a concave mirror as a shaving mirror.
   (ii) Explain with the aid of a ray diagram why a convex mirror is used as a car driving mirror.
8. Show with the aid of a ray diagram, that the radius of curvature of a concave mirror is twice the focal length of the mirror
9. Use a geometrical ray diagram to derive the relation
10. Derive the relation connecting the radius of curvature **r** object distance **u** and image distance **v** of a diverging mirror.
11. An object is placed perpendicular to the principal axis of a concave mirror of focal length f at a distance **(f+ x)** and a real image of the object is formed at a distance **(f + y).** Show that the radius of curvature **r** of the mirror is given by

    $$\mathbf{r = 2\sqrt{xy}}$$
12. (i) Define the term **linear magnification.**

    (ii) Show that in a concave mirror, **linear magnification**$=\frac{\textbf{image distance}}{\textbf{object distance}}$

    (iii) A concave mirror of focal length 15cm forms an erect image that is three times the size of the object. Determine the object and its corresponding image position.[u =10cm, v = 30cm]

    (iv) A concave mirror of focal length 10cm forms an image five times the height of its object. Find the possible object and corresponding image positions.

    [u = 12cm, v = 60cm OR u = 8cm, v =40cm]
13. A concave mirror forms on a screen a real image which is twice the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is 30cm, determine the

(i) focal length of the mirror[10cm]
(ii) shift of the object [3cm]

14. A concave mirror of radius of curvature **20cm** faces a convex mirror of radius of curvature **10cm** and is **28cm** from it. If an object is placed midway between the mirrors, find the nature and position of the image formed by reflection first at the concave mirror and then at the convex mirror.

[A final virtual image is **17.5cm** behind the convex mirror]

15. A small convex mirror is placed **100cm** from the pole and on the axis of a large concave mirror of radius of curvature **320cm.** The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole.

(a) (i) Draw a ray diagram to show how a convex mirror forms an image of a non-axial point of a distant object

(ii) Suggest a practical application for the arrangement of mirrors in **a (i)** above**.**

(iii) Calculate the radius of curvature of the convex mirror [150cm]

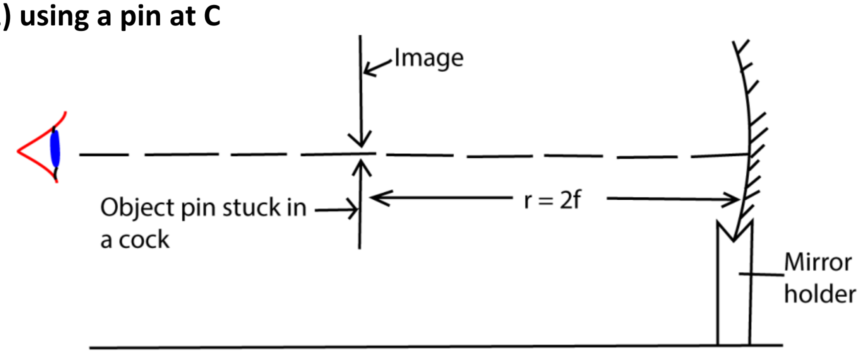
(b) If the distant object subtends an angle of **3×10^{-3} radians** at the pole of the concave mirror, calculate the

(i) size of the real image that would have been formed at the focus of the concave mirror. [0.48cm]

(ii) size of the image formed by the convex mirror. [0.8cm]

Determination of the focal length of a concave mirror

Method (1) using a pin at C



An object pin is placed in front of a mounted concave mirror so that its tip lies along the axis of the mirror. The position of the pin is adjusted until it coincides with its image such that there is no parallax between the pin and its image. The distance **r** of the pin from the mirror is measured.

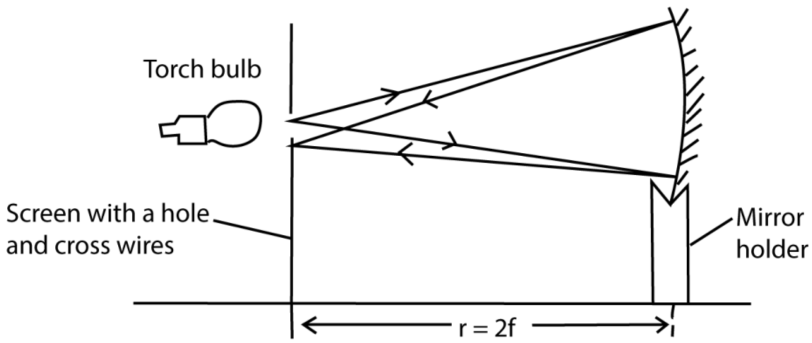
The required focal length **f =** $\frac{r}{2}$

Note

(i) In the position where there is no parallax between the object pin and its image, there is no relative motion between the object and its image when the observer moves the head from side to side.

(ii) When the pin coincides with its image, the rays are incident normal to the mirror and are thus reflected along their own path. Therefore the pin coincides with its image at the centre of curvature of the mirror.

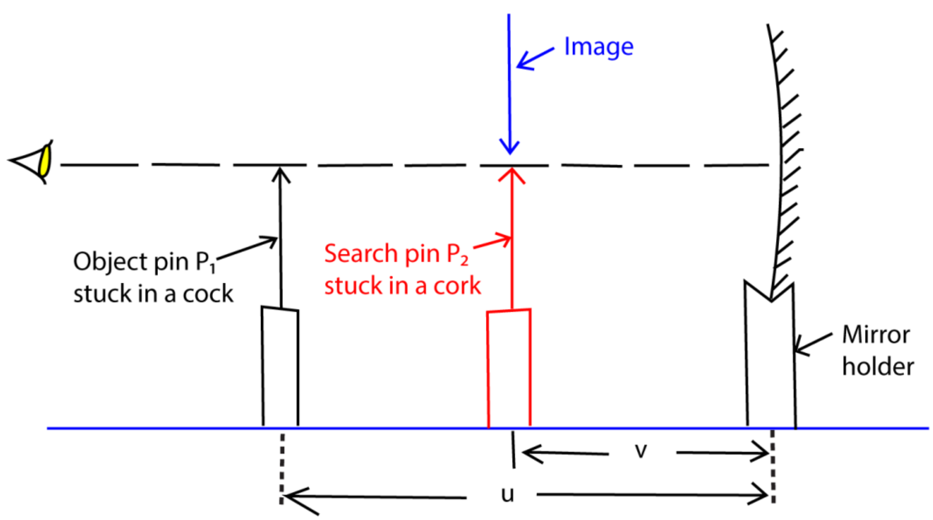
Method (2) using an illuminated object at C



Align an illuminated bulb, a screen and a concave mirror mounted in a holder as shown above. The mirror position is adjusted to or from the screen until a sharp image of the cross-wire is formed on the screen besides the object. The distance **r** of the mirror from the screen is measured.

The required focal length **f** = $\frac{r}{2}$

Method (3) Using no parallax method in locating V



An object pin **P1** is placed at a distance **u** in front of a mounted concave mirror so that its tip lies along the axis of the mirror. A search pin **P2** placed between the mirror and pin **p1** is adjusted until it coincides with the image of pin **p1** by no-parallax method. The distance **v** of pin **p2** from the mirror is measured**.** The procedure is repeated for several values of **u** and the results are tabulated including values of **uv**, and **u+v.**

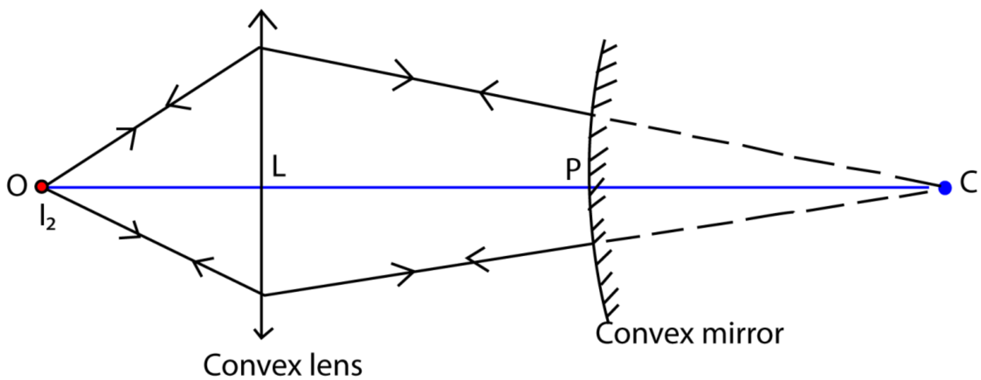
A graph of **uv** against **u+v** is plotted and the slope **s** of such a graph is equal to the focal length **f** of the mirror.

NOTE:

If a graph of $\frac{1}{u}$ against $\frac{1}{v}$ is plotted, then each intercept C = $\frac{1}{f}$

Determination of focal length of a convex mirror

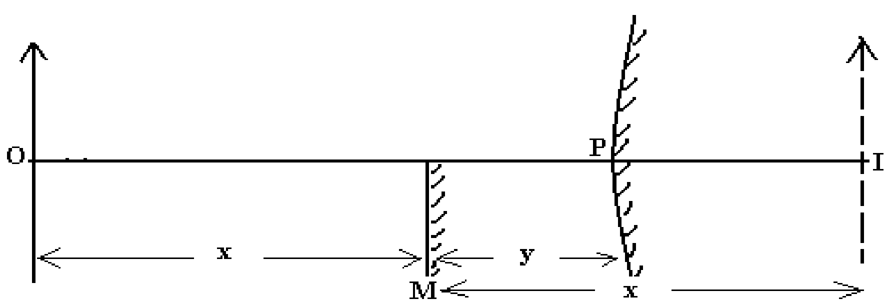
Method (1) Using a convex lens.



(i) The apparatus is arranged as shown above
(ii) An object, O is placed in front of a convex lens L and its image formed at C
(iii) The distance LC is measured and recorded.
(iv) The convex mirror whose focal length, f, is required is placed is placed between L and C with its reflecting surface facing the lens.
(v) The lens is then moved along the axis, OC until a converging beam incident normally on the mirror forms its image at O
(vi) Distance LP is measured

PC = LC – LP thus, f can be determined from f = $\frac{PC}{2}$

Method (2) using No parallax.



(i) An object pin **O** is placed in front of a convex mirror as shown in the diagram above
(ii) A virtual diminished image is formed at **I**.

(iii) A plane mirror **M** is placed between **O** and **P** so as to intercept half the field of view of the convex mirror.

(iv) Mirror **M** is adjusted until its own image of **O** coincides with **I** by no parallax method.

(v) Measure the distances **x** and **y.**

(vi) The focal length of the mirror is calculated from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Where u = (x + y) and v = (y-x)

Note:

(i) The two images coincides when they are as far behind the plane mirror as the object is in front.

(ii) Substituting for u = x + y and v = y - x in the mirror formula gives $f = \frac{y^2 - x^2}{2y}$

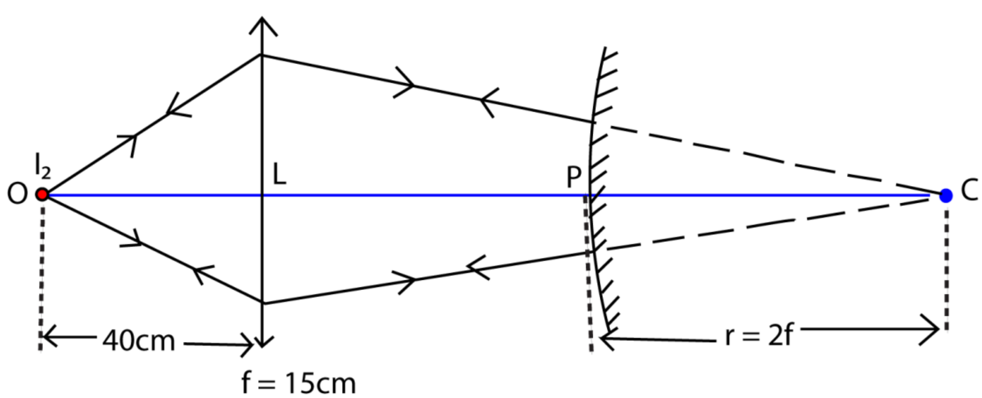
Examples 13

An object **O** is placed **40cm** in front of a convex lens of focal length **15cm** forming an image on the screen. A convex mirror situated **4cm** from the lens in the region between the lens and the screen forms the final image besides object **O**.

(i) Draw a ray diagram to show how the final image is formed.

(ii) Determine the focal length of the convex mirror.

Solution



Consider the action of a convex lens u =40cm, and f = 15cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ give

Image distance v of the lens = $\frac{fu}{u-f} = \frac{15 \, x \, 40}{40-15} = 24cm$

The radius of curvature **r = (24 – 4)cm = 20cm**

Using the relation r = 2f

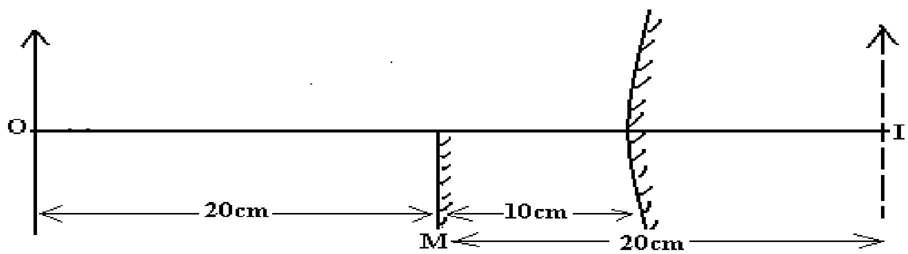
⇨ 2f = 20cm

∴ f = 10cm

Thus f = -10cm "The centre of curvature of a convex mirror is virtual"

Examples 14

A plane mirror is placed **10cm** in front of a convex mirror so that it covers about half of the mirror surface. A pin **20cm** in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. Find the focal length of the convex mirror.



Consider the action of a convex mirror

u = 30cm and v = - (20 - 10) = -10cm "The image formed is virtual "

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ give

Image distance v of the lens = $\frac{fu}{u-f} = \frac{-10\ x\ 30}{30-10} = -15cm$

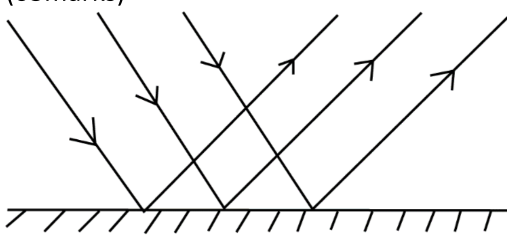
Revision exercise 3

1. Describe an experiment to determine the focal length of a concave mirror.
2. You are provided with the following pieces of apparatus: A screen with cross wires, a lamp, a concave mirror, and a meter ruler. Describe an experiment to determine the focal length of a concave mirror using the above apparatus.
3. Describe an experiment, including a graphical analysis of the results to determine the focal length of a concave mirror using a no parallax method.
4. Describe an experiment to measure the focal length of a convex mirror
5. Describe how the focal length of a diverging mirror can be determined using a convex lens.
6. Describe how the focal length of a convex mirror can be obtained using a plane mirror and the no parallax method.
7. A plane mirror is placed at a distance **d** in front of a convex mirror of focal length **f** such that it covers about half of the mirror surface. A pin placed at a distance **L** in front of the plane

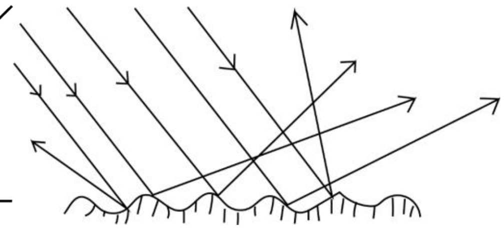
mirror gives an image in it, which coincides with that of the pin in the convex mirror. With the aid of an illustration, Show that $2df = d^2 - L^2$.

Topical revision questions

1. (a) (i) With the ais of digrams, differentiatebetween regular and irregular refelection (03marks)



Regular reflection



Irregular reflection

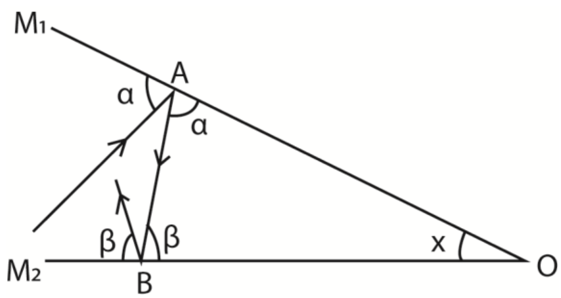
Differences between regular and irregular reflection

Regular reflection	Irregular reflection
Occurs on smooth surface	Occurs on a rough surface
Parallel incident beam is reflected parallel	Parallel incident beam is scattered after reflection
Reflected beam is very bright	Reflected beam is dull

(ii) Show that if two plane mirrors are inclined at an angle, x, then the total deviation of a light ray produced is 2x after reflection once in each mirror. (03 marks)

Deviation by successive reflections at two inclined mirrors

Consider an incident ray of light reflected successively from two mirrors M_1 and M_2 inclined at an angle x to each other at O as shown



Let the glancing angle at and B be α and β respectively

Deviation by $M_1 = 2\alpha$ (clockwise direction)

Deviation by $M_2 = 2\beta$ (clockwise direction)

Total deviation $= 2\alpha + 2\beta$

$= 2(\alpha + \beta)$ …………………. (i)

But $\alpha + \beta + x = 180$ (Angle sum of a triangle)

$\alpha + \beta = 180 - x$ ……………………...(ii)

Combining (i) and (ii)

Total deviation $= 2(180 - x)$ (clockwise)

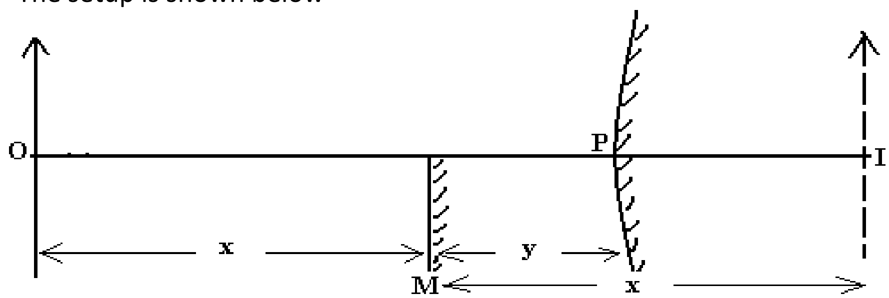
$= 360 - 2x$ (clockwise)

$= 2x$ (anticlockwise)

Thus, the deviation produced by two inclined mirrors is twice the angle between the mirrors when a ray under goes two successive reflections.

(b) Describe an experiment to determine the focal length of a convex mirror using a plane mirror and a pin. (04marks)

The setup is shown below



(i) An object pin **O** is placed in front of a convex mirror as shown in the diagram above

(ii) A virtual diminished image is formed at **I**.

(iii) A plane mirror **M** is placed between **O** and **P** so as to intercept half the field of view of the convex mirror.

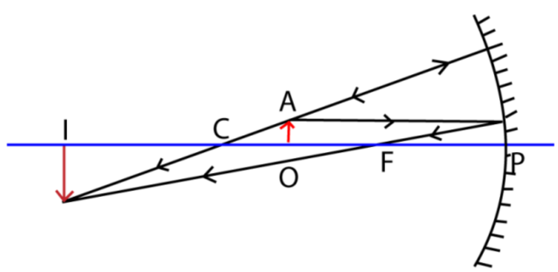
(iv) Mirror **M** is adjusted until its own image of **O** coincides with **I** by no parallax method.

(v) Measure the distances **x** and **y.**

(vi) The focal length of the mirror is calculated from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Where u = (x + y) and v = (y-x)

(c) (i) Sketch a ray diagram to show how a concave mirror forms a magnified real image of a real object. (02marks)



Note that the object is between F and C while the image is real and inverted beyond C

(ii) A concave mirror of radius of curvanture 48cm produces a real image whose size is three times that of the object,. Determine the position of th object.m(02marks)

$\frac{u}{f} = 1 + \frac{1}{m}$

⇨ $\frac{u}{24} = 1 + \frac{1}{3}$

⇨ u = 32cm

Or

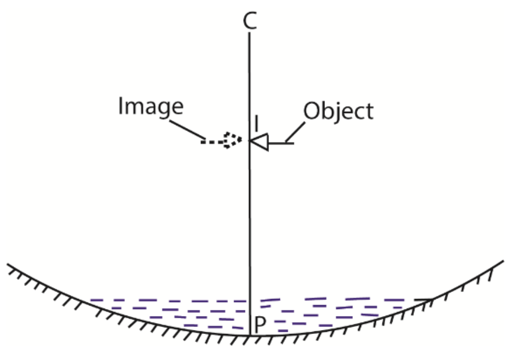
$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{24} = \frac{1}{u} + \frac{1}{3u}$

u = 32cm

(d) Describe an experiment to determine the refractive index of a liuid using a concave mirror. (04marks)

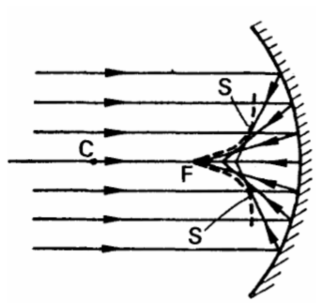
An experiment to determine refractive index of a liquid using a concave mirror



- A concave mirror is placed on a horizontal table with its reflecting surface facing upwards.
- A clamped pin with its tip along the principal axis above the concave mirror is coincided with its image at C and distance PC is measured
- A small quantity of the liquid under test is poured into a concave mirror and a new point I at which the object pin coincides with its image is obtained.
- Distance IP is measured.
- The required refractive index of a liquid, $\boldsymbol{n_l} = \frac{\boldsymbol{PC}}{\boldsymbol{IP}}$

(e) Explain the meaning of a caustic curve. (02 marks)

When a wide parallel beam of light is incident on a converging mirror, the reflected rays converge at different points along the axis of the mirror. A curve, S, called a caustic curve, with an apex or cusp at principal focus, F is formed which passes through the points of intersection of the reflected and incident rays.

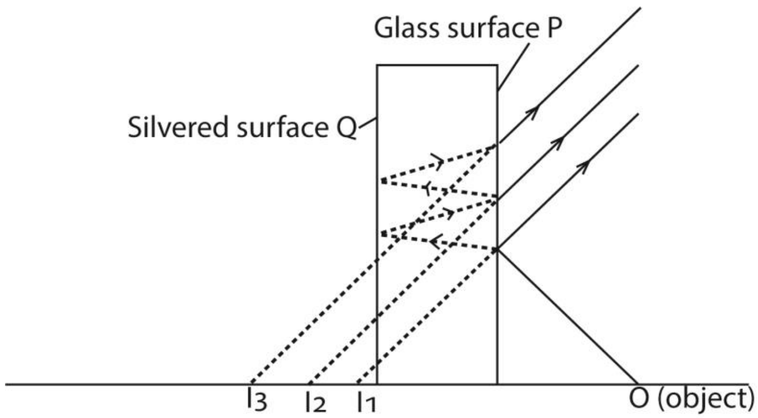


2. (a) (i) State two differences between real and virtual images (02marks)

A real image is formed by actual intersection of rays and can be formed on a screen while virtual image is formed by apparent intersection of rays and cannot be formed on a screen.

(ii) Explain with the aid of a diagram how thick plane mirror forms multiple images (04marks)

Formation of multiple images in thick plane mirror



Multiple images are formed due to partial reflection and refraction at the non-silvered surface of the mirror.

- İmage I_1 is formed by reflection on the glass surface P
- The image I_2 (the brightest is formed by reflection of the most light on the silvered surface Q
- Others by partial refraction

(b) A convex mirror forms a real image which is three times the linear size of the object. When the object is displaced though a distance y, the real mage formed is four times the linear size of he object. If the distance between the two image positions is 20 cm, find the

(i) focal length of the mirror (03marks)

$M_1 = \frac{v_1}{f} - 1$ and $M_2 = \frac{v_2}{f} - 1$

$M_2 - M_1 = \frac{v_2 - v_1}{f}$

$4 - 3 = \frac{20}{f}$

f = 20cm

(ii) distance, y. (03marks)

Also $\frac{1}{M} = \frac{u}{f} - 1$

$\frac{1}{3} = \frac{u_1}{f} - 1$

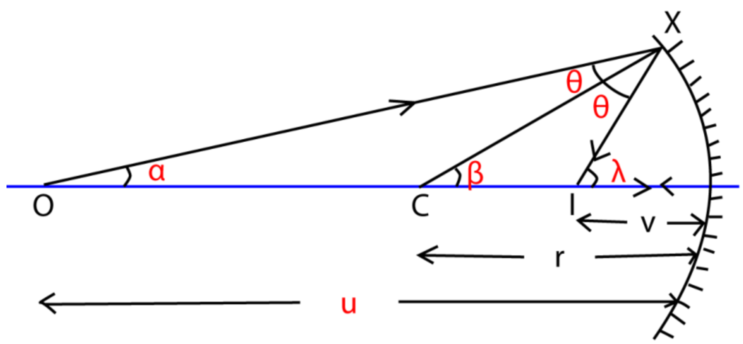
$\frac{1}{4} = \frac{u_2}{f} - 1$

$\frac{1}{3} - \frac{1}{4} = \frac{u_1 - u_2}{f} = \frac{y}{20}$

y = 1.67cm

(c) Use a geometrical ray diagram to derive the relation, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ for concave mirror. (05marks)

Consider the incidence of ray **OX** on to a concave mirror from a point object **O** placed along the principal axis and then suddenly reflected in the direction XI making an angle ⍰ with the normal CX.



Ray OP strikes the mirror incident normally at P and thus reflected back along its own path. The point of intersection **I** of the two reflected rays is the image position.

From Δ OXC, α + θ = β ………………. (i)

From Δ CXI β + θ = λ ………………… (ii)

Eliminating θ from eqn. (i) and eqn. (ii)

α + λ = 2β ………………………………….. (iii)

If X is very close to P, then

α ≈ tan α = $\frac{XP}{u}$, β ≈ tan β = $\frac{XP}{r}$, λ ≈ tan λ= $\frac{XP}{v}$Equation (a) becomes = $\frac{XP}{u} + \frac{XP}{v} = \frac{2XP}{r}$

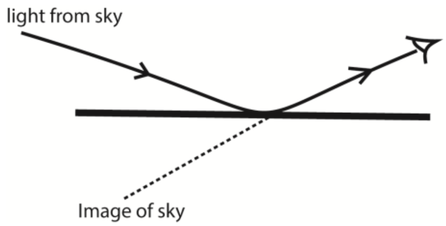
$= \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$

But 2 = 2f

$= \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

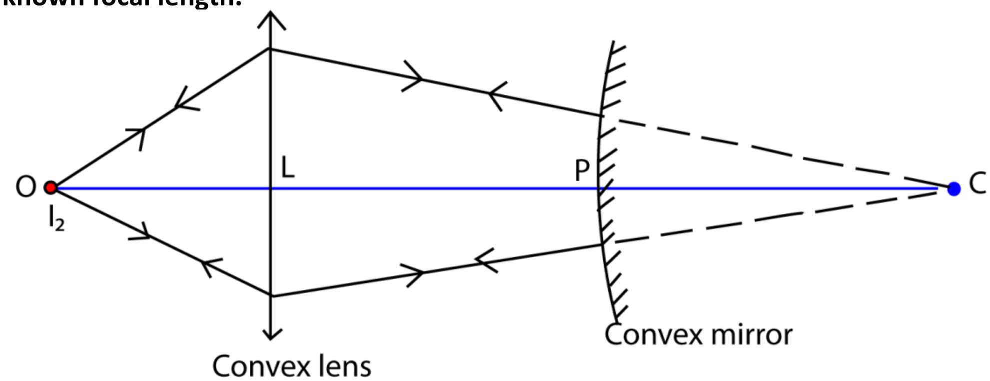
(d) Explain how mirage is formed. (03marks)

On a hot day, layer on air near the ground are hotter and lens dense than layers above. This leads to total internal reflection of rays of light from the sky. And mirage is the image of the sky to the eye by total internal reflection



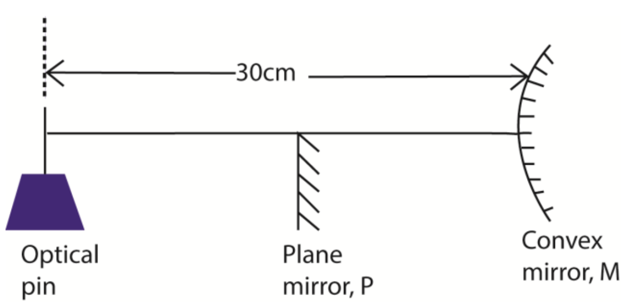
3. (a) (i) Describe how the focal length of a convex mirror can be measured using a convex lens of known focal length. (04marks)

Determination of the focal length of a convex mirror can be measured using a convex lens of known focal length.

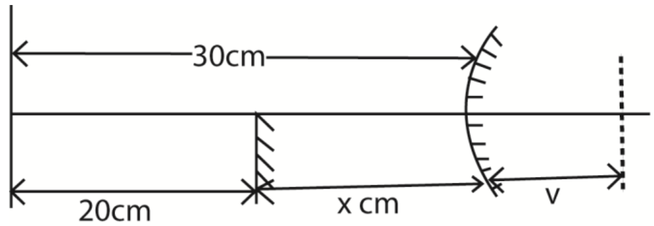


(i) The apparatus is arranged as shown above
(ii) An object, O is placed in front of a convex lens L and its image formed at C
(iii) The distance LC is measured and recorded.
(iv) The convex mirror whose focal length, f, is required is placed is placed between L and C with its reflecting surface facing the lens.
(v) The lens is then moved along the axis, OC until a converging beam incident normally on the mirror forms its image at O
(vi) Distance LP is measured
PC = LC – LP thus, f can be determined from f = $\frac{PC}{2}$

(ii) The plane mirror, P, in the figure below is adjusted to a position20cm from optical pin, the image of the pin in P coincides with its image in M.



Calculate the focal length of the convex mirror. (04marks)



v = 20 – x

= 20 – (30-20) = 10cm

Using $\frac{1}{f} = \frac{1}{u} + \frac{1}{V}$

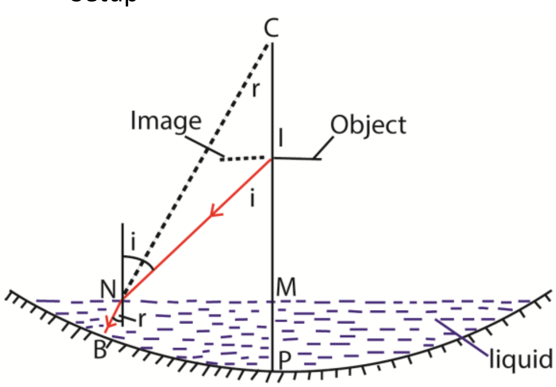
$= \frac{1}{30} + \frac{1}{-10}$

f = 15cm

(b) A pin is clamped horizontally above a concave mirror with its tip along the principal axis. When the pin is adjusted, it coincides with its image at a distance R from the mirror. When a small liquid of refractive index, n, is put on the mirror, the pin again coincides with its image at a distance R' from the mirror. Show that the refractive index, n, is given by

$n = \frac{R}{R'}$ (04marks)

Setup



From $n = \frac{\sin i}{\sin r}$

From the diagram,

$\sin i = \frac{NM}{NI}$ and $\sin r = \frac{NM}{NC}$

$n = \frac{NM}{NI} \div \frac{NM}{NC} = \frac{NM}{NI} x \frac{NC}{NM} = \frac{NC}{NI}$

For small angle, i, and small liquid

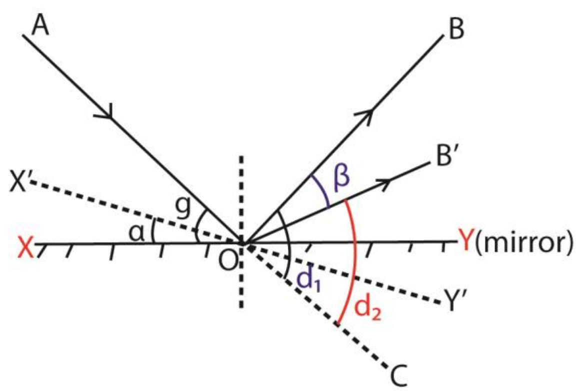
NC $\approx PC = R$ and NI $\approx PI = R'$

Thus, $n = \frac{R}{R'}$

4. (a) (i) State the laws of reflection of light. (02marks)
   - The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane
   - The angle of incidence is equal to the angle of reflection

(ii) A ray of light is incident on a plane mirror. The mirror is then turned through an angle α keeping the direction of the incident ray constant. If a reflected ray turned through angle β, find the relationship between α and β.

Let XY be the initial position of the mirror with ray AO making a glancing angle g. By keeping the direction of the incident ray fixed, the mirror is rotated through an angle **α** to a new position X'Y' as shown.



Case 1 (mirror in position XY)

Glancing angle = g

Deviation $d_1 = 2g$……… (i)

Case 2**(mirror in position X'Y')**

Glancing angle = $(g - \alpha)$

Deviation $d_2 = 2(g - \alpha)$ …..(ii)

$\beta = d_1 - d_2$

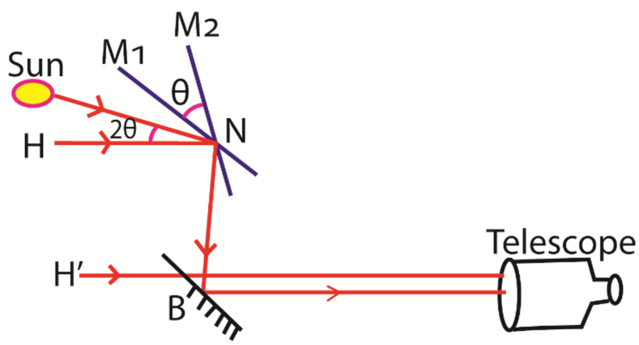
$= 2g - 2(g - \alpha)$

$= 2g - 2g + 2\alpha$

$= 2\alpha$

(b) Describe how a sextant is used to determine the angle of elevation of a star. (05marks)

Setup



( ) A sextant consists of a fully silvered mirror M1which can be rotated about a horizontal axis and a fixed half silvered mirror B.

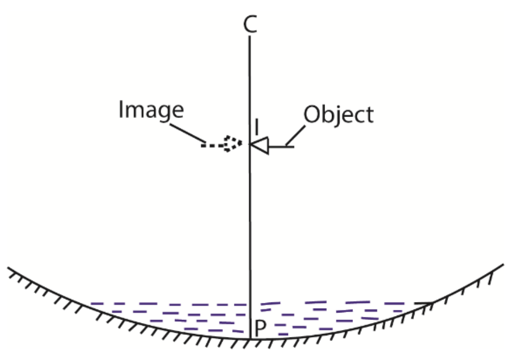
(i) Mirror M1 is adjusted to become parallel to B by rotating it until the image of the horizon, H' is seen directly through the unsilvered part of mirror B by successive reflection in mirror M_1 and B respectively

(ii) The mirror M_1 is rotated to position M_2 such that the image of the horizon H, and the sun coincides at H'

(iii) The angle of rotation is measured from the scale on the instrument. The elevation of the sun is 2θ.

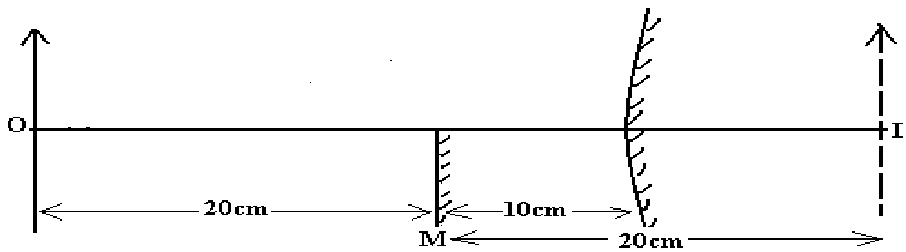
(c) Describe an experiment to determine the refractive index of a small quantity of a liquid using a concave mirror. (05marks)

An experiment to determine refractive index of a liquid using a concave mirror



- A clamped pin with its tip along the principal axis above the concave mirror is coincided with its image at C and distance PC is measured
- A small quantity of the liquid under test is poured into a concave mirror and a new point I at which the object pin coincides with its image is obtained.
- Distance IP is measured.
- The required refractive index of a liquid, $\boldsymbol{n_l} = \frac{\boldsymbol{PC}}{\boldsymbol{IP}}$

(d) A plane mirror is placed 10cmin front of a convex mirror so that it covers about half of the convex mirror surface. A pin placed 20cm in front of the plane mirror gives an image which coincides with that of the pin in the convex mirror. Find the focal length of the convex mirror. (04marks)



Consider the action of a convex mirror

u = 30cm and v = - (20 - 10) = -10cm "The image formed is virtual "

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ give

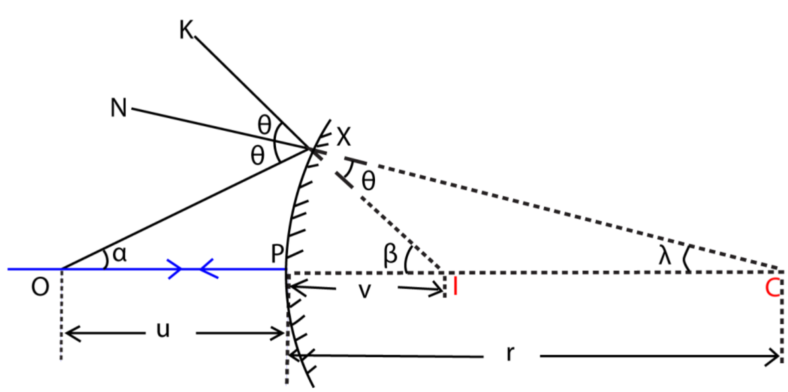
Image distance v of the lens = $\frac{fu}{u-f} = \frac{-10\ x\ 30}{30-10} = -15cm$

5. (a) Distinguish between a real and virtual image (02marks)

A real image is formed by actual intersection of rays and can be formed on a screen while a virtual image is formed by apparent intersection of rays and cannot be formed on a screen.

(b) Derive an expression relating the focal length, f, of a convex mirror to the object distance, u and image distance, v.

Consider the incidence of ray **OX** on to a convex mirror from a point object **O** placed along the principle axis and then suddenly reflected in the direction **XK** making an angle q with the normal **XN**.



From triangle XIC, θ + λ = β

⇨ θ = β – λ .... (i)

From triangle OXI, α + β = 2θ ……………………………… (ii)
Substituting (i) into (ii)
α + β = 2(β – λ)

⇨ α – β = -2λ ……………………………………………………………. (iii)

If X is very close to P. then

α ≈ tan α =$\frac{XP}{u}$, β = $\frac{XP}{-v}$ (I is virtual) and λ ≈ tan λ = $\frac{XP}{-r}$ (C is virtual)

Equation (iii) becomes

$\frac{XP}{u} - \frac{XP}{-v} = \frac{-2XP}{-r}$

$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$

But 2 = 2f

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

(c) A convex mirror forms an image half the size of the object. The object is then moved towards the mirror until the image is three quarter that of the object. If the image is moved by a distance of 0.6cm, calculate

(i) focal length of the mirror. (03marks)

Using $\frac{v}{f} = m + 1$

Initially, $\frac{v}{f} = \frac{1}{2} + 1 = \frac{3}{2}$ …………………… (i)

When the object was displaced

$\frac{v+0.6}{f} = \frac{3}{4} + 1 = \frac{7}{4}$ …………………………… (ii)

Eqn. (i) and Eqn. (ii)

$\frac{0.6}{f} = \frac{7}{4} - \frac{3}{2} = \frac{1}{4}$; f= 2.4cm

(ii) new potion of the object (03marks)

v + 0.6 = $\frac{7}{4}\ x\ f = \frac{7}{4}\ x$ 2.4 = 4.2 cm

$\frac{4.2}{u'} = \frac{3}{4}$

u' = 5.6cm

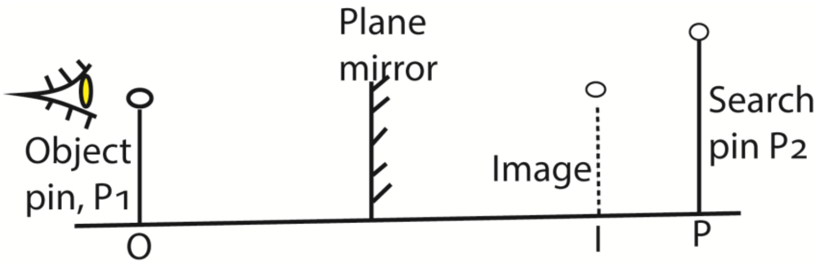
The object is 5.6 cm from the mirror.

6. (a) (i) Distinguish between real and virtual images. (02marks)

A real image is one formed by actual intersession of rays and can be formed on the screen while a virtual image is formed by apparent intersession of rays and cannot be formed on a screen.

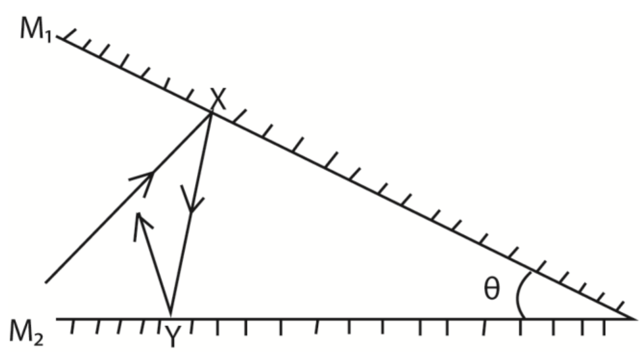
(ii) Describe how the position of an image in a plane mirror can be located (03marks)

Method 1: IMAGE LOCATION BY NO PARALLAX



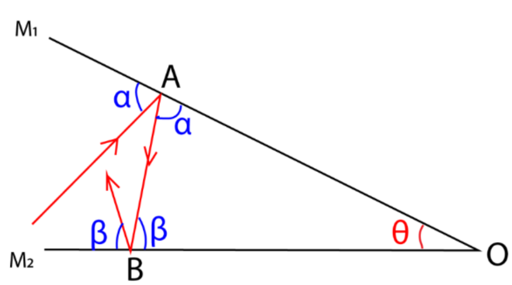
- An object pin **O** is placed in front of a plane mirror to form a virtual image **I**.
- A large search pin **P** is placed behind the mirror and moved to such a position that there is no parallax between pin **P2** and the image **I**.
- The image position has therefore been located.

(b) The diagram in the figure below shows a ray of light undergoing two successive reflections at points X and Y in two mirrors M_1 and M_2 inclined at an angle θ.



Show that the ray is deviated through an angle 2θ (03marks)

Consider an incident ray of light reflected successively from two mirrors M_1 and M_2 inclined at an angle θ to each other at O as shown



Let the glancing angles at A and B be α and β respectively.
Deviation by $M_1 = 2\alpha$ (clockwise direction)

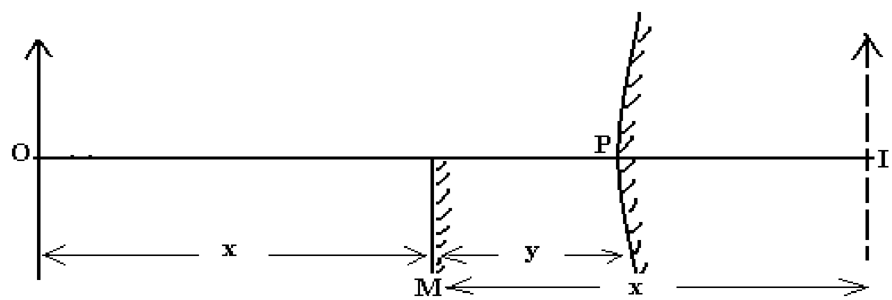
Deviation by M_2 = 2β   (clockwise direction)
Total deviation   = 2α + 2β
= 2 (α + β) ---------------------- (i)
But, α + β + θ = 180^0 (Angle sum of a triangle)
⇨   α + β = (1800 - θ) ---------------------(ii)

Combining equation (i) and (ii) gives
Total deviation = 2 (180^0 - θ)   (clockwise direction)
= 360^0 - 2θ   (clockwise direction)
= 2θ   (anti-clockwise direction)

(c) (i) What is a radius of curvature of a convex mirror? (01mark)

Radius of curvature of a convex mirror is the radius of a hollow sphere of which the mirror forms part

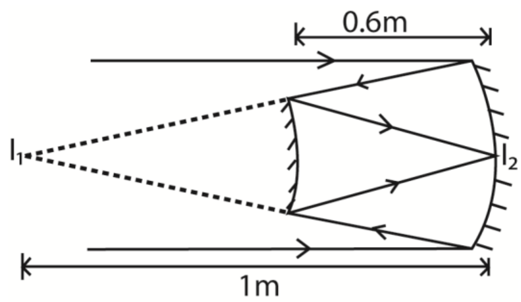
(ii) Describe the experiment to determine the focal length of a convex mirror using a plane mirror. (05marks)



(i) An object pin **O** is placed in front of a convex mirror as shown in the diagram above
(ii) A virtual diminished image is formed at **I**.
(iii) A plane mirror **M** is placed between **O** and **P** so as to intercept half the field of view of the convex mirror.
(iv) Mirror **M** is adjusted until its own image of **O** coincides with **I** by no parallax method.
(v) Measure the distances **x** and **y.**
(vi) The focal length of the mirror is calculated from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Where u = (x + y) and v = (v-x)(virtual)

(d) A small convex mirror is placed 0.60m from the pole and on the axis of a large concave mirror of radius of curvature 2.0m. The position of the convex mirror is such that a real image of a distant object is formed in the pane of a hole drilled through the concave mirror at its pole. Calculate the radius of curvature of the convex mirror. (04marks)



I1 is the virtual object of convex mirror that produces a real image, I2

Action of convex mirror;

u = -(1-0.6) = -0.4; v = 0.6

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{-0.4} + \frac{1}{0.6}$

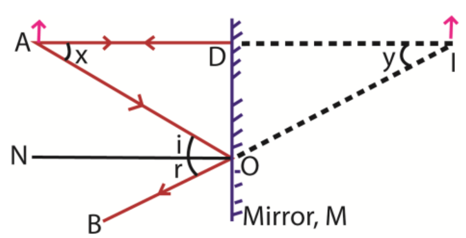
f = -1.2m

but radius of curvature = 2f = 2 x 1.2 = 2.4m

7. (a) (i) State the laws of reflection (02marks)
   - Incident ray, refracted ray and the normal at the point of incidence all lie in the same plane
   - Angle of reflection is equal to the angle of incidence

(ii) Show that the image formed in a plane mirror is as far behind the mirror as the object is in front. (04marks)

Consider an object A placed in front of a mirror **M**.



A ray AD from A incident normally on the mirror at D is reflected back along DA. The Ray AO is reflected along OB and appears to come from point I behind the mirror. The intersection **I** of the rays AD and BO is the image position.

From above angles,
x = i (alternating angles)
i = r (2nd law of reflection)
r = y (corresponding angles)
Combining all the equations gives
x = i = r = y
⇨ angle x = angle y

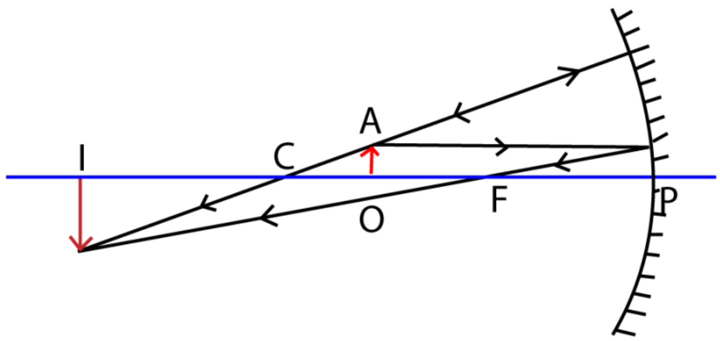
Tan x = t a n y

∴ $\frac{DO}{AD} = \frac{DO}{DI}$

Thus AD = ID.

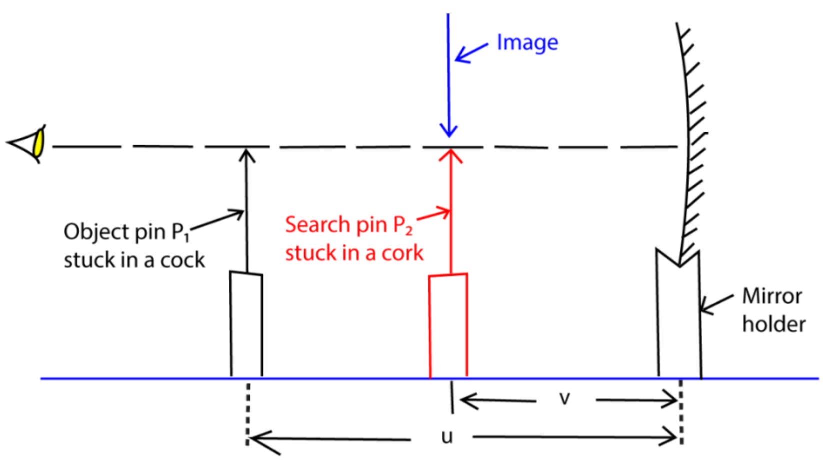
∴ The **image is as far behind the mirror as the object is in front**

(b) (i) Draw a ray diagram to show how a concave mirror forms a real image of a real object placed perpendicular to its principal axis. (01mark)



(ii) Describe an experiment, including a graphical analysis of the results to determine the focal length of a concave mirror using the No-parallax method. (06marks)

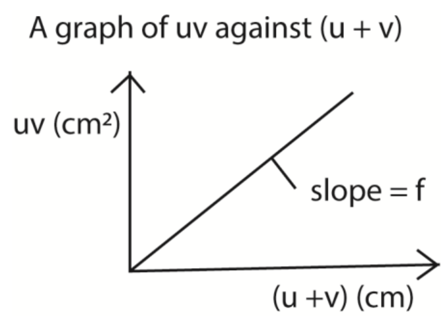
Experiment to determine focal length of concave mirror using no parallax method



- An object pin $\mathbf{P_1}$ is placed at a distance **u** in front of a mounted concave mirror so that its tip lies along the axis of the mirror.
- A search pin $\mathbf{P_2}$ placed between the mirror and pin $\mathbf{P_1}$ is adjusted until it coincides with the image of pin $\mathbf{p_1}$ by no-parallax method.
- The distance **v** of pin $\mathbf{P_2}$ from the mirror is measured**.**

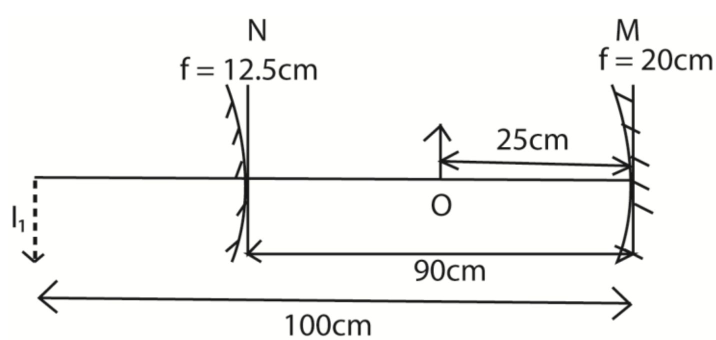
- The procedure is repeated for several values of **u** and the results are tabulated including values of **uv**, and **u+v.**

A graph of **uv** against **u+v** is plotted and the slope **s** of such a graph is equal to the focal length **f** of the mirror.



(c) A concave mirror M of focal length 20.0cmis placed 90cm in front of a convex mirror, N, of focal length 12.5cm. An object is placed on the common axis of M and N at a point 25.0 cm in front of M.

(i) Determine the distance from N of the image formed by reflection, first in M and then in N. (05marks)



For M

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{20} = \frac{1}{25} + \frac{1}{v}$; v = 100cm

For mirror N

u = -10cm, f = -12.5cm, v = ?

$\frac{-1}{12.5} = \frac{-1}{10} + \frac{1}{v}$; v = 50cm

So the distance of the final image from N is 50cm

(ii) find the magnification of the image formed in (c)(i) above (02marks)

Let the magnification due to M and N be M_1 and M_2 respectively

$M_1 = \frac{100}{25} = 4$ and $M_2 = \frac{50}{10} = 5$

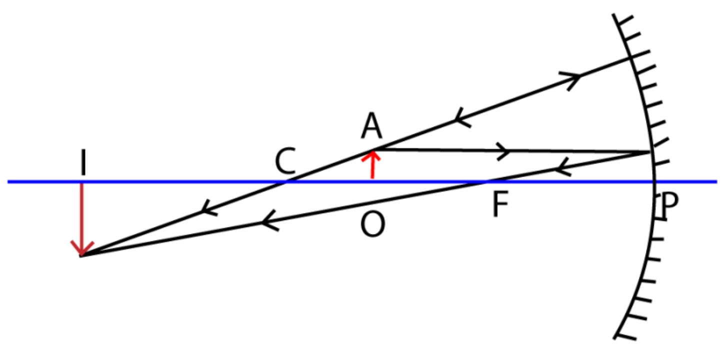
Resultant magnification, $M = M_1 \times M_2 = 4 \times 5 = 20$

8. (a) Define the focal length of a concave mirror. (01mark)

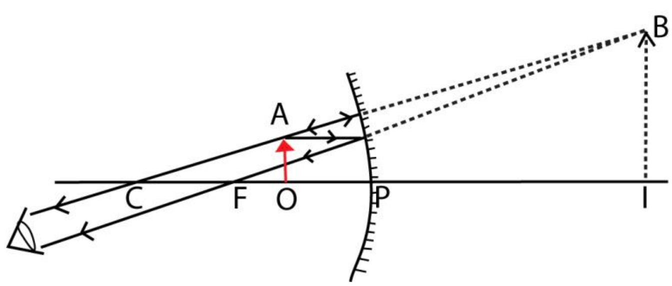
Focal length "f" of a concave mirror: it is the distance from the pole of the mirror to the point where paraxial rays incident and parallel to the principal axis converge after reflection by the mirror.

(b) An object is placed at a distance u from a concave mirror, the mirror forms an image of the object at a distance v. Draw diagrams to show the path of light rays when an image formed is

(i) real (02marks)

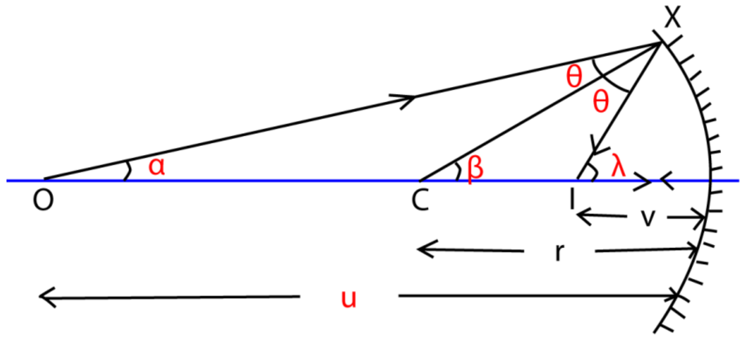


(ii) virtual (02marks)



(c) Use a geometrical diagram to derive the relation, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ (05marks)

Consider the incidence of ray **OX** on to a concave mirror from a point object **O** placed along the principal axis and then suddenly reflected in the direction XI making an angle ⍰ with the normal CX.



Ray OP strikes the mirror incident normally at P and thus reflected back along its own path. The point of intersection I of the two reflected rays is the image position.
From Δ OXC; α + θ = β ……………………………….. (i)
From Δ CXI; β + θ = λ ………………………………… (ii)
Eqn. (i) and (ii)
α + λ = 2β …………………………………………………….. (a)
If X is close to P then.
$\alpha \approx \tan\alpha = \frac{XP}{u}$; $\beta \approx \tan\beta = \frac{XP}{r}$; $\lambda \approx \tan\lambda = \frac{XP}{v}$

Equation (a) becomes

$$\frac{XP}{u} + \frac{XP}{v} = \frac{2XP}{r}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

But r = 2f

Thus, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

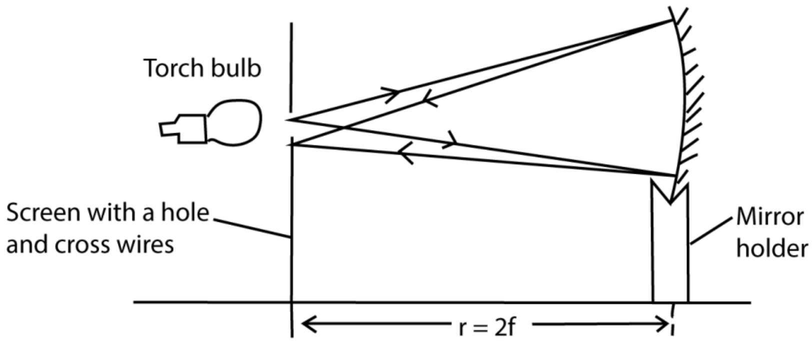
(d) A concave mirror of radius of curvature 40.0cm contains a liquid to a height 2.0cm. A pin clamped horizontally and viewed from above is observed to coincide with its image when it is 27.0 above the surface of the liquid. Calculate the refractive index of the liquid. (04marks)

R = 40.0cm, d = 2.0cm and h=27.0cm

$$n = \frac{R-d}{h} = \frac{40-2}{27} = 1.41$$

(e) You are provided with the following pieces of apparatus:

A screen with a cross wire, a lamp, a concave mirror, and a meter rule. Describe an experiment to determine the focal length of the concave mirror using the above apparatus.



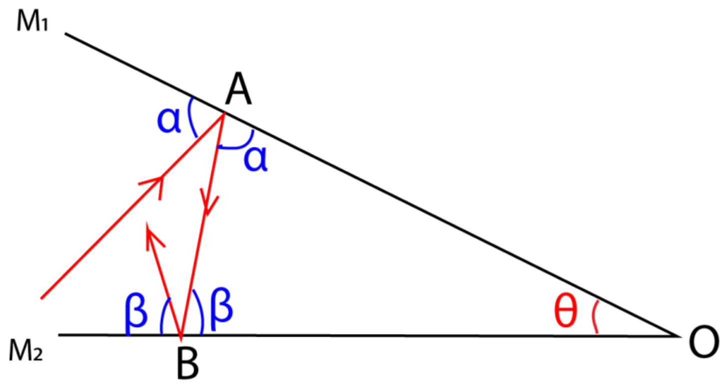
Align an illuminated bulb, a screen and a concave mirror mounted in a holder as shown above. The mirror position is adjusted to or from the screen until a sharp image of the cross-wire is formed on the screen besides the object. The distance **r** of the mirror from the screen is measured.

The required focal length **f** = $\frac{r}{2}$

9. (a) (i) State the laws of reflection of light. (02marks)
   - The incident ray reflected ray and the normal at the point of incidence all lie in the same plane
   - Angle of incidence is equal to the angle of reflaction.

(ii) Show that an incident ray of light reflected successively from two mirrors inclined at an angle θ to each other, is rotated through 2θ. (04marks)

Consider an incident ray of light reflected successively from two mirrors M_1 and M_2 inclined at an angle θ to each other at O as shown



Let the glancing angles at A and B be α and β respectively.

Deviation by M_1 = 2α (clockwise direction)

Deviation by M_2 = 2β (clockwise direction)

Total deviation = $2\alpha + 2\beta$

= $2(\alpha + \beta)$ ----------------------(i)

But, $\alpha + \beta + \theta = 180°$ (Angle sum of a triangle)

$\Rightarrow$ $\alpha + \beta = (180° - \theta)$ ---------------------(ii)

Combining equation (i) and (ii) gives

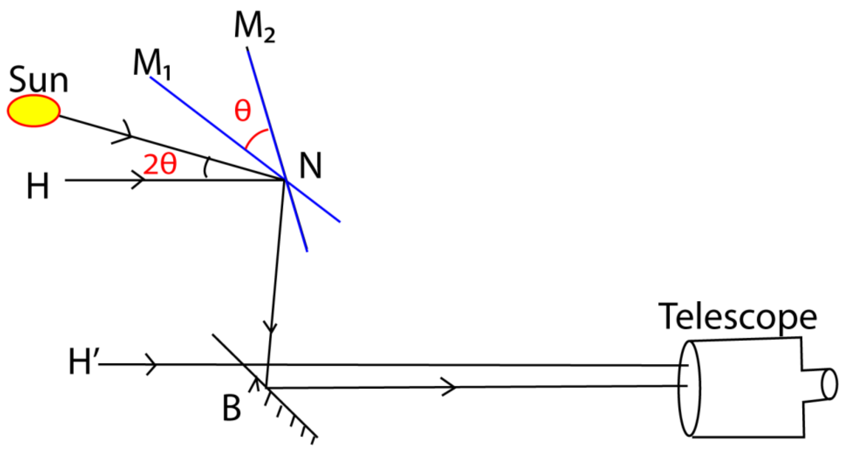
Total deviation = $2(180° - \theta)$ (clockwise direction)

= $360° - 2\theta$ (clockwise direction)

= 2θ (anti-clockwise direction)

(b) Describe how a sextant is used to measure the angle of elevation of a sun. (05marks)

The principle of a sextant is that, when the ray of light is reflected from two mirrors in succession in the same plane, then the angle between the incident and reflected ray is two times the angle between the mirrors.



(i) A sextant consists of a fully silvered mirror M1which can be rotated about a horizontal axis and a fixed half silvered mirror B.

(ii) Mirror M1 is adjusted to become parallel to B by rotating it until the image of the horizon (the line at which the earth's surface and the sky appear to meet), H' is seen directly through the unsilvered part of mirror B by successive reflection in mirror M_1 and B respectively

Note that, if the two mirrors are parallel, the incident ray from any observed body must be parallel to the observer's line of sight through the horizon glass

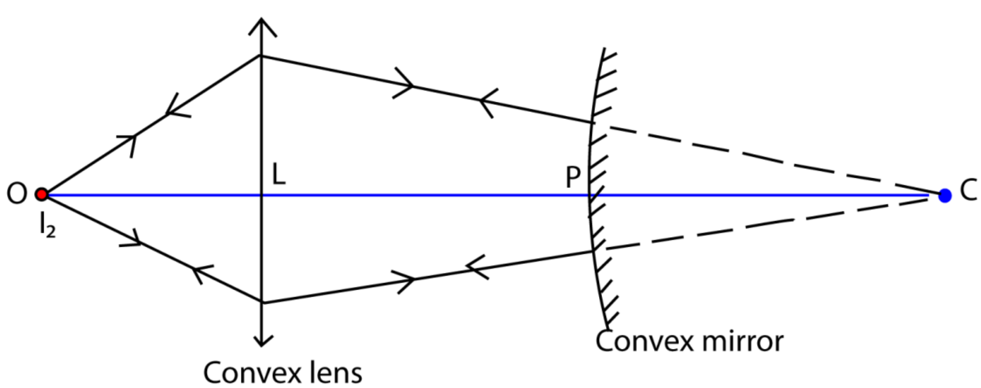
(iii) The mirror M_1 is rotated to position M_2 such that the image of the horizon H, and the sun coincides at H'

(iv) The angle of rotation is measured from the scale on the instrument. The elevation of the sun is 2θ.

(c) (i) Describe an experiment to measure the focal length of a convex mirror. (05marks)

Determination of focal length of a convex mirror.

Method (1) Using a convex lens.



(vii) The apparatus is arranged as shown above

(i) An object, O is placed in front of a convex lens L and its image formed at C

(ii) The distance LC is measured and recorded.

(iii) The convex mirror whose focal length, f, is required is placed is placed between L and C with its reflecting surface facing the lens.

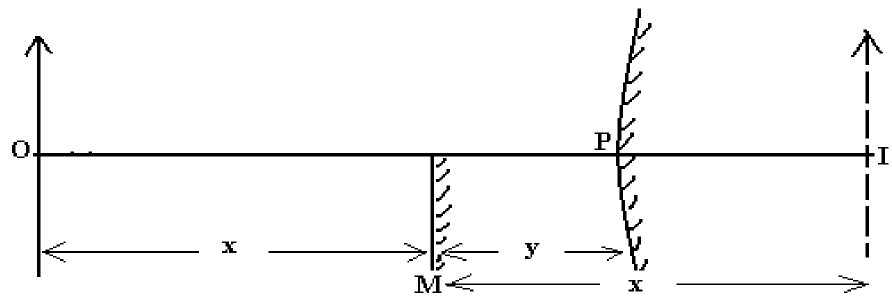
(iv) The lens is then moved along the axis, OC until a converging beam incident normally on the mirror forms its image at O

(v) Distance LP is measured

PC = LC – LP thus, f can be determined from f = $\frac{PC}{2}$

Or

Method (2) using No parallax.



(i) An object pin **O** is placed in front of a convex mirror as shown in the diagram above

(ii) A virtual diminished image is formed at **I**.

(iii) A plane mirror **M** is placed between **O** and **P** so as to intercept half the field of view of the convex mirror.

(iv) Mirror **M** is adjusted until its own image of **O** coincides with **I** by no parallax method.

(v) Measure the distances **x** and **y.**

(vi) The focal length of the mirror is calculated from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Where u = (x + y) and v = (y-x)

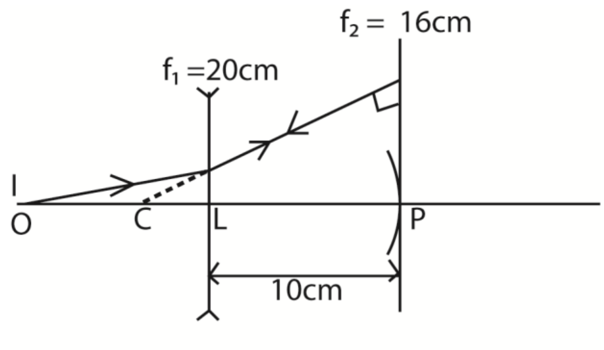
Note:

(i) The two images coincides when they are as far behind the plane mirror as the object is in front.

(ii) Substituting for u = x ? y and v = y ? x in the mirror formula gives $f = \frac{y^2 - x^2}{2y}$

(ii) A concave lens of focal length 20cm is placed 10cm in front of a concave mirror of focal length 16cm. Calculate the distance from the lens at which an object would coincide with its image. (04marks)

Ray diagram



The object to coincide with the image if the rays strike the concave mirror normally

i.e. CP = $2f_2$ = 16 x 2 = 32cm

but LP = 10cm

⇨ CL = 22cm
For concave mirror
Object distance be = u
Image distance = -22cm
From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

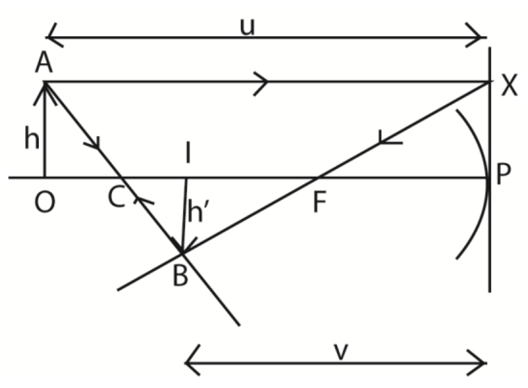
$$-\frac{1}{22} = \frac{1}{u} - \left(\frac{-1}{22}\right);\ u = -220cm$$

10. (a) Define the terms radius of curvature and focal length of a converging mirror. (02marks)

Radius of curvature of a converging mirror is the distance from the pole of the converging mirror to the center of curvature.

Focal length of converging mirror is the distance from the pole of the mirror to the principal focus.

(b) (i) Draw a ray diagram to show the formation of a real image of a real object in a converging mirror (02marks)



(ii) Use the ray diagram in (b)(i) to derive the expression, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$; where u. v and f are the object distance, image distance and focal length respectively. (05marks)

PC = r and PF = f

Triangle OAC and triangle ICB are similar

$\frac{OA}{IB} = \frac{u-2f}{2f-v}$

Also triangle

Triangle

XRF and PQF are similar

$\frac{XP}{IB} = \frac{PF}{IF} = \frac{PF}{IP-PF}$

$= \frac{f}{v-f}$

But XP = OA

$\therefore \frac{u-2f}{2f-v} = \frac{f}{v-f}$

$uv - uf - 2vf + 2f^2 = 2f^2 - vf$

$uv - uf - vf = 0$

after dividing by uvf

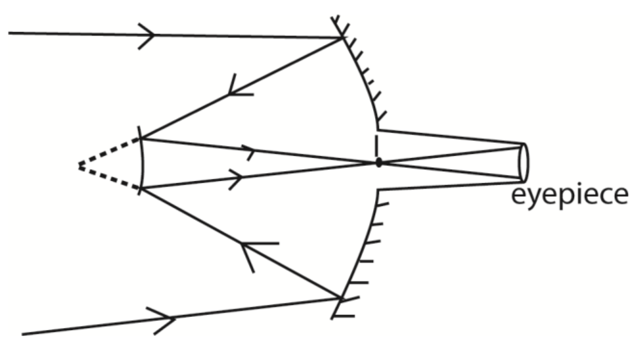
$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

(c) (i) With the aid of ray diagram, describe the structure and action of a reflecting telescope in normal adjustment. (05marks)

The objective is a large parabolic mirror with a circular aperture of long focal length. A beam of from a distant object is reflected and intercepted by a convex mirror.

The convex mirror forms a real and inverted image at I, the pole of the objective mirror and principal focus of the eyepiece

The eyepiece forms a virtual, magnified image at infinity.



(ii) State two advantages of reflecting telescope over an astronomical telescope. (02marks)

- has high resolving power

- has no chromatic aberration
- produce brighter images
- cheap
- spherical aberration is minimized
-

(d) An astronomical telescope has objective of focal length 100cm and eyepiece of focal length 10cm. Calculate the separation of the objective if the lenses are arranged in such a way that the final image is formed at 25cm from the eye. (04marks)

Action of the eyepiece

f = 10cm, v = -25cm, u= ?

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{10} = \frac{1}{u} - \frac{1}{25}$, u = 7.1cm

Separation of lenses = f_0 + u

=100 + 7.1 = 107.1cm

Thank you
Dr. Bbosa Science