SENIOR FIVE TERM 3

TOPIC 3/6: OPTICAL INSTRUMENTS

Competency: The learner explores the optical operation of different optical instruments that can be applied in a variety of fields such as medicine, military and navigation.

Optical instruments

An optical instrument is a device that aids vision working on the principles of reflection and refraction of light.

Examples are the human eye, microscope, telescope, projector and lens cameras.

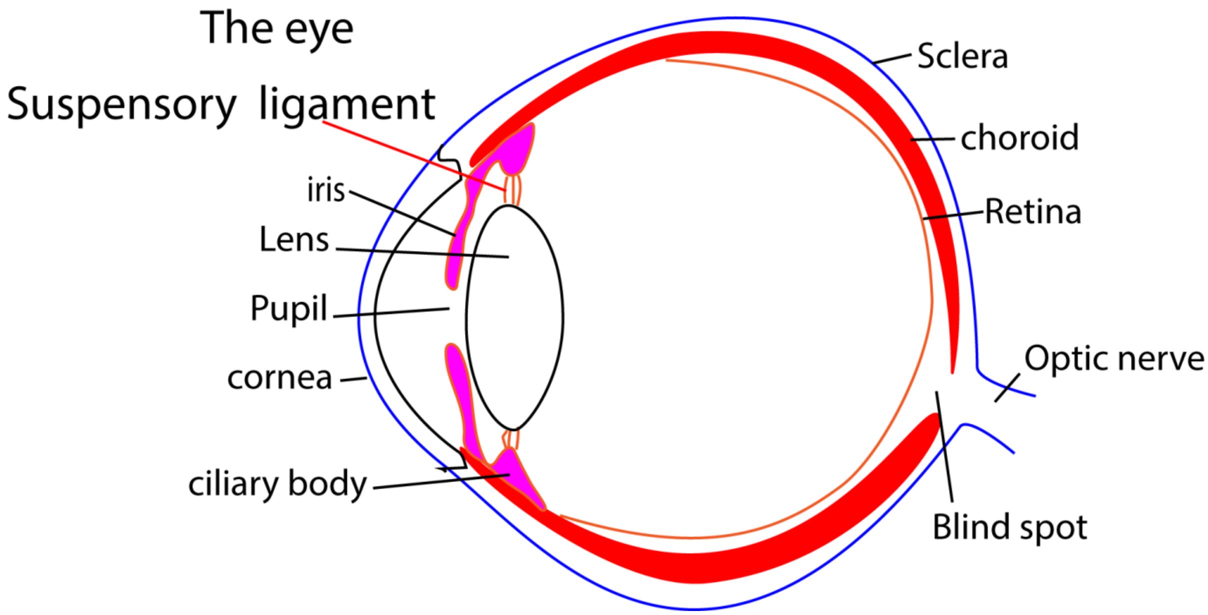
Terminology in optical instrument

1. Near point.
   This is the nearest position that can be focused distinctly by the unaided eye. The distance from the eye to the near point is known as the least distance of distinct vision (LDDV), and it is usually denoted by D.
   The least distance of distinct vision of a normal eye is 25cm.
2. The far point
   This is the furthest point which can be seen distinctly by the unaided eye. For normal eye, the farthest distance is infinity.
3. Accommodation
   This is the ability of the eye to clearly see near and far object.
4. Visual angle (α)
   This is the angle subtended by the object at the unaided eye.
5. Angular magnification (magnifying power, M)
   This is the ratio of the angle subtended at the eye by the image when using an instrument to the angle subtended at unaided eye by the object.
   M = $\frac{\alpha'}{\alpha}$ where α' and α are in radians

Note that the height of the image formed by the eye on the retina is proportional to the angle subtended at the eye by the object. (I.e. the greater the visual angle, the greater is the apparent size of the object).

The mammalian eye

This an organ that helps us to see



Parts of the eye

The iris controls the size of the pupil

The pupil allows in light into the eye

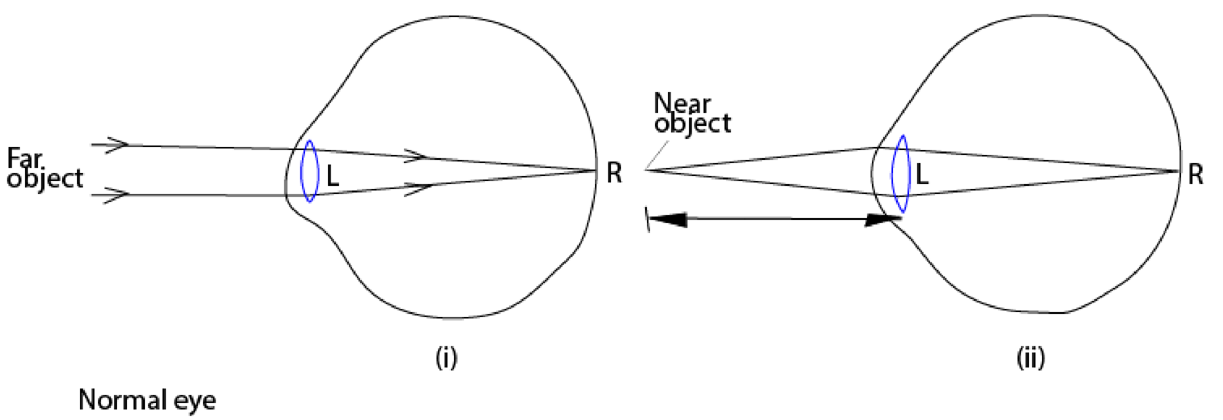
Ciliary body regulate the size of the lens

Cornea and lens refract light to the retina where the image forms

Sclera protects and maintains the shape of the shape of the eye.

Accommodation

This is the ability of the eye focus objects at different distances on the retina



To view near object the ciliary muscle contract creating less tension on suspensory ligaments the lens fattens and its refractive power increases.

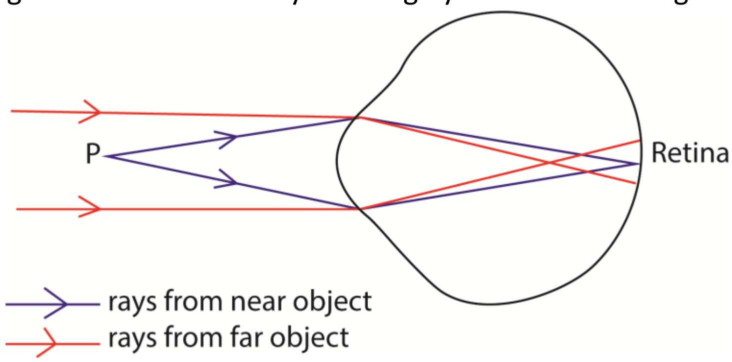
To view distant objects the ciliary relax creating a tension in the suspensory ligament, the lens this reducing its refractive power.

Eye defects

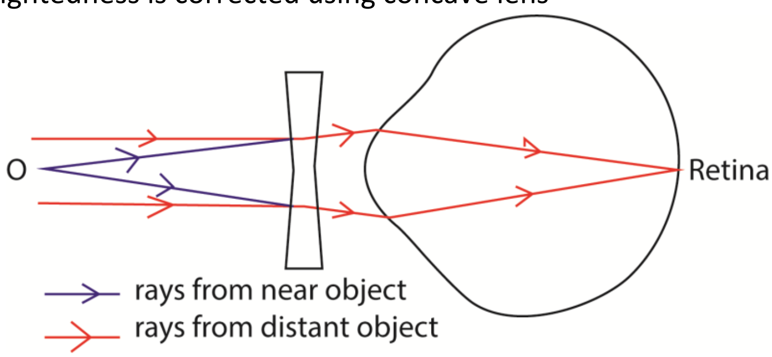
(a) Short sight/myopia

A person with short sightedness does not see distant objects clearly because rays of light from a distant object are focus in front of the retina

Short sightedness is caused by too long eye-ball or to strong lens



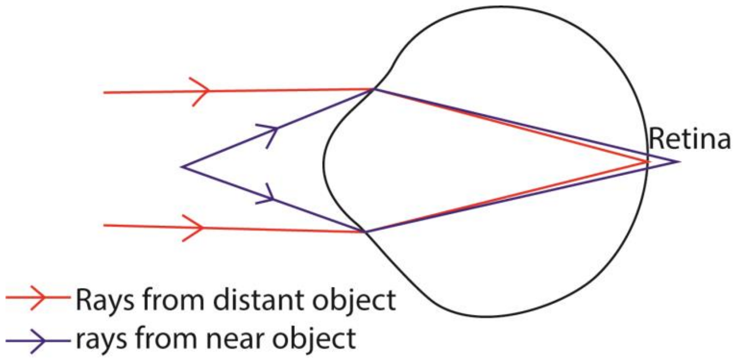
Short sightedness is corrected using concave lens



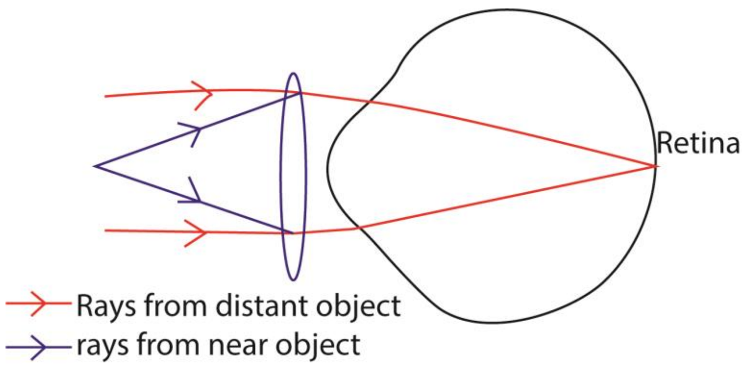
(b) Long sight/hyper myopia

A person with long sightedness does not see near objects clearly because rays of light from a near object are focus behind of the retina

Long sightedness is caused by too short eye-ball or to weak lens



Long sightedness is corrected using convex lens



Microscope

These are used to magnify near objects when in normal use. The image formed by microscope is at the least distance of distinct vision from the eye.

Types of microscope

(i) Simple microscopes (magnifying glasses)

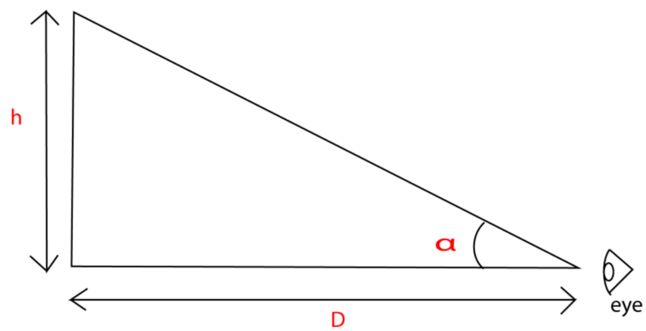
(ii) Compound microscope

Simple microscope

A magnifying glass consists of a converging lens which forms a virtual, upright image of the object placed inside its focal point.

(a) Final image in normal adjustment

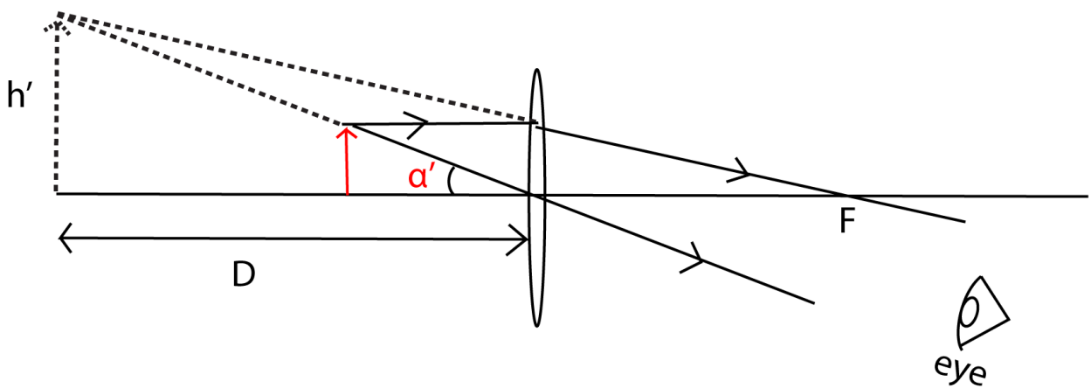
Consider an object subtending an angle α at the eye when the object is at the near point.



tanα ≈ α for small angle in radians

α =$\frac{h}{D}$ ………………………………… (i)

If a convex lens is used to view the object



For small angle tanα’ ≈ α’ for small angle in radians

α’ =$\frac{h\prime}{D}$ ………………………………… (ii)

Linear magnification from (i) and (ii), m = $\frac{\alpha'}{\alpha}$ = $\frac{h\prime}{D}$ $\div$ $\frac{h\prime}{h}$ = $\frac{h\prime}{h}$

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ and multiplying through by v

$\frac{v}{f} = \frac{v}{u} + 1$

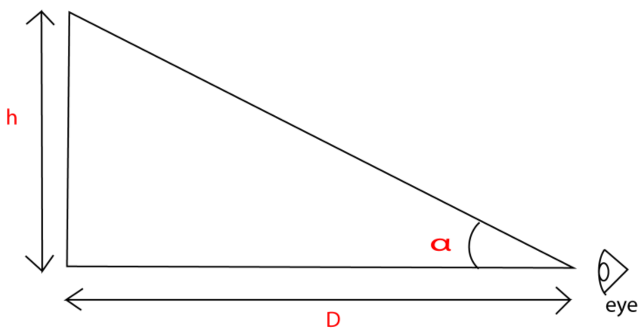
Since, v = D

$\frac{D}{f} = m + 1$ or m = $\frac{D}{f}$ -1 where f is the focal length

(b) Final image at infinity

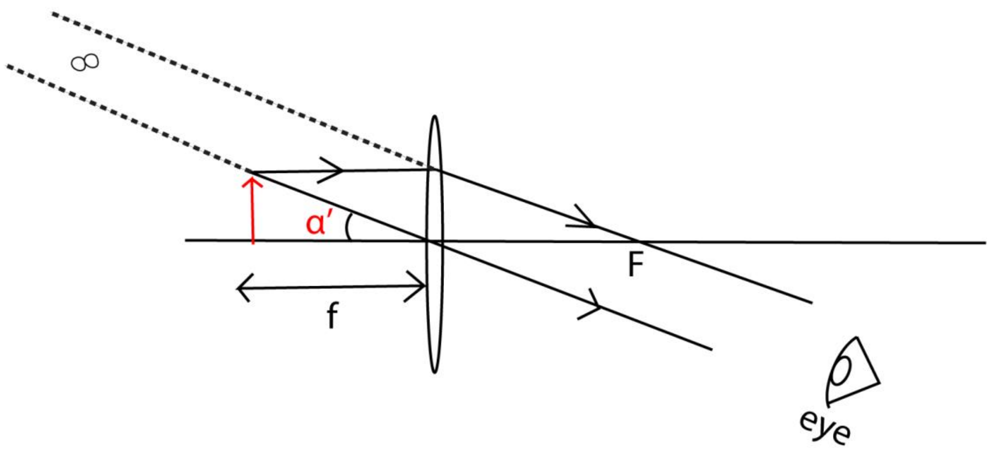
For the image to be formed at infinity, the object must be at, f.

Using unaided eye



tanα ≈ α for small angle in radians

α =$\frac{h}{D}$ ………………………………… (i)



For small angle tanα' ≈ α' for small angle in radians

α' =$\frac{h\prime}{f}$ ………………………………… (ii)

Linear magnification from (i) and (ii), m = $\frac{\alpha'}{\alpha}$ = $\frac{h\prime}{D}$ $\div$ $\frac{h\prime}{f}$ = $\frac{h\prime}{f}$

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ and multiplying through by v

$\frac{v}{f} = \frac{v}{u} + 1$

Since, v = f

$\frac{D}{f} = m + 1$ or m = $\frac{D}{f}$ -1 where f is the focal length

It should be noted that higher magnification is obtained from a lens of short focal length.

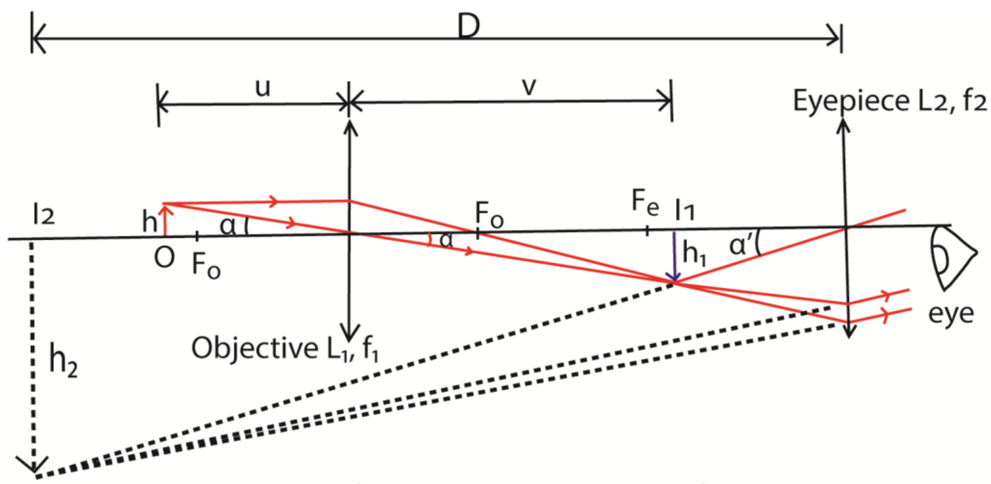
Compound microscopes

(i) These utilize two converging lenses of short focal lengths are used.

(ii) The lens closer to the object is called objective lens while that through which the final image is viewed is called the eyepiece.

(iii) When the microscope is used, the object O is placed at a distance slightly bigger than the principal focus, F, of objective lens, to form an inverted image I_1.

(iv) I_1 the a real object of the eye piece forms between its principal focus F_0 and its optical center so that a large virtual image I_2 is viewed by the eyes at least distance D.



Compound microscope in normal use

For an aided eye (using the instrument)

For small angle, tanα' ≈ α' for small angle in radians

$\alpha' = \frac{h_2}{D}$ .... (i)

For unaided eye

$\alpha = \frac{h_1}{D}$ .... (ii)

Magnifying power, $m = \frac{\alpha'}{\alpha} = \frac{h_2}{D} \div \frac{h_1}{D} = \boldsymbol{\frac{h_2}{h_1}}$

Or

Linear magnification can also be expressed as, $m = \frac{h_2}{h} \; x \; \frac{h}{h_1} = m_e \text{ x } m_o$.

Where, m_e and m_o are linear magnifications of the eye piece and objective lenses respectively.

$m_o = \frac{v}{f_0} - 1$ and $me = \frac{-D}{fe} - 1$

$m = (\frac{-D}{fe} - 1)(\frac{v}{f_0} - 1)$

Examples 1

The objective of a compound microscope has a focal length of 2cm while the eye piece has a focal length of 5cm. an object is paced a distance of 2.5cm in front of objective lens. The distance of the eye piece from the objective is adjusted so that the final image is 25cm in front of the eyepiece. Find the separation of the lenses.

Solution

u_0 =2.5cm, f_0 = 2.0cm, v_0?,

v_e = D = -25cm, u_2 ?

For objective lens

$\frac{1}{f_0} = \frac{1}{u_0} + \frac{1}{v_0}$

$\frac{1}{2} = \frac{1}{2.5} + \frac{1}{v_0}$ => v_0 = 10cm

$Also\ \frac{1}{fe} = \frac{1}{u_e} + \frac{1}{v_e}$

$\frac{1}{5} = \frac{1}{u_e} + \frac{1}{-25}$ => u_e = -4.2cm

$Separation = v_0 - u_e$ = 10 - $^-$4.2 = 14.2cm

Example 2

The objective and eye piece of a compound microscope have focal lengths of 1cm and 5cm respectively. the distance of lens separation is 25 cm. The object is placed1.05cm from the objective lens.

(a) Find the position, and nature of the final image
(b) Calculate the magnification

Solution

In objective lens

$\frac{1}{f_0} = \frac{1}{u_0} + \frac{1}{v_0}$

$\frac{1}{1} = \frac{1}{1.05} + \frac{1}{v_0}$; v_o = 21

$using\ eyepiece\ u_e = 25 - 21$ = 4cm, f_e = 5

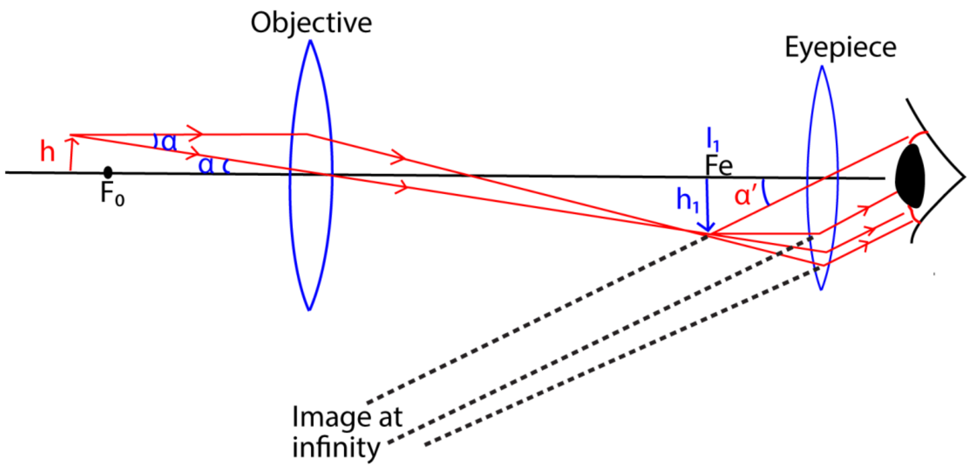
$\frac{1}{5} = \frac{1}{4} + \frac{1}{v_e}$; v_e = -20cm

Nature of image, virtual, magnified and inverted.

(b) Magnifying power, m = ($\frac{20}{5}$ – 1)($\frac{21}{1}$ – 1) =80

Microscope with image at infinity

To form the final image at infinity, the image of the objective lens must be formed at the principal focus. F_e, of eyepiece



Microscope with image at infinity

For unaided eye (using the instrument)

For small angle, tanα' ≈ α' for small angle in radians

$\alpha = \frac{h}{D}$ ……………………………………… (i)

For aided eye

$\alpha' = \frac{h_1}{f_e}$ ……………………………………… (ii)

Magnifying power, $m = \frac{\alpha'}{\alpha} = \frac{h_1}{f_e} \div \frac{h}{D} = \boldsymbol{\frac{h_1}{f_e} x \frac{D}{h}}$

But $\frac{h_1}{h}$ is the linear magnification of objective lens

$m_o = \frac{h_1}{h} = \left(\frac{v}{f_0} - 1\right)$

$m = \left(\frac{v}{f_0} - 1\right) \frac{D}{f_e}$

NB: D is always 25cm for this case

Example 3

An object is placed at a distance of 5cm from the objective of focal length 4cm. if the eye piece has focal length of 15cm and final image is formed at infinity, calculate

(i) Separation of the lens

(ii) Final magnification of the microscope.

Solution

For objective lens, u =5cm, f_0 = 4cm, v =?

From $\frac{1}{f_0} = \frac{1}{u_0} + \frac{1}{v_0}$

$\frac{1}{4} = \frac{1}{5} + \frac{1}{v_0}$, vo = 20cm

Separation of lenses = $v_o + f_e$ = 20 + 15 = 35cm.

(ii) D is always 25cm

Magnification, m = $(\frac{v}{f_0} - 1)\ \frac{D}{f_e}$ = $(\frac{20}{4} - 1)\ x\ \frac{25}{15}$ =6.7

Telescopes

These are instruments focus the light and make distant objects appear brighter, clearer and magnified

They are mainly divided into two

- Refractive telescope
- Reflective telescope

Refractive telescope

These are telescopes which use lenses and they include

(a) Astronomical telescope
(b) Terrestrial telescope
(c) Galilean telescope

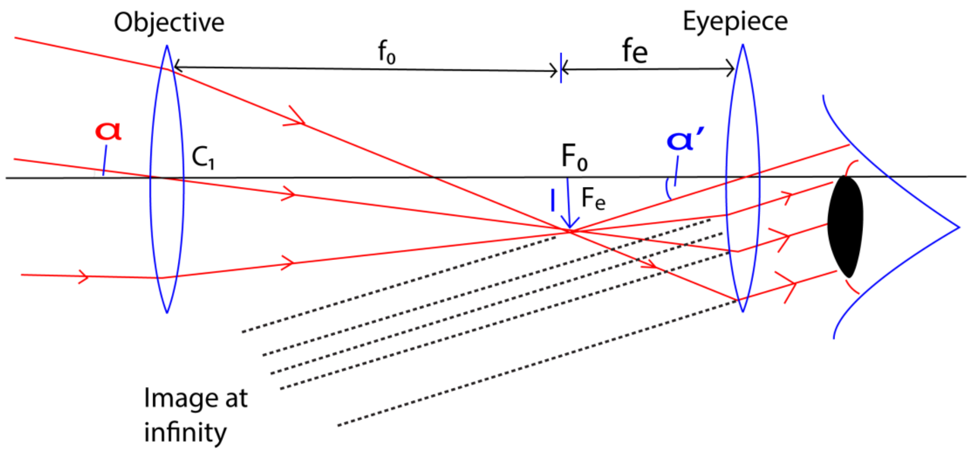
Astronomical telescope

This consists of two convex lenses: the objective of a long focal length and eye-piece of short focal length. The objective forms a real diminished image of a distant object at its focal length.

(a) Astronomical telescope in normal adjustment (image at infinity)

Note,

- when an astronomical telescope is in normal adjustment, the final image is formed at infinity
- the focal length of the objective is bigger than that of eye piece
- the principal focuses of the two lenses coincide



Telescope in nornal use

To obtain the magnification, m, we assume the eye is very close to the eye piece.

For an aided eye (using the instrument)

For small angle, tanα' ≈ α' for small angle in radians

$\alpha' = \frac{h}{f_e}$ .... (i)

Where, h, is the height of image I, f_e is the focal length of eye piece

For unaided eye

$\alpha = \frac{h}{f_0}$ .... (ii)

f_0 is the focal length of objective lens

Combining equations (i) and (ii)

Magnifying power, $m = \frac{\alpha'}{\alpha} = \frac{h}{f_e} \div \frac{h}{f_o} = \frac{\boldsymbol{f_o}}{\boldsymbol{f_e}}$

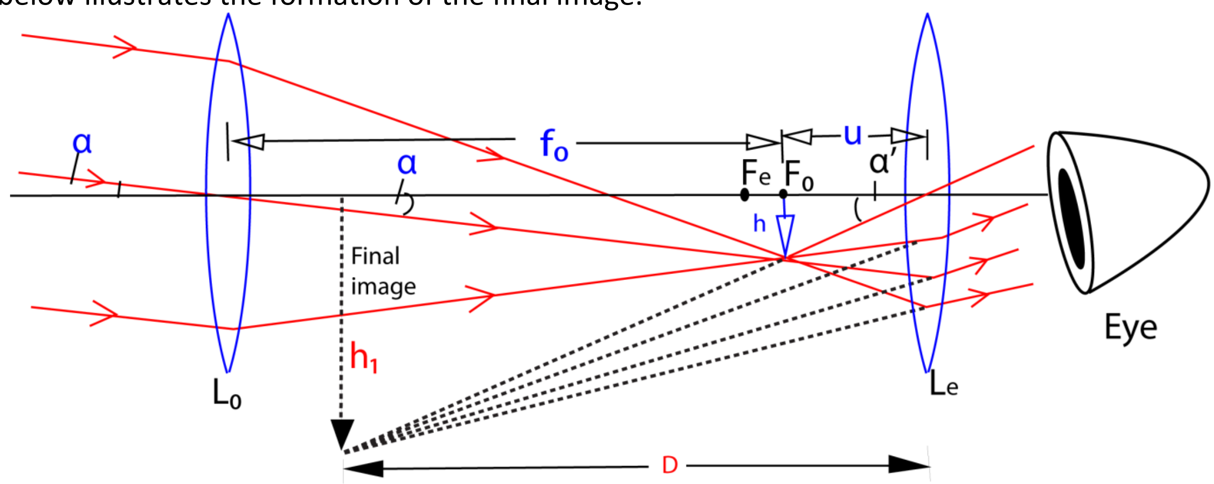
Thus the angular magnification is the ratio of the focal length of objective lens to the focal length of the eyepiece.

Note that, bigger magnification power is obtained when the focal length of objective lens is very big compared to the focal length of eye piece.

Secondly the separation of the lenses is the sum of the focal length of the objective and eye-piece lenses, (f_0 + f_e).

(b) Astronomical telescope with final image at near point

When the image of a telescope forms an image at near point, the telescope is not in normal adjustment, the eye is accommodated, although the image is still clearly seen. The diagram below illustrates the formation of the final image.



Final image at near point

The objective lens forms a real image of the distant object at its the principal focus F_0, and the

eye piece is moved so that the image is nearer to it than its focus, F_e, thus acting as a magnifying glass. The separation of the lens is (f_o + u).

To obtain the magnification, m, we assume the eye is very close to the eye piece.

For an aided eye (using the instrument)

For small angle, tanα' ≈ α' for small angle in radians

$\alpha' = \frac{h_1}{D}$ .... (i)

Where, h, is the height of image I, f_e is the focal length of eye piece

For unaided eye

$\alpha = \frac{h}{f_0}$ .... (ii)

f_0 is the focal length of objective lens

combining equations (i) and (ii)

Magnifying power, $m = \frac{\alpha'}{\alpha} = \frac{h_1}{D} \div \frac{h}{f_0} = \frac{\boldsymbol{h_1}}{\boldsymbol{h}}\ \boldsymbol{x}\ \frac{\boldsymbol{f_0}}{\boldsymbol{D}}$

Since $\frac{h_1}{h}$ is linear magnification

$\frac{\boldsymbol{h_1}}{\boldsymbol{h}} = \mathbf{m_e} = \left[\frac{\boldsymbol{D}}{\boldsymbol{f_e}} - \mathbf{1}\right]$

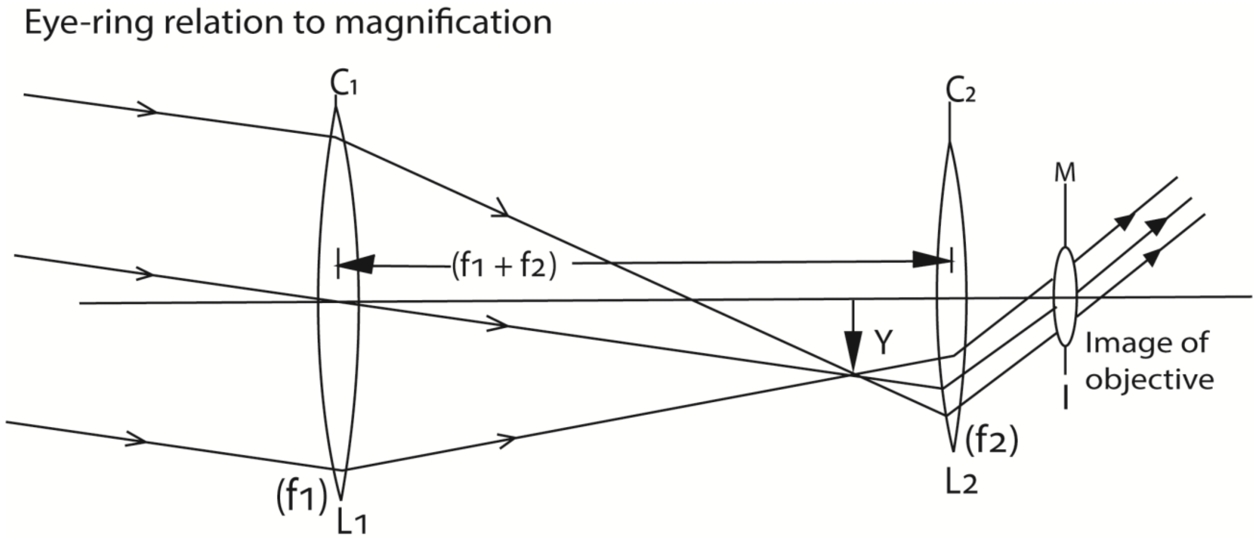
$\boldsymbol{m} = \left[\frac{\boldsymbol{D}}{\boldsymbol{f_e}} - \mathbf{1}\right]\frac{\boldsymbol{f_0}}{\boldsymbol{D}}$

Note that the magnifying power of astronomical telescope in normal use is greater than when in its normal adjustment.

Eye ring

The eye-ring the best position for the eye, is the circular image of the objective in the eye-piece.

The figure below illustrates three rays from a point on the distant object which pass through the objective, forming an image at Y.



The position of the eye ring, M, can be calculated as follow position as

$C_1C_2 = f_1 + f_2$

The focal length of eye piece, $L_2 = f_2$

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

The distance C_2M or v is given by

$\frac{1}{f_2} = \frac{1}{v} + \frac{1}{(f_1+f_2)}$

$v = \frac{f_2}{f_1}(f_1 + f_2)$

$\frac{Objective\ diameter}{eye-ring\ diameter} = \frac{C_1C_2}{C_2M} = \frac{u}{V}$

$= (f_1 + f_2) \div \frac{f_2}{f_1}(f_1 + f_2)$

$= \frac{f_1}{f_2}$

But angular magnification of telescope in normal adjustment = $\frac{f_1}{f_2}$

Thus anglar magnification, M = $\frac{Objective\ diameter}{eye-ring\ diameter}$

Advantages of refractive astronomical telescope

- It is less bulky since it is more compact
- Refractor telescopes are rugged. After the initial alignment, their optical system is more resistant to misalignment than the reflector telescopes.
- The glass surface inside the tube is sealed from the atmosphere so it rarely needs cleaning.
- Since the tube is closed off from the outside, air currents and effects due to changing temperatures are eliminated. This means that the images are steadier and sharper than those from a reflector telescope of the same size.

Disadvantages of astronomical telescope to view objects on earth

- form inverted images

Example 4

An astronomical telescope consists of two thin converging lenses of focal length 100m and 10cm respectively. if the final image of a distant object is virtual and is 20cm from eyepiece. Find the separation of the lenses.

Solution

f_0 = 100cm, f_e=cm, v = -20cm

From, $\frac{1}{f_e} = \frac{1}{u_e} + \frac{1}{v_e}$

$\frac{1}{10} = \frac{1}{u} + \frac{1}{-20}$, u = 7cm

Separation = f_e + u = 100 + 7 = 107cm

Example 5

(a) What is the meant by the following as applied to a telescope

(i) Magnifying power

The ratio between the dimensions of the image and the object.

(ii) Eye ring

This is the ideal position of the eye for observing the image through the telescope.

(b) (i) Draw a ray diagram to show how the formation of the final image by an astronomical telescope in normal use.

(ii) With the aid of the diagram in (b)(i), derive an expression for the magnifying power of astronomical telescope in normal use.

(iii) Give the disadvantage of the telescope in (b)(i) when used to view distant object on earth. Describe how the telescope can be modified to overcome this disadvantage.

Disadvantage: it forms inverted image. This can be overcome by using an erect lens or converting it into a terrestrial telescope.

(c) Find the separation of eyepiece and objective of astronomical of magnifying power 20 and in normal adjustment if its eyepiece has a focal length of 5cm

Solution

Separation = f_0 + fe

m = 20, f_e = 5cm

but m = $\frac{f_0}{f_2}$

20 = $\frac{f_0}{5}$

f_o =100cm

Separation = 100 +5 = 105cm

Terrestrial telescope

This uses one more convex lens than astronomical refracting telescope. It has an erecting lens of focal length, f, placed between the objective and the eyepiece lens.

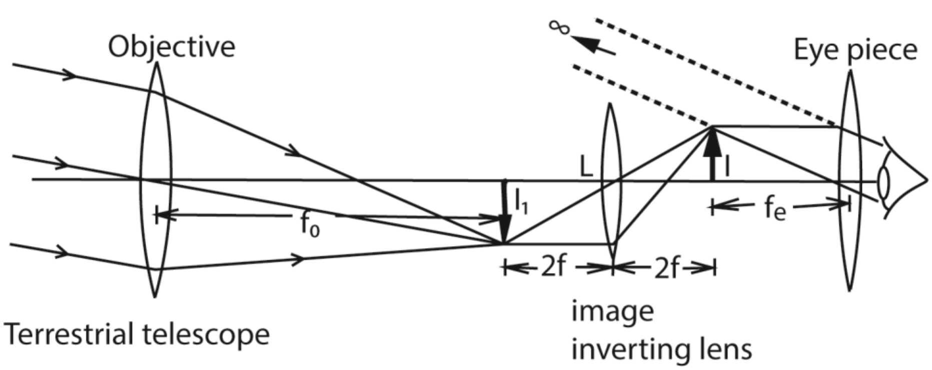
The erecting lens is positioned such that image, I, of the object is formed at 2f.

The terrestrial telescope is longer than the astronomical telescope by 4f; where f is the focal length of the erecting lens.

Angular magnification, m = $\frac{f_0}{f_e}$

It is used to view different objects on the earth surface

The objective lens forms a real image I1



Action

(i) The objective lens forms a real image I_1 of the distant object at its principal focus, F_o. which is at 2f of erecting lens L
(ii) The erecting lens L form an erect image I at its 2f and at principal focus, Fe, of the eye piece. The images I_1 and I have equal sizes
(iii) The eye piece forms a virtual image at infinity.
(iv) The angular magnification of terrestrial and astronomical telescopes is the same in the same adjustments.

Advantages of a terrestrial telescope

It forms upright images

Disadvantages of a terrestrial telescope

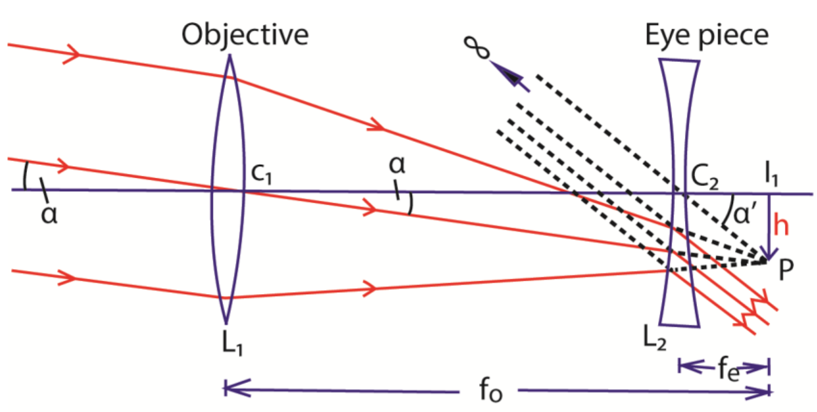
- Long and bulky
- Extra diffraction occurs in erecting lens reducing the clarity of the final image

Galilean telescope

It consists of two lenses, the objective which is converging lens of long focal length and the eye piece which is a diverging lens of short focal length. It is intended to produce an erect final image. Always the final image produced by the objective beyond the eyepiece.

The separation of the lenses is equal to the difference in the magnitude of focal lengths, i.e. $(f_0 - f_e)$.

(i) Galilean telescope in normal adjustment (final image is at infinity)



Galilean telescope in normal adjustment

Light from distant object incident on objective lens is refracted to form real inverted image at its principal focus f_0. In normal adjustment, the principal focuses of both lenses coincide. The eye piece therefore forms the final image of the object at infinity.

To obtain the magnification, m, we assume the eye is very close to the eye piece.

m = $\frac{\alpha'}{\alpha}$

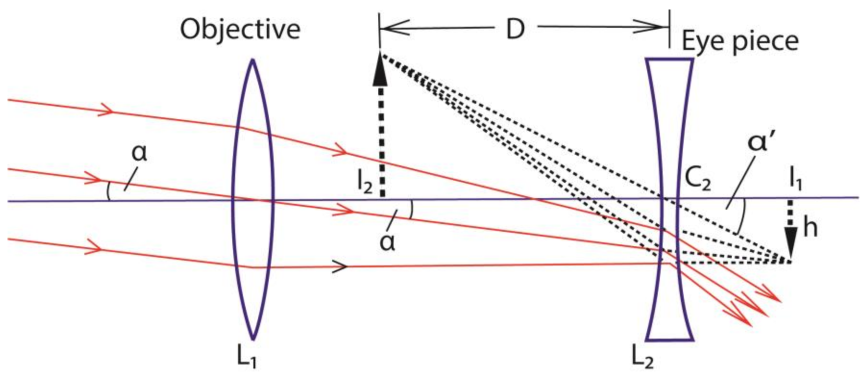
For small angles, α and α' are measured in radians

α ≈ tan α = $\frac{h}{f_e}$ and α' ≈ tan α = $\frac{h}{f_0}$

Substituting for α and α'

Magnifying power, m = $\frac{\alpha'}{\alpha} = \frac{h}{f_e} \div \frac{h}{f_o} = \frac{\boldsymbol{f_0}}{\boldsymbol{f_e}}$

(ii) Galilean telescope not in normal adjustment (final image at near point)



Galilean telescope image at near point

To obtain the magnification, m, we assume the eye is very close to the eye piece.

For an aided eye (using the instrument)
For small angle, tanα' ≈ α' for small angle in radians
α' =$\frac{h'}{D}$ ………………………………………… (i)
Where, h, is the height of image I, f_e is the focal length of eye piece
For unaided eye
α =$\frac{h}{f_0}$ …………………………………………. (ii)
f_0 is the focal length of objective lens

Combining equations (i) and (ii)
Magnifying power, m = $\frac{\alpha'}{\alpha} = \frac{h'}{D} \div \frac{h}{f_o} = \frac{h'}{h} \boldsymbol{x} \frac{\boldsymbol{f_0}}{\boldsymbol{D}}$
Since $\frac{h_1}{h}$ is linear magnification
$\boldsymbol{\frac{h_1}{h}}$ **= $\mathbf{m_e}$ =** $\left[\frac{\boldsymbol{D}}{\boldsymbol{f_e}} - \mathbf{1}\right]$

$$m = \left[\frac{D}{f_e} - 1\right]\frac{f_0}{D}$$

Advantages of a Galilean telescope

It forms a final erect image.

It is shorter than the terrestrial and astronomical telescopes because the separation of lenses is f_0-f_e.

Disadvantages of a Galilean telescope

It has a small field of view.

It has a virtual eye ring not accessible to the observer.

Example 6

A convex lens of focal length 60m is arranged coaxially with a diverging lens of focal length 5cm to view a distant star. Calculate the magnifying power if the image of the star is formed at a distance of 25cm in front of the eyepiece.

Solution

f_0 = 60cm, f_e = -5cm, D -25cm

From, $m = \frac{f_0}{f_e}\left[1 - \frac{f_e}{D}\right] = \frac{60}{-5}\left[1 - \frac{-5}{-25}\right] = 9.6$

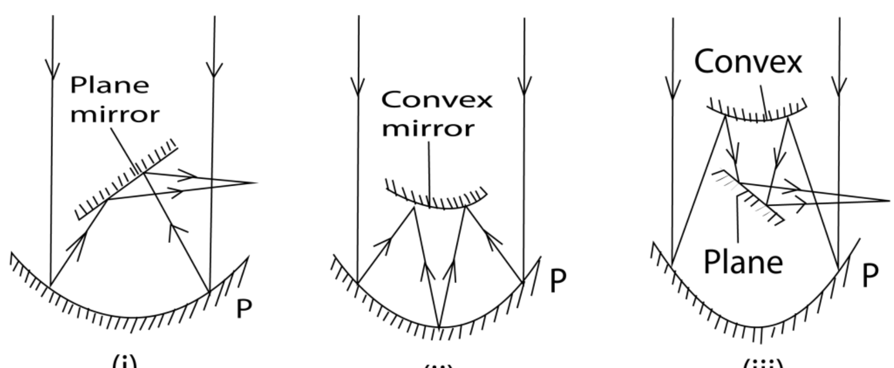
Eye ring of a telescope [Exit pupil]

This is the best position where the eye can be placed so as to receive all rays that are refracted by the objective lens

Reflecting telescope

In these telescopes, a concave mirror of long focal length is used instead of convex lenses as objectives.

They include



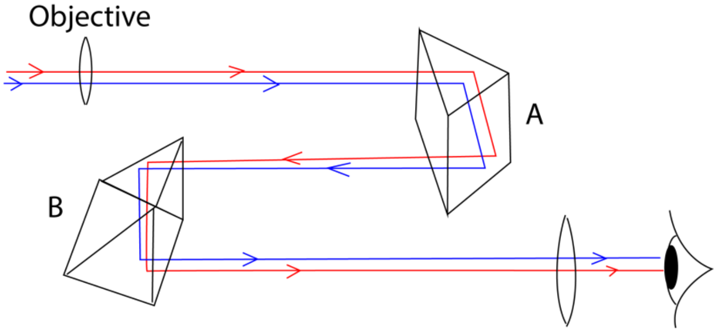
(i) Newton reflector (ii) Cassegrain reflector (iii) Coude reflector

Advantages of reflecting telescopes

(i) There is no chromatic aberration since no refraction occurs at the objective
(ii) There is no spherical aberration since a paraboloidal mirror is used.
(iii) It is cheaper to construct since only one surface requires grinding.
(iv) When curved mirrors of large diameter are used, a greater resolving power is obtained.

The prism binoculars

It consists of a pair of a refracting astronomical telescope with two reflecting prisms between each objective and eye piece



Prism binoculars

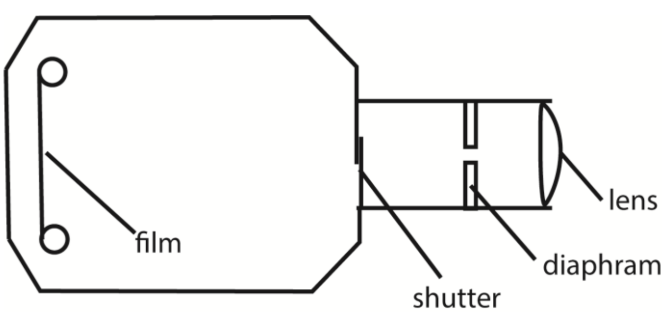
Action

(i) The objective lens forms a real inverted of a distant object at its principal focus in from of prism A
(ii) By total internal reflection, the prism A causes a lateral inversion direction through 180^0.
(iii) The right angled prism B, then inverts the image such that an erect image is formed in front of the eye piece.
(iv) The eye piece then forms a virtual magnified image of the object.

The lens camera

It consists of a lens system, a focusing device, an exposure system and light sensitive film.

Lens camera

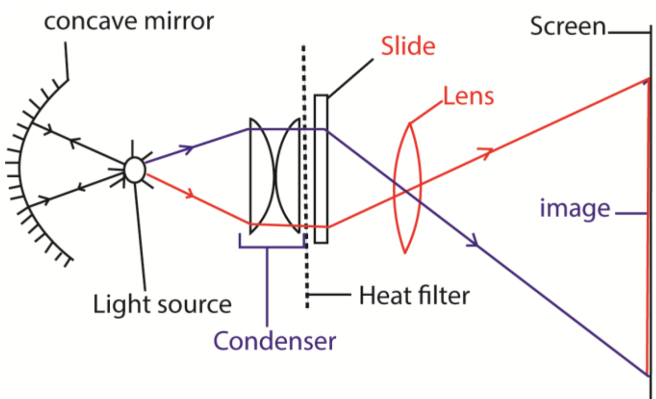


- The lens focused light from the object on to film
- The diaphragm controls the amount of light reaching the film
- The shutter controls the exposure time of light reaching the film
- Film contain photosensitive chemicals and is where the image is stored.

Projection lantern

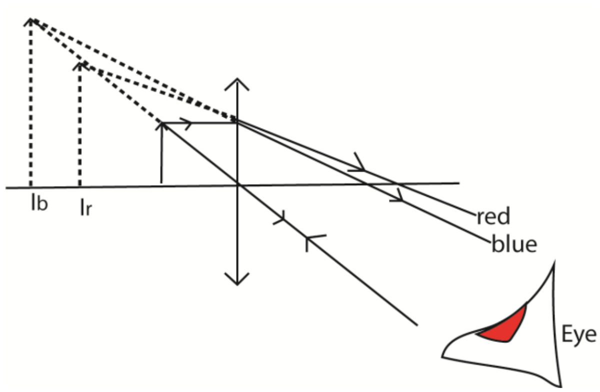
A **projector** is an instrument used to produce a large image of a small object.

- The slide, or film, is placed behind the **projector** lens outside its focal length and is illuminated buy a small but powerful source of light from concave reflector through condensing lenses.
- A magnified, real and inverted image on the screen



Area magnification = $\frac{Area\ of\ the\ image}{Area\ of\ the\ object}$

Elimination of chromatic aberration in images formed by magnifying lens



In the lens the light rays from the object are dispersed to form colored images I_b and I_r due the different refractive indices; but when the eye is placed close to the lens, the mages subtend the same angle to the eye. The images overlap reducing chromatic aberration

Revision exercise 1

1. (a) What is meant by

(i) Visual angle

It an angle subtended by an object or image at the eye.

(ii) Near point.

It is the nearest point at which the eye is able to view the object clearly.

(iii) Linear magnification

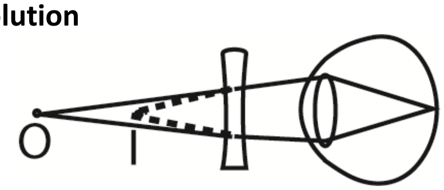
This a ratio of image height to object height

(iv) Angular magnification

It a ratio of the angle subtended by the final image to eye when using a lens or optical instrument to the angle subtended by the object to unaided eye.

(b) A person with a normal near point distance of 25cm away wears spectacles with a diverging lens of focal length 200cm in order to correct the far point distance to infinity. Calculate the near point when viewing using spectacles.

Solution



Object O appears to be at I, the near point of the eye.

Now, v = -25, f= 200cm

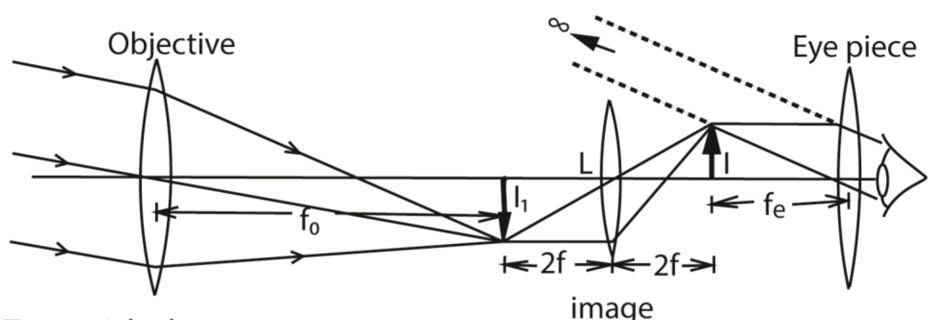
From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{-200} = \frac{1}{u} + \frac{1}{-25}$

u = 28.6cm

Therefore, the near point = 28.6cm.

(c) Draw a ray diagram to show the formation of an image of a distant object in terrestrial telescope in normal adjustment.



Terrestrial telescope

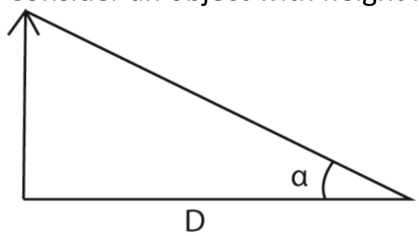
(d) State two disadvantages of terrestrial telescope.

- It long and bulky due extra image inverting lens
- Extra diffraction occurs in erecting lens reducing the clarity of the final image

2. Derive an expression for magnifying power of magnifying glass when the final image is formed at near point.

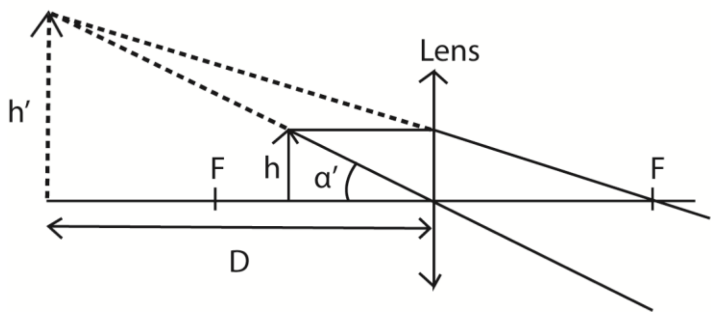
Solution

Consider an object with height h viewed at near point by unaided eye



The visual angle α measured in radians is given by α ≈tan α = $\frac{h}{D}$

If the final image at near point subtends an angle α' to the eye when a magnifying glass is used as in the diagram below



If α' is small angle, in radian, α' = $\frac{h\prime}{D}$

Magnification = $\frac{\alpha\prime}{\alpha}$ = $\frac{h\prime}{D} \div \frac{h}{D}$

Or

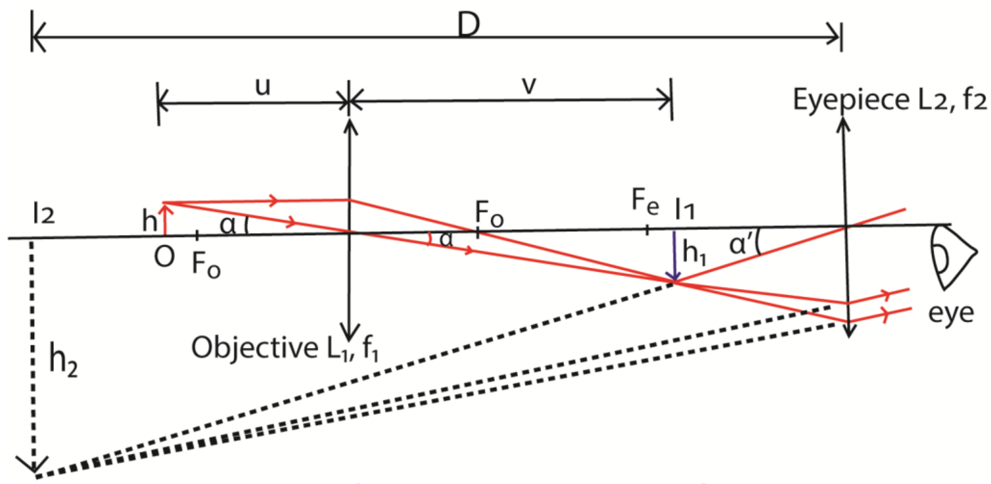
Magnification, M = $\frac{v}{f} - 1$ but v= -D (image is virtual)

= $\frac{-D}{f} - 1$

= $-\left(\frac{D}{f} + 1\right)$

$|M|$ = $\left(\frac{D}{f} + 1\right)$

3. (a) Draw a diagram to show the formation of an image by a compound microscope in in normal adjustment and use it to derive an expression for magnifying power



Compound microscope in normal use

The object is placed a distance u slightly greater than the focal length f_0 of objective lens to form a real image I_1 between the principal focus Fe of the eyepiece and its optical center to form a virtual image I_2.

For an aided eye (using the instrument)

For small angle, tanα' ≈ α' for small angle in radians

$\alpha' = \frac{h_2}{D}$ .... (i)

For unaided eye

$\alpha = \frac{h_1}{D}$ .... (ii)

Magnifying power, $m = \frac{\alpha'}{\alpha} = \frac{h_2}{D} \div \frac{h_1}{D} = \boldsymbol{\frac{h_2}{h_1}}$

Or

Linear magnification can also be expressed as, $m = \frac{h_2}{h} x \frac{h}{h_1} = m_e \times m_o$.

Where, m_e and m_o are linear magnifications of the eye piece and objective lenses respectively.

$m_o = \frac{v}{f_0} - 1$ and $me = \frac{-D}{fe} - 1$

$m = (\frac{-D}{f_e} - 1)(\frac{v}{f_0} - 1)$

(e) A microscope has an objective of focal length 10.0cm and eyepiece of focal length 20cm. if the distance between the objective and eyepiece is 20cm, calculate the magnifying power of the microscope. (03marks)

Solution

Given f_0 = 10cm, f_e = 20cm, D = 25cm, $v_0 + u_e$ = 20cm

Using eye piece lens

$\frac{1}{fe} = \frac{1}{u_e} + \frac{1}{-D}$

$\frac{1}{20} = \frac{1}{u_e} + \frac{1}{-25}$; u_e = 11.11cm

but $v_0 + u_e$ = 20cm

v_o = 20 – 11.11 = 8.89cm

From m = $(\frac{-D}{f_e} - 1)(\frac{v}{f_0} - 1)$

m = $(\frac{-25}{20} - 1)(\frac{8.89}{10} - 1)$ = 0.02775

4. (a) A compound microscope consists of two thin lenses, an objective of focal length 1.0cm and eye piece of focal length 5.0 cm. the objective forms an image of an object placed in front of it at a point 16.0cm away. If the final image is formed at the near point of the eye, calculate

(i) Separation of the lenses (03marks)

Solution

Considering the eye piece; from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{1} = \frac{1}{u} + \frac{1}{25}$; u = 4.2cm

Separation of lenses = $v_0 + v_e$ = 16.0 + 4.2 = 20.2cm

(ii) Magnifying power of the instrument (03marks)

Magnifying power, M = $-\left(\frac{v_0}{f_0} - 1\right)\left(\frac{D}{f_e} - 1\right) = -\left(\frac{16}{1} - 1\right)\left(\frac{25}{5} - 1\right)$ = -90

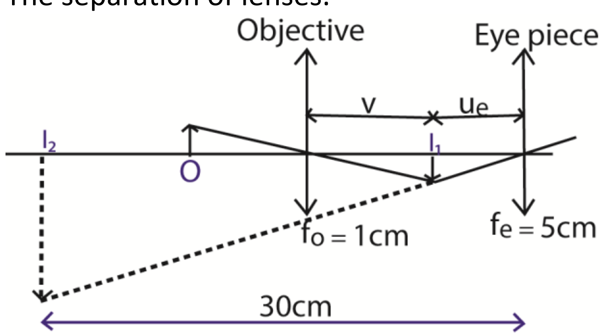
Negative magnification because the image is virtual

(b) Why should the objective and eye piece for compound microscope have short focal length?

From the expression of magnification i.e. M = $-\left(\frac{v_0}{f_0} - 1\right)\left(\frac{D}{f_e} - 1\right)$ high magnification is achieved when the focal lengths f_o and f_e are small.

5. (a) A compound microscope consist of two converging lenses of focal lengths 1.0cm and 5.0cm respectively. An object is placed 1.1cm from objective and the microscope is adjusted so that the final image is formed 30cm from eye piece. Calculate

(i) The separation of lenses.



From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Objective

$\frac{1}{1.0} = \frac{1}{1.1} + \frac{1}{v}$; v = 11cm

For eye piece

$\frac{1}{5} = \frac{1}{u_e} + \frac{1}{-30}$; u_e = 4.3cm

$separation = v + u_e = 11 + 4.3 = 15.3cm$

(ii) The magnifying power of the lenses.

Magnifying power, M = $-\left(\frac{v_0}{f_0} - 1\right)\left(\frac{v_e}{f_e} - 1\right)\frac{D}{v_e}$

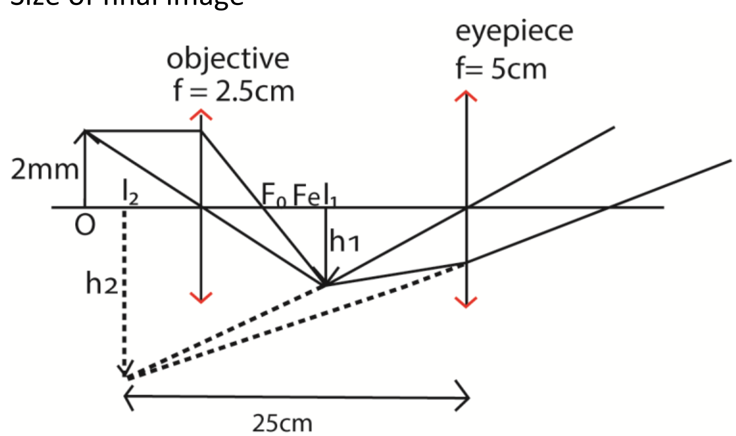
M = $-\left(\frac{11}{1} - 1\right)\left(\frac{30}{5} - 1\right)\frac{25}{-30}$ = 58.3

(b) State two differences between a compound microscope and an astronomic telescope

Compound microscope	Astronomical telescope
- View near objects - Objective lens has smaller focal length - In normal adjustment the final image is at near point - Has greater resolving power	- View distant object - Objective lens has longer focal length - In normal adjustment, final image at infinity - Has lower resolving power

6. An object of size 2.0mm is placed 3.0cm in front of objective of compound microscope. The focal length of objective is 2.5cm while that of the eye piece is 5.0cm. The microscope forms a virtual image of the object at near point of the eye. Find the

(i) Size of final image



The objective forms a real magnified image I_1 of height, h_1 at a distance less than f_e from the eye piece. The eye piece forms a magnified image $I_2$of height h_2 at near point.

Action of objective

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{2.5} = \frac{1}{3} + \frac{1}{v}$; v =15

M0 = $\frac{h_1}{2} = \frac{15}{3} = 5$; h1 = 10mm

Action of the eye piece

$\frac{1}{5} = \frac{1}{u} + \frac{1}{-25}$; u = 4.2cm

$M_e = \frac{25}{4.2} = \frac{h_2}{10}$; h_2 = 59.5mm

$the\ size\ of\ final\ image = 59.5$mm

(ii) Position of eye-ring

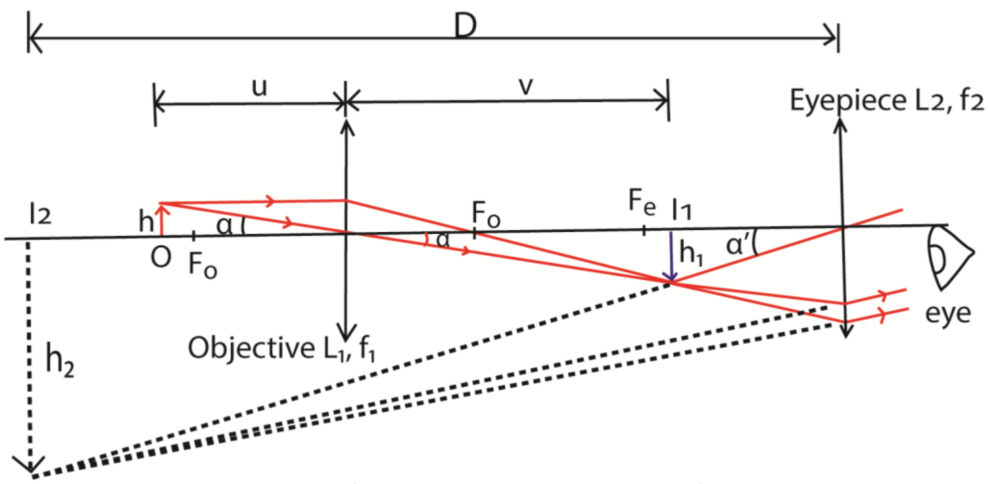
Separation = 15 +4.2cm = 19.2cm

The distance, v, of the image of objective in eye piece is given by

$\frac{1}{5} = \frac{1}{19.2} + \frac{1}{v}$; v = 6.8cm

$the\ position\ of\ eye\ ring = 6.8$cm from eye piece.

7. Describe with aid of a diagram, a compound microscope forms image at near point



Compound microscope in normal use

Note that

(i) A compound microscope forms an image at the near point when the image of the objective is formed between principal focus and optical center of the eyepiece.

(ii) In normal adjustment and image at infinity, the image of objective is formed on the principal focus of the eyepiece.

8. (a) Define the following as applied to a telescope

(i) Eye-ring (01mark)

This is the image of objective formed by the eyepiece and is the best position to place an eye to view the image through a telescope.

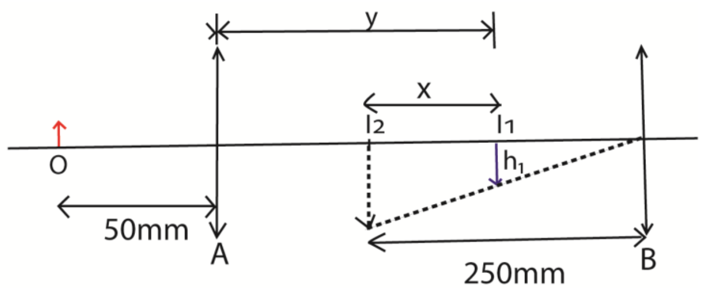
(ii) Magnifying power

It is the ration of the angle subtended by the final image at the eye when using the telescope to the angle subtended by the object at the unaided eye.

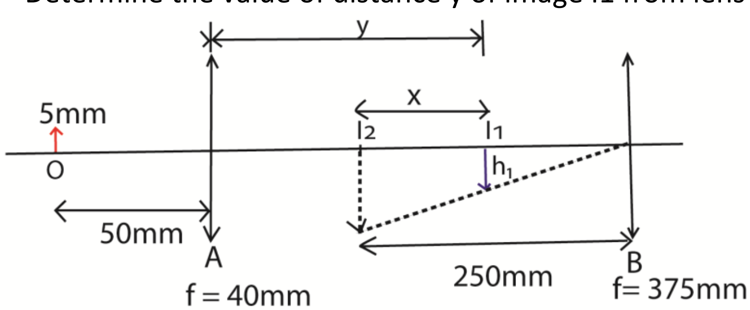
(b) What is the significance of the eye-ring of an astronomical telescope?
It is the best position for placing the eye, to view the image through the telescope; here the image is clearer and bright.

(c) State two advantages of reflecting telescope over refracting telescope
- There is no chromatic aberration
- There is no spherical aberration
- High resolution
- The image is brighter
- It is cheap

(d) The figure below shows an optical system consisting of two thin converging lenses arranged coaxially. Lens A has a focal length of 40mm and lens B has focal length of 375mm. An object O of height 5mm is placed 5mm from A, I_1 is a real image of O in A and I_2 is a virtual image of I_1 in B and is 250mm from B.



(i) Determine the value of distance y of image I1 from lens A (02marks)



Using $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{40} = \frac{1}{50} + \frac{1}{y}$; y = 200mm

(ii) Calculate the distance x between the image I_1 and I_2. (02marks)
Let the object distance in lens B be u

$\frac{1}{375} = \frac{1}{u} + \frac{1}{-250}$; u = 150mm

x = v-u = 250 -150 = 100mm

(iii) Find the magnification of the system.
M = m_A x m_B = $\frac{200}{50} x \frac{250}{150}$ = 6.7

(e) Name one defect of image formed by a lens and explain how the defect is minimized in practice

(i) Chromatic aberration is minimized by
- Placing the eye close to the lens such that images due to different colours subtend the same angle at the eye.
- Use of a chromatic doublet such that the dispersion produced by one lens is reversed by another.

(ii) Spherical aberration is minimized by
- Using a stopper such that only light incident on the middle of the lens pass.
- Using a lens of aspheric surface
- Using narrow aperture lens.

(f) Explain the following

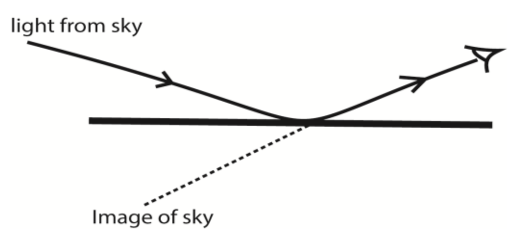
(i) Total internal reflection

It is the bouncing back of light moving from an optically denser medium to less dense medium when the angle of incidence exceeds the critical angle.

(ii) Formation of mirages

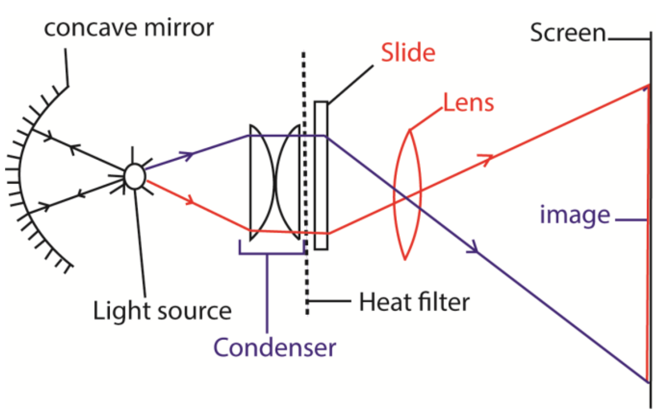
Mirage is an image of the sky formed by total internal reflection of light rays from the sky at air layer near the ground.

This occurs because the hotter layers of air near the ground are less optically dense than those above.



9. (a) Describe with aid of a labelled diagram the structure and operation of a projection lantern. (04marks)

Solution



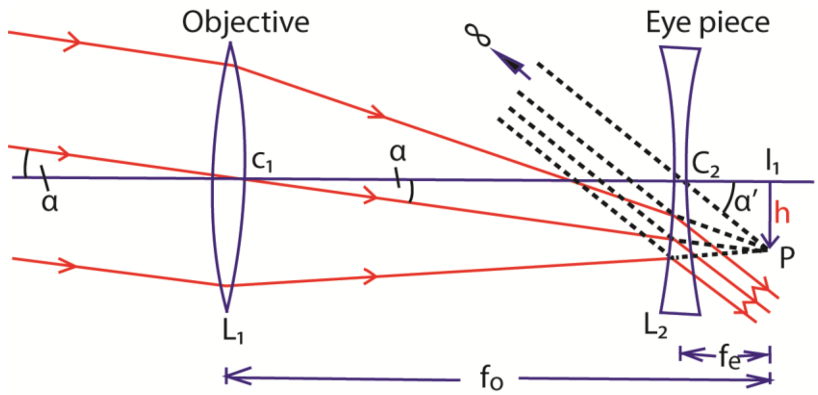
a. The slide, or film, is placed behind the **projector** lens outside its focal length and is illuminated buy a small but powerful source of light from concave reflector through condensing lenses.
b. A magnified, real and inverted image on the screen

(b) A projector produces an image of are $2m^2$ on the screen placed 5m from the projection lens. If the area of the object slide is $8cm^2$, calculate the focal length of the projected lens. (02marks)

Solution
$1m^2 = 10000cm^2$
Area magnification = $\frac{area\ of\ the\ image}{area\ of\ the\ object} = \frac{20000}{8} = 2500$
Linear magnification = $\sqrt{2500} = 50$
From $m = \frac{v}{f} - 1$
$50 = \frac{5}{f} - 1;$
$f = \frac{5}{51} = 0.098m = 9.8cm$

10. The slide of a projection lantern has dimensions 36mm by 24mm. find the focal length the lens required to project an image 1.44m x 0.98m on the screen placed 4.0m from the lens.
Solution
Image area = 1.44 x 0.98 = $1.4112m^2$
Object area = $(36 \times 10^{-3} \times 24 \times 10^{-3}) = 8.64 \times 10^{-4} m^2$.
Area magnification= $\frac{Image\ area}{object\ area} = \frac{1.4112}{8.64 \times 10^{-4}} = 1633.3$
Linear magnification = $\sqrt{1633.3} = 40.4$
From $M = \frac{v}{f} - 1$, v = 4.0m
$40.4 = \frac{4}{f} - 1$; f = 0.097m

11. (a) Explain the term eye-ring as applied to a telescope
This is the image of objective formed by the eyepiece and is the best position to place an eye to view the image through a telescope.
(b) (i) Draw a ray diagram to show the formation of a final image in Galilean telescope in normal adjustment.



Galilean telescope in normal adjustment

(ii) Derive an expression for angular magnification of the telescope in (b)(i) above

The angular magnification is the ratio of the angle subtended by the final image at the aided eye to the angle subtended by the object at the unaided eye.

To obtain the magnification, m, we assume the eye is very close to the eye piece.

For an aided eye (using the instrument)

For small angle, tanα' ≈ α' for small angle in radians

$\alpha' = \frac{h}{f_e}$ .... (i)

Where, h, is the height of image I, f_e is the focal length of eye piece

For unaided eye

$\alpha = \frac{h}{f_0}$ .... (ii)

f_0 is the focal length of objective lens

Combining equations (i) and (ii)

Magnifying power, $m = \frac{\alpha'}{\alpha} = \frac{h}{f_e} \div \frac{h}{f_o} = \frac{\boldsymbol{f_0}}{\boldsymbol{f_e}}$

(c) Explain two advantages and one disadvantage of the telescope in (b) above

Advantages

- Forms an erect image with only two lenses.
- It short and potable because the distance between the lenses is f_o-f_e.

Disadvantage

- It has a virtual (inaccessible) eye ring hence limited field of view.

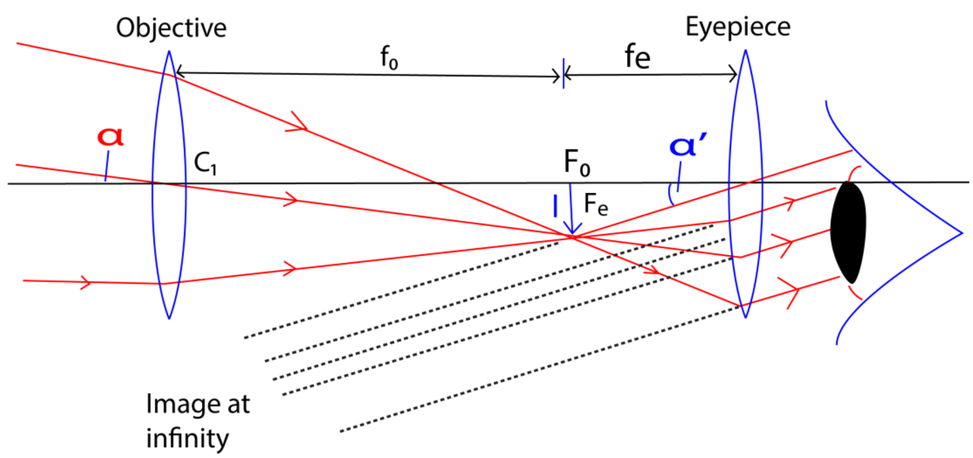
12. (a) What is meant by reversibility of light as applied to formation of real image by convex lens.

Reversibility of light means that light travels along its original path when its direction is reversed.

(b) (i) Draw the action of astronomical telescope in normal adjustment.

Note,

- when an astronomical telescope is in normal adjustment, the final image is formed at infinity
- the focal length of the objective is bigger than that of eye piece
- the principal focuses of the two lenses coincide



Telescope in nornal use

To obtain the magnification, m, we assume the eye is very close to the eye piece.

For an aided eye (using the instrument)
For small angle, tanα' ≈ α' for small angle in radians
$\alpha' = \frac{h}{f_e}$ …………………………………………. (i)
Where, h, is the height of image I, f_e is the focal length of eye piece
For unaided eye
$\alpha = \frac{h}{f_0}$ ……………………………………………. (ii)
f_0 is the focal length of objective lens
Combining equations (i) and (ii)
Magnifying power, m = $\frac{\alpha'}{\alpha} = \frac{h}{f_e} \div \frac{h}{f_o} = \frac{\boldsymbol{f_o}}{\boldsymbol{f_e}}$

(c) The objective and eyepiece of an astronomical telescope have focal lengths of 75.0cm and 2.5cm respectively. Find the separation of lenses if the final image is 25cm from the eyepiece.

Object distance in eye piece
From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$
$\frac{1}{2.5} = \frac{1}{u} + \frac{1}{-25}$; u =2.3vm
Separation of lenses = f_0 + u = 75 + 2.3 =77.3cm

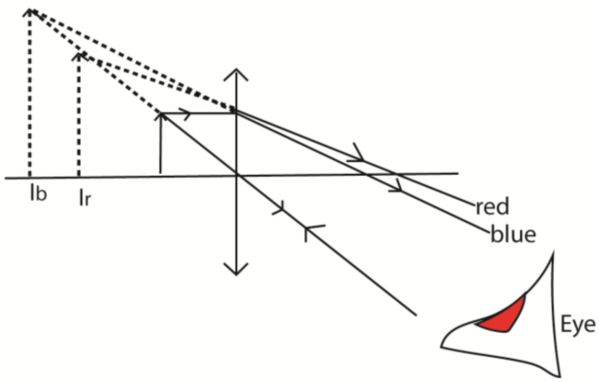
(d) (i) What is the significance of eye ring in astronomical telescope

It is the position where an eye gets the clearest image

(ii) State two advantages of a reflecting telescope over a refracting telescope

- No spherical aberration
- No chromatic aberration since there is no refraction
- Higher resolving power due bigger surface
- Cheap

(e) Explain why chromatic aberration is not observed in simple microscope.



In the lens the light rays from the object are dispersed to form colored images I_b and I_r due the different refractive indices; but when the eye is placed close to the lens, the mages subtend the same angle to the eye. The images overlap reducing chromatic aberration

13. (a) The objective of an astronomical telescope in normal adjustment has a diameter of 150mm and focal length of 3.0m. The eye piece has a focal length of 25.0mm. Calculate

(i) Position of the eye ring (03marks)

For eye-ring

$u = f_0 + f_e = 3000 + 25 = 3025$mm

$f = f_e = 25$mm

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{25} = \frac{1}{3025} + \frac{1}{v}$; v = 25.2mm.

(ii) Diameter of the eye ring (03mark)

$$m = \frac{diameter\ of\ objective}{diametr\ of\ the\ eye-ring} = \frac{f_0}{f_e}$$

$$= \frac{150}{De} = \frac{3000}{25}$$

Diameter of eye-ring = 1.25mm

(iii) Give the advantage of placing the eye at the eye ring (01mark)

Clearest image is observed

(b) (i) Give the disadvantage of using astronomical telescope to view distant object on earth
Images are inverted

(ii) Describe how astronomical telescope may be modified to overcome the disadvantage in (b)(i).

A converging lens (erecting lens) of focal length, f, is placed between the objective and eye piece. The distances of the erecting lens from objective and eye piece are 2f + fo and 2f +fe where fo and fe are the focal lengths of objective and eye piece respectively.

Or

Two right-angle isosceles prisms are used between the lenses. One placed with its refracting edge vertical another with its refracting edge horizontal.

(c) Find the separation of eye-piece and objective lenses in astronomical telescope of magnifying power 20 in normal adjustment, if its eyepiece has a focal length of 5.0cm.

M = $\frac{f_o}{f_e} = 20$

f_o = 20 x 5 = 100cm

Separation of lenses = $f_o + f_e$ = 100 + 5 = 105cm

14. An astronomical telescope has objective of focal length 100cm and eye piece of 10cm.find the separation of the objective and eyepiece lenses if the lenses are arranged such that the final image is formed 25cm away from the eyepiece.

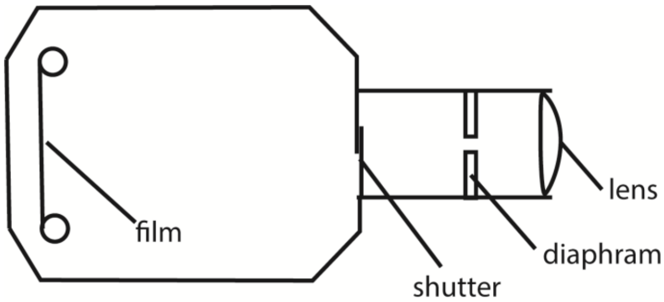
Action of the eyepiece

$\frac{1}{10} = \frac{1}{u} + \frac{1}{-25}$= u = 7.14cm

Separation of lenses = f_0 +u = 100 + 7.14 =107.14cm

15. (a) With the aid of a labelled diagram describe the essential parts of photographic camera

Lens camera

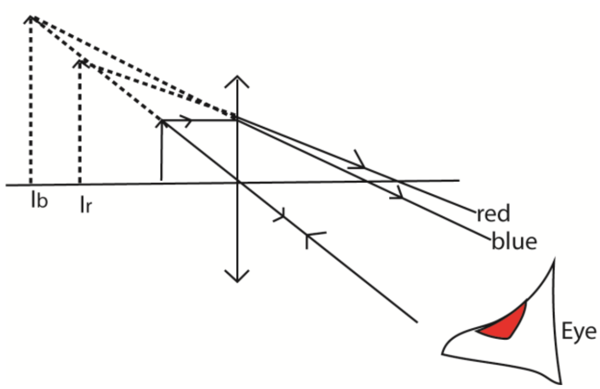


a. The lens focused light from the object on to film
b. The diaphragm controls the amount of light reaching the film
c. The shutter controls the exposure time of light reaching the film
d. Film contain photosensitive chemicals and is where the image is stored.

(b) Explain how chromatic and spherical aberration are minimized in photographic camera.

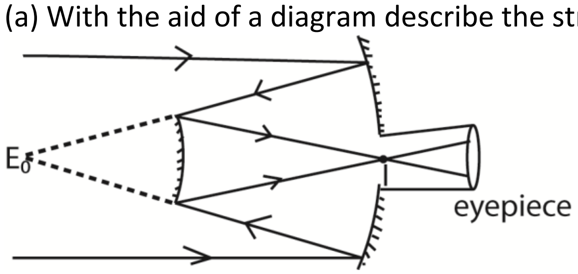
a. Chromatic aberration is minimized by using achromatic doublet made of convex and concave mirror such that the dispersion caused by one lens is nullified by another
b. Spherical aberration is reduced by using a stopper that prevents marginal rays from reaching the film.

(c) With aid of a diagram, explain why images seen in magnifying lens is almost free from chromatic aberration



In the lens the light rays from the object are dispersed to form colored images I_b and I_r due the different refractive indices; but when the eye is placed close to the lens, the mages subtend the same angle to the eye. The images overlap reducing chromatic aberration

16. (a) With the aid of a diagram describe the structure and action of reflecting telescope



- The objective is large parabolic mirror with circular aperture along the focal length.

- A beam of light parallel to the principal axis of telescope from distant object to the convex mirror.
- The convex mirror the forms a real, inverted image at, I, the principal focus of eyepiece.
- The eye piece forms a virtual magnified image at infinity.

(c) State the advantages of reflecting over refracting telescopes

- No chromatic aberration because there is no refraction
- No spherical aberration
- High resolution power due to larges surface area of the mirror.
- Cheap
- Produce brighter images

Dr Bbosa Science

Thank you
Dr. Bbosa Science