SENIOR FIVE TERM 3

TOPIC 4/6: ELECTROSTATICS

Electrostatics

This is the study of electrical charges at rest.

There are two kinds of charges i.e., unlike charges which attract and like charges which repel.

Positive and negative charges

Glass rubbed with silk develops a positive charge whereas an ebonite rod rubbed with fur develops a negative charge. This is because during rubbing electrons move from glass to silk whence the glass become positively charged whereas silk becomes negatively charged.

Similarly, electrons move from fur to rubber during rubbing and rubber becomes negatively charged while fur becomes positively charged.

Insulators and conductors

In insulators, electrons in the atoms are firmly bound to the nucleus and the removal or addition of electrons at a place does not cause flow of electrons elsewhere.

In conductors, the electrons are free to move from individual atoms and if such materials gain electrons these can move about in them. The loss of electrons by conductors cause a redistribution of those left. A charge on the conduct therefore spreads over entire surface.

Examples of insulators include glass, rubber and plastics.
Examples of conductors include metals, water, and electrolytic solutions

Charging insulators by contact rubbing or friction

Insulators can only be charged by contact rubbing or friction. Consider two bodies A and B of different work functions.

NB. Work function is the minimum energy required by an electron to escape from the surface of an atom.

When a body A has a low work function compared to B, then, during rubbing, body A will lose electron to body B due to heat energy that increases the kinetic energy of electrons.
Body A becomes positively charged while body B becomes negatively charged.

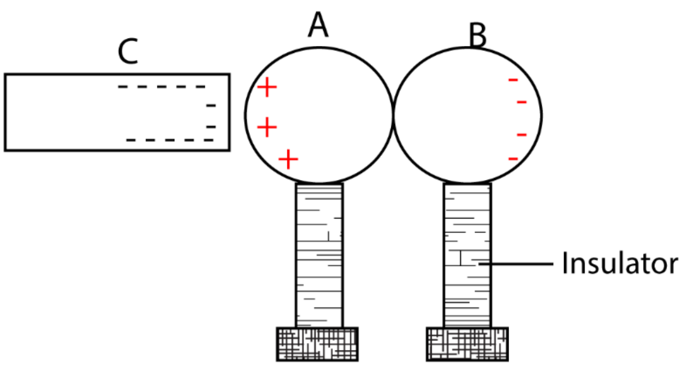
Example 1
Explain how charging by rubbing occurs.

Electrostatic induction

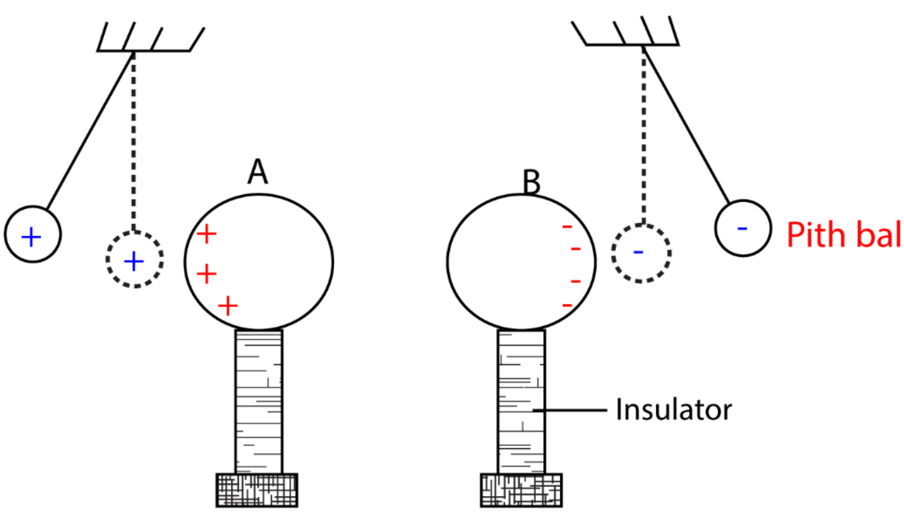
This is the process by which a neutral body places near a charged body gets charged with contact.

Induction is used when charging conductors where induced charges are obtained without contact with other charges.

Consider two insulated metal spheres A and B, arranged such that they touch one another. A negatively charged rod C is brought near A.



While keeping the inducing rod C in position, the spheres are separated and tested with a charged pith ball. It is discovered that A has a positive charged and B has a negative charge.



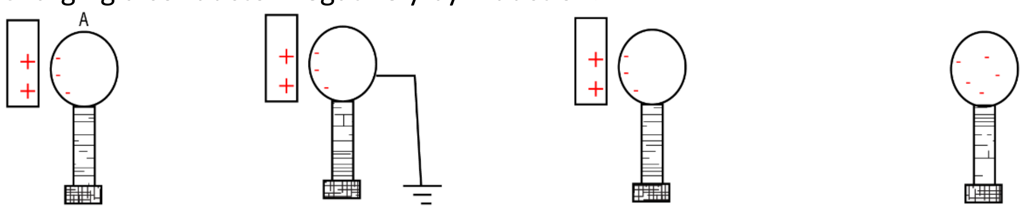
If the spheres are brought back in contact, it is found that they have no effect on the pith ball. Their charges have neutralized each other. This shows that

(i) positive and negative charges are created.

(ii) the number of electrons which move is equal to the number of positive charges that are created.

2. **Charging by induction**

(a) Charging a conductor negatively by induction.



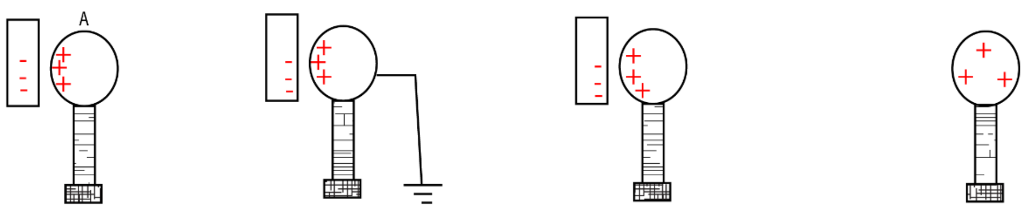
1. Bring a positively charged rod near an insulated conductor

2. Earth the conductor while the inducing rod is still in position

3. Remove the earthing while the inducing rod is still in position

2. Remove the inducing rod. Negative charges redistribute evenly

(b) Charging a conductor positively by induction



1. Bring a negatively charged rod near an insulated conductor

2. Earth the conductor while the inducing rod is still in position

3. Remove the earthing while the inducing rod is still in position

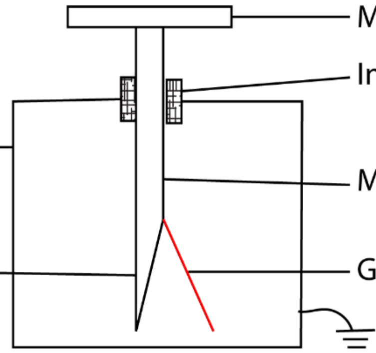
2. Remove the inducing rod. Positive charges redistribute evenly

Gold leaf electroscope

This is an instrument for testing the presence, the sign and the magnitude of the charge.

It consists of a circular metal disc (cap) attached to a metal rod with a brass plate to which is attached a thin foil of gold or aluminium.

It is fitted in a metal case with help of a plug (insulator) using perspex windows. The metal case is earthed in order to screen the electroscope from outside influences other than those brought nearer the cap and it is insulated from the ground.



Mode of action

When a charged body id brought near or in contact with the cap of the electroscope, the cap will acquire an opposite charge by induction. The charge on the body will repel all charges similar to it down to the rod, to the plate and the leaf.

Due to presence of like charges on the plate and gold leaf, the leaf diverges as it is repelled by the plate.

Leaf divergence indicates that the body brought nearer or in contact with the cap carries a charge.

Uses of the gold leaf electroscope

(i) Detecting of charge on a body
(ii) Testing the nature and sign of charge on the body
(iii) Comparing the magnitude of charge on various bodies
(iv) Test the insulating and conducting properties of various substances
(v) Measure potential difference

1. Testing for the nature or sign of charge

Charge a gold electroscope (GLE) negatively. Bring the body under test near the cap of GLE, it the leaf diverges further, then the body has a negative charge but if the gold leaf collapses, then that body is either has a positive or a neutral conductor.

The experiment is repeated is repeated where the GLE is charged is charged positively.
A charged body is brought near the cap; if the gold leaf diverges more, the body is positively charged. If the gold leaf collapses, the charged body is either negatively charged or neutral conductor.

Note that an increase in divergence occurs when the charge on electroscope and the tested charge are the same. Therefore, an increase in divergence is the only sure test for a sign of charge on the body.

2. To detect the presence of a charge on a body

Bring the b0dy to be tested near the metal cap of a neutral gold leaf electroscope. When the leaf deflected, then the body has got a charge. However, it the leaf remain undeflected, then a charge is absent on the body.

3. To compare and measure potentials

Two bodies which are similarly charged are brought into contact with the metal cap of the gold electroscope one after another. The body that cause a big divergence has a big charge.

4. To classify conductors and insulators

Bring the body to be tested in contact with the metal cap of a charged gold leaf electroscope. When the leaf collapses suddenly, then the body is a good conductor. If it collapses gradually, the body is a bad conductor. However, if it does not collapse then, it is an insulator.

Example

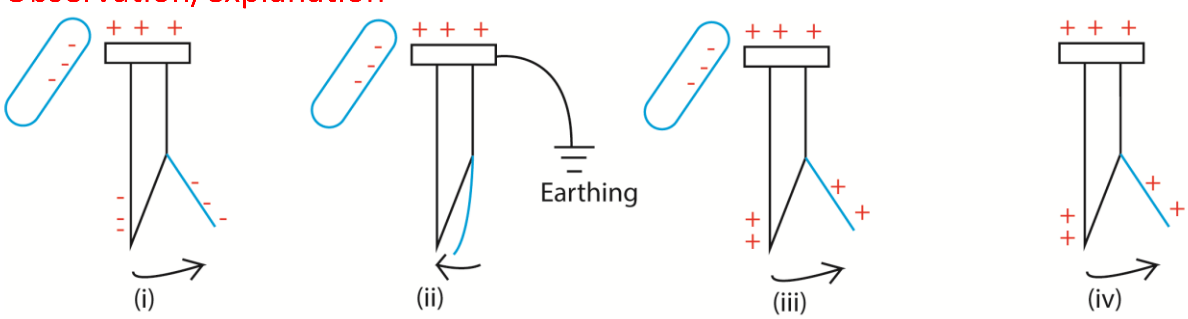
Describe how a gold leaf can be charged positively and negatively by induction.

(a) Charging gold leaf electroscope positively by induction

Procedures

- A negatively charged rod is brought a cap of GLE
- The cap is earthed while the charged body is still in place.
- Earth connection is removed.
- Lastly the charged body is removed.

Observation/explanation



(i) When a negatively charged body is brought near a cap of GLE, positive charges are induced on the cap while electrons are repelled to the metal plate and the gold leaf. The gold leaf is repelled and diverges.

(ii) When the cap is earthed say by touching it with a finger, electrons flow to the earth and the leaf collapses.

(iii) When the earth connection is removed, the remaining positive charge on the cap redistributes itself on the Gold leaf and the leaf diverges again.

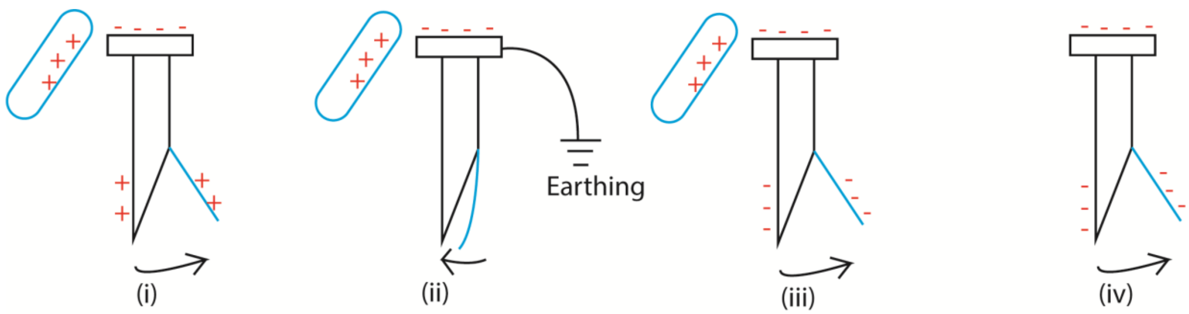
(iv) When the charged body is removed, the GLE acquires a permanent positive charge.

(b) Charging gold leaf electroscope negatively by induction

Procedures

- A Positively charged rod is brought a cap of GLE
- The cap is earthed while the charged body is still in place.
- The earth connection is removed.
- Lastly the charged body is removed.

Observation/explanation



(i) When a positively charged body is brought near a cap of GLE, positive charges are induced on the cap while electrons are repelled to the metal plate and the gold leaf. The gold leaf is repelled and diverges.

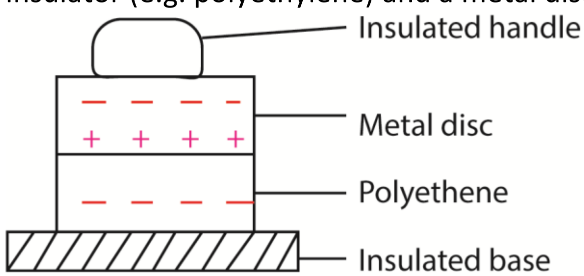
(ii) When the cap is earthed say by touching it with a finger, electrons flow from the earth and the leaf collapses.

(iii) When the earth connection is removed, the acquired negative charge on the cap redistributes itself on the Gold leaf and the leaf diverges again.

(iv) When the charged body is removed, the GLE acquires a permanent negative charge.

Electrophorus

This a device which provides large quantities of charge by induction. It consists of an insulator (e.g. polyethylene) and a metal disc with an insulated metal handle.



Procedure

The polyethene is charged negatively by rubbing it vigorously with a duster, when the metal disc is laid up on it. It acquires induced positive charge after earthing it with a finger. Very little negative charge from the polyethene to disc because the material has uneven surface preventing them from touching at more than a few point. Little charge escapes from these points only because the polyethene is a nonconductor. On removing the disc, it has a sufficient positive charge.

The disc can be discharged and charged again until the charge on the polyethene has disappeared by linkage.

Electrophorus is a device used for converting mechanical energy into electrical energy. Since work is done in raising the disc against the attraction of the opposite charge.

Electrophorus and the advantage of charging by induction

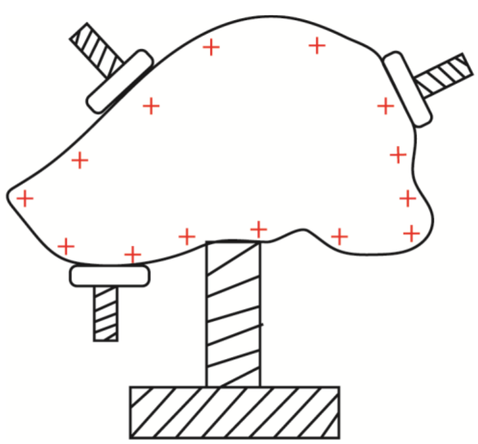
(i) Supply of charge is almost unlimited because the origin of charge in not carried away.
(ii) A greater charge nearly equal to that of the whole of polyethene can be concentrated on the conducting disc.
(iii) Only a very small charge can be transferred by contact leakage because the polyethene is not a conductor.
(iv) The disc can be discharged and recharged.

Charge distribution

Surface density is a charge per unit area in a conductor's surface. Surface density increases with the curvature of the body. The charge density is higher on the pointed and sharp surfaces than on round ones.

Testing charge distribution using proof planes

A proof plane consists of an aluminium foil (leaf) which can be modeled to fit any shape. It is made from a small sheets of the different shapes but to the same size.

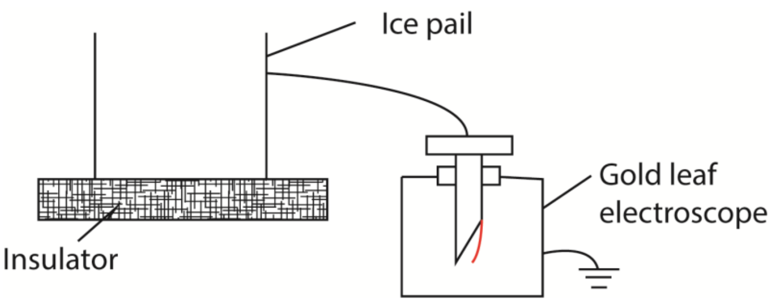


When a proof plane is placed at any point when contact with any surface, it carries away the charge which is directly proportional to the charge per unit area of the body at this point.

This charge is tested by lowering the proof plane in the can connected to an electrophorus. Different shapes of the conductors are investigated, it found that the charge per unit area or surface density increases with the curvature of the body, i.e. the leaf divergence is directly proportional to the charge density at the point of the body.

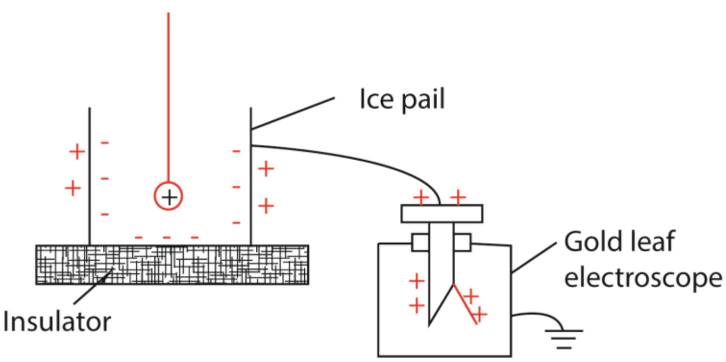
Faraday's ice pail experiments

The ice pail is placed on an insulator and connected to a gold leaf electroscope



A metal ball held on a long silk thread is charged positively and then lowered into the pail without allowing it to touch the pail side or the bottom of the pail. The positive charge is induced on the outside of the pail and the leaf diverges

Once the ball is inside the pail, the divergence of the leaf does not change when the ball is moved around, near to or further from the bottom of the pail which shows that the amount of induced positive charge does not the position of the ball once it is in the pail



The metal ball is allowed to touch the pail and no leaf divergence is observed. This means that when the ball touches the pail no charge is given to or taken away from outside of the pail.

The metal ball is then removed and tested for the charge with gold leaf electroscope. It was found that the ball had no charge, hence, the induced charge on the inside of the pail must have been equal in magnitude to the original positive charge on the ball

These conclusion apply only to hollow closed conductors

1. When a charged body is enclosed in a hollow conductor, it induces on the inside of a conductor a charge equal and opposite its own and on outside a charge equal and similar to its own
2. The total charge inside the conductor is always zero, either there is equal and opposite charges on the inside (before the ball touches) or there is zero no charge at all after the ball has touched.

Example 3

A charged body of 10C is lowered in a charged ice pail which carries a charge of -3C in the inside and +5C on the outside. Find the total charge on the pail when

(i) The ball is lowered without touching

(ii) The ball is allowed to touch the bottom of the pail.

Solution

(i) Without touching the pail

Outside = +10C + 5C = +15C

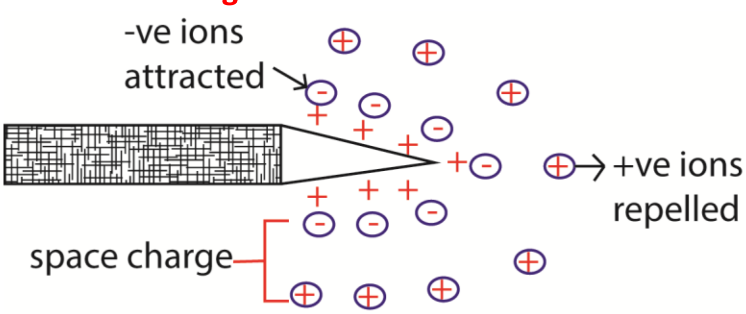
Inside = -10C – 3C = -13C

(ii) When the ball touches the bottom

Inside = -13C + 10C = -3C

Outside = +15C – 3C = +12C

Corona Discharge



- On a pointed charged conductor, there is high charge density and thus high electric field at the sharp point.
- This causes air molecules around the sharp point to ionize and form positive and negative ions.
- The similar charges to those on the conductor are repelled while those opposite are attracted to neutralize some of the charges on the conductor.
- The apparent loss of charge from a sharp point of the conductor in this way is called Corona discharge.

Definition

Corona discharge is a phenomenon where a strong electric field of a conductor causes the surrounding air (or another gas) to become ionized, leading to a faint glow, hissing sound, and sometimes ozone formation.

Or

Corona discharge is ionization of the surrounding medium caused by intense electric fields at sharp points of a conductor.

Importance corona discharge in Electrostatics

Demonstrates field concentration: Corona occurs more easily at sharp points because electric fields are strongest there.

Practical relevance: **of corona discharge**

- In **high-voltage transmission lines**, corona discharge causes power loss and radio interference.
- Corona discharge cause apparent loss of charge from sharp points.
- In **electrostatic devices** (like photocopiers or ozone generators), corona is deliberately used to ionize air.
- In **insulation testing**, corona discharge helps detect weak spots

Example 4

Explain what is meant by Action at points in electrostatics (Corona discharge)

Lightning

Lightning is a sudden, powerful electrical discharge that occurs during thunderstorms, caused by the buildup of static electricity between regions with opposing charges within a cloud or between a cloud and the ground. This massive energy release creates a bright flash and a sound called thunder, which is the rapid expansion of air heated by the lightning strike.

How lightning forms

- **Charge buildup:** Thunderstorms form when warm, moist air rises and cools, condensing into water droplets and ice crystals. Collisions between these ice crystals and water droplets generate static electricity. Lighter, positively charged particles are carried higher in the cloud, while heavier, negatively charged particles gather at the bottom.

- **Opposite charges attract:** This separation of charge creates a potential difference between different parts of the cloud and between the cloud and the ground, which becomes positively charged underneath a storm cloud.
- **Electrical discharge:** When the opposite charges build up enough to overcome the air's resistance, the electrical potential difference becomes too strong, and an electrical discharge occurs to equalize the charges.
- **The flash:** This discharge is the lightning flash, a giant spark that travels through the air. The intense heat from the electrical current causes the air to expand rapidly, creating the shockwave we hear as thunder.

How lightening causes damages to buildings and trees

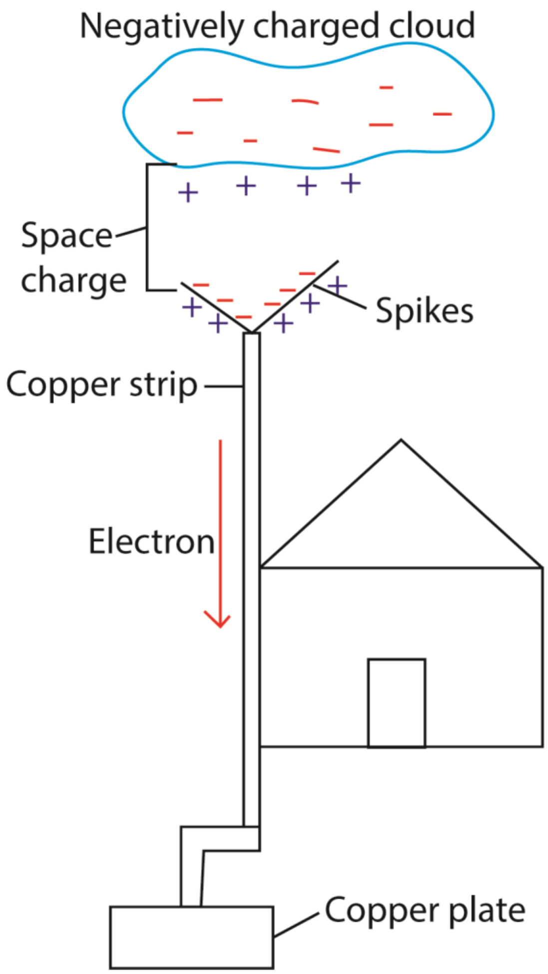
Lightning damages buildings through three primary mechanisms: fire, explosive shock waves, and power surges.

- **Fire:** The extreme heat of a lightning bolt (up to 50,000°F) can easily ignite flammable building materials like wood framing, roofing shingles, or insulation.
- **Shock Waves:** The explosive shock wave created by the rapid heating and expansion of air (which we hear as thunder) can cause physical destruction. This force can crack or shatter hard materials like brick, stone, concrete, and cinderblock, causing damage to chimneys, walls, and foundations, and can also shatter windows.
- **Power Surges:** Lightning current travels through conductive paths such as electrical wiring, plumbing, and communication lines. The resulting massive power surge can instantly destroy appliances, computers, and entire electrical systems, even if they are not directly struck. These surges can also melt wiring, posing a serious latent fire hazard

The lightning conductor

A **lightning conductor** protects a building by safely directing the enormous electrical energy of a lightning strike into the ground instead of letting it pass through the structure.

A lightning conductor consists of a pointed metal rod fixed at the highest point of a building connected to a thick metal strip or cable that runs down the building into the earth. Because of its sharp tip, it attracts the lightning discharge when a storm occurs and conducts the massive electrical current harmlessly into the ground. This prevents damage to the building, its occupants, and electrical systems.



How a lightning conductor works

(i) **Attraction & Interception**: The metal rod, positioned at the building's peak, is the most likely point for lightning to strike because it's the highest and most conductive point.

(ii) **Conduction**: Once struck, the electrical current flows through the thick metal cable connected to the rod.

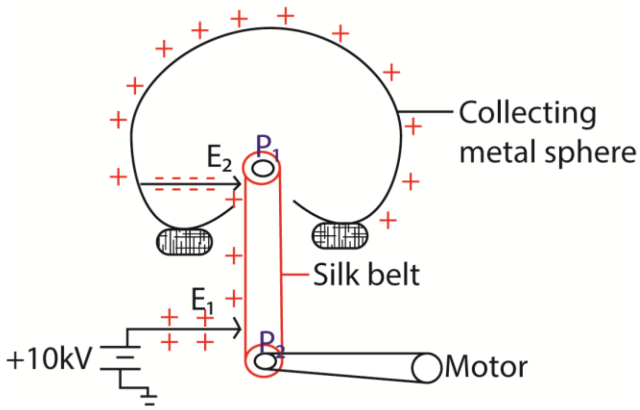
(iii) **Grounding**: This cable runs down the side of the building to a copper plate buried deep in the earth, safely dissipating the energy into the ground.

(iv) **Prevention**: By providing this dedicated, safe pathway, the conductor prevents the lightning's intense heat and current from passing through the building itself, averting fires, explosions, and electrical damage.

Example 5

Explain how a lightening conductor works.

Van der Graff generator



Main features

a. It consists of a large hollow metal sphere (collecting sphere) supported on insulating stand.
b. A silk belt inside the tube driven by an electric motor possesses the sharply pointed electrode metal E_1, held at electric potential of about 10kV relative to the earth.
c. As the belt moves up, it passes another sharply pointed metal electrode E_2 connected inside the hollow sphere.

Mode of operation

The generator works by continuously transferring electric charge from a lower potential to a higher potential using a moving belt.

(i) **Charge Production**: A comb or brush, E_1, near the bottom of the belt sprays charges (usually positive) onto the belt using a high-voltage source or corona discharge. This is often achieved by ionizing air molecules near the comb.
(ii) **Charge Transport**: The insulating belt (made of rubber or fabric) carries the charges upward as it moves.
(iii) **Charge Collection**: At the top, another comb or brush E2, is positioned close to the belt transfers the charges from the belt to the large hollow metal dome (sphere). The high electric field intensity around E_2 ionizes air there, repelling negative charge onto the belt. The negative charge neutralizes positive charge on the belt before it goes over the upper pulley.
(iv) **Charge Accumulation**: The dome accumulates charge, and because it is large and smooth, it can hold a very high potential (millions of volts). The electric field around the dome increases until limited by leakage or breakdown of air.

Main Uses of the Van de Graaff Generator

(i) **Physics Education & Demonstrations**
- Commonly used in classrooms and science museums to illustrate electrostatics.
- Demonstrations include hair standing on end, sparks, and charge distribution on conductors.

(ii) **Particle Acceleration**
- Early nuclear physics experiments used Van de Graaff generators to accelerate protons and other ions.
- Helped in studying nuclear reactions and discovering subatomic particles.

(iii) **X-ray Production**
- High voltages from the generator can be used to produce energetic X-ray beams.
- Applications in nuclear medicine and material testing.

(iv) **Industrial & Scientific Applications**
- Used to sterilize food and process materials by accelerating electrons.
- Helps in insulation testing of materials under high-voltage stress.

(v) **Entertainment & Public Outreach**
- Popular in science shows for dramatic demonstrations of sparks and static electricity effects.

Why It's Important

- **Safe for demonstrations**: Produces very high voltage but very low current, making it safe for controlled experiments.
- **Foundation for modern accelerators**: The principle of charge accumulation and acceleration paved the way for more advanced particle accelerators.
- **Versatile tool**: From classrooms to laboratories, it bridges theory and practice in electrostatics and high-voltage physics.

Example 6

Describe with aid of a diagram the mode of operation of a van der Graff generator.

Or

Describe the application of Corona discharge in Van der Graff generator.

Or

Describe the applications of electrostatics

Example 7

The belt of a van der Graff generator is of width 20 cm and travels at a speed of $15ms^{-1}$. The charge density is $1.5 \times 10^{-4}Cm^{-2}$. The generator is connected to a resistor of $6.0 \times 10^{9}\Omega$.

Determine

(i) The maximum current that can be drawn from the generator

(ii) Maximum possible power output of the motor that drives the belt.

(iii) Maximum potential difference

Solution

(i) Area per second = speed x width

$= 15 \times \frac{20}{100} = 3m^2s^{-1}$

Charge carried per second = charge density x width

$= 1.5 \times 10^{-4} \times 3$

$= 4.5 \times 10^{-4} Cs^{-1}$

But rate of flow charge is current

Maximum stead current, $I_{max} = 4.5 \times 10^{-4} Cs^{-1}$

$= 4.5 \times 10^{-4} A$

(ii) From ohm's law

$V_{max} = I_{MAX} \times R$

$= 4.5 \times 10^{-4} \times 6.0 \times 10^9$

$= 2.7 \times 10^6 V$

(iii) Power output = I_{max}^2 R = $(4.5 \times 10^{-4})^2 \times 6.0 \times 10^9$ = 1.215kW

Example 8.

A large van der Graff generator has a top terminal in the form of a sphere of diameter 3.5m. When the terminal is at the operating potential of $3.5 \times 10^6 V$, what is

(i) The stored charge

(ii) The energy stored

(iii) The electric field intensity of the surface of the sphere.

Solution

(i) Potential, $V = \frac{Q}{4\pi\varepsilon_0 r}$

⇨ $Q = 4\pi\varepsilon_0 rV$

$= \frac{4\pi\varepsilon_0 dV}{2}$

$= \frac{3.5 \times 3.5 \times 10^6}{2 \times 9 \times 10^9}$

$= 6.0806 \times 10^{-4} C$

(ii) Energy stored = $\frac{1}{2}QV$

$= \frac{1}{2} \times 6.0806 \times 10^{-4} \times 3.5 \times 10^6$

= 1191J

(iii) $E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{4Q}{4\pi\varepsilon_0 r^2} = \frac{4 \times 9 \times 10^9 \times 6.806 \times 10^{-4}}{3.5^2} = 2.0 \times 10^6 NC^{-1}$

Force between two point charges

Consider two electrically charged bodies at a separation r which is much larger than the dimension of the bodies. The bodies at such a separation behave like point charges.

- The electrostatic force, F, between two pints charges Q_1 and Q_2 is directly proportional to the product of the magnitude of the charge.

  $F \propto Q_1 Q_2$

- It is inversely proportional to the square of the distance between the two equation

  $F \propto \frac{Q_1 Q_2}{r^2}$

  $F = \frac{kQ_1 Q_2}{r^2}$

  Where k is a constant of proportionality

  $k = \frac{1}{4\pi\varepsilon_0}$ is called the permittivity of the medium.

  ε_0 = permittivity of free space (vacuum) = $8.855 \times 10^{-12} Fm^{-1}$.

  The permittivity of air = 1.005 x permittivity of free space.

  $\therefore F = \frac{1}{4\pi\varepsilon_0} x \frac{Q_1 Q_2}{r^2}$

  $$k = \frac{1}{4\pi\varepsilon_0} = 9.0 \, x \, 10^9 \, Fm^{-1}$$

Permittivity of a medium

Permittivity (ϵ\epsilon) is the measure of a medium's ability to store electrical energy in an electric field.

Permittivity of a medium is a fundamental concept in electrostatics and electromagnetism. It describes how much a material allows electric field lines to pass through it, or in other words, how easily the medium can be polarized by an electric field.

Coulomb's Law

The force between any two point charges is directly proportional to the magnitude of the product of the charges and inversely proportional to the square of the distance between them.

Example 9

Calculate the value of two equal charges if they repel each other with a force of 0.1N when placed 50cm apart in the vacuum. What would be the size of the charge if they were situated in an insulating liquid whose permittivity was ten times that of the vacuum?

$(\frac{1}{4\pi\varepsilon_0} = 9.0 \, x \, 10^9 \, Fm^{-1})$

Solution

(a) From $F = \frac{1}{4\pi\varepsilon_0} x \frac{QQ}{r^2}$

$0.1 = \frac{9.0 \, x \, 10^9 Q^2}{(50 \, x \, 10^{-1})^2}$

$$Q = \sqrt{\frac{0.1 \, x \, (50 \, x \, 10^{-2})^2}{9 \, x \, 10^9}}$$

$= 1.667 \times 10^{-6}C$

(b) Permittivity of the insulating liquid = $10\varepsilon_0$

$$F = \frac{1}{10 \, x \, 4\pi\varepsilon_0} x \, \frac{QQ}{r^2}$$

$$0.1 = \frac{9.0 \, x \, 10^9 Q^2}{10 \, x \, (50 \, x \, 10^{-1})^2}$$

$$Q = \sqrt{\frac{0.1 \, x \, 10 \, x \, (50 \, x \, 10^{-2})^2}{9 \, x \, 10^9}}$$

$= 5.27 \times 10^{-6}C$

Relative permittivity ε_r

is the ratio of the permittivity of any medium to the permittivity of free space

Subunits

10^{-3} - mill (m)

10^{-6} – micro(μ)

10^{-9} – nano (n)

10^{-12} – pico (p)

10^{3} – kilo (k)

10^{6} – Mega (M)

10^{9} – Giga (G)

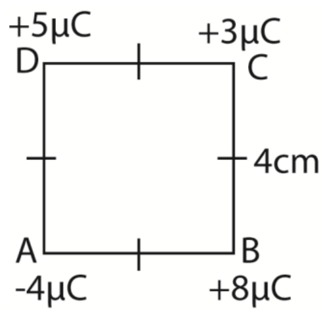
Example 10

Find the force between two point charges of +4μC and -3μC placed at a distance of 10cm apart in air.

$$F = \frac{1}{4\pi\varepsilon_0} x \, \frac{QQ}{r^2} = \frac{9.0 \, x \, 10^9 \, x \, 4 \, x \, 10^{-6} \, x \, 3 \, x \, 10^{-6}}{(10 \, x \, 10^{-2})^2} = 10.0N \text{ (attraction)}$$

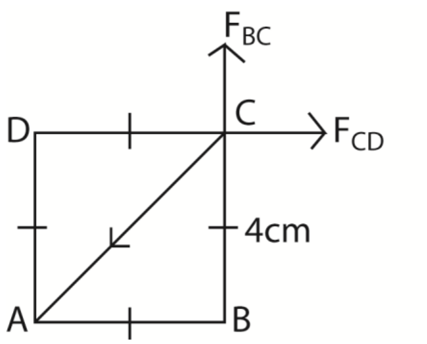
Example 11

Four charges are placed at the corner of a square ABCD as shown below.



Find the resultant force on the charge at C

Solution



From F = $\frac{1}{4\pi\varepsilon_0} x \frac{QQ}{r^2}$

Force, F_{BC}, at C due to charge at B

$F_{BC} = \frac{9.0 \; x \; 10^9 \; x \; 3 \; x \; 10^{-6} \; x \; 8 \; x \; 10^{-6}}{(4.0 \; x \; 10^{-2})^2}$ = 135N

Force, F_{CD}, at C due to charge at D

$F_{BC} = \frac{9.0 \; x \; 10^9 \; x \; 3 \; x \; 10^{-6} \; x \; 5 \; x \; 10^{-6}}{(4.0 \; x \; 10^{-2})^2}$ = 84.375N

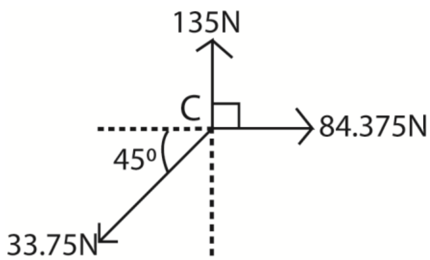
Distance AC

$(AC)^2 = (AB)^2 + (BC)^2$

AC = $\sqrt{4^2 + 4^2}$ = 5.66

Force, F_{CA}, at C due to charge at A

$F_{CA} = \frac{9.0 \; x \; 10^9 \; x \; 3 \; x \; 10^{-6} \; x \; 4 \; x \; 10^{-6}}{(5.66 \; x \; 10^{-2})^2}$ = 33.75N

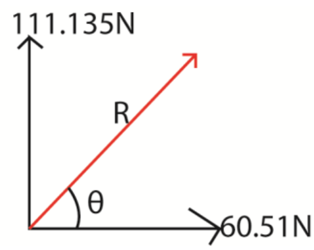


Resolving vertically

F_y = 135 – 33.75sin45 = 111.135N

Resolving horizontally

F_x = 84.375 – 33.75cos 45 = 60.51N



$R^2 = F_x^2 + F_y^2$

= $111.135^2 + 60.51^2$

R = 126.52N

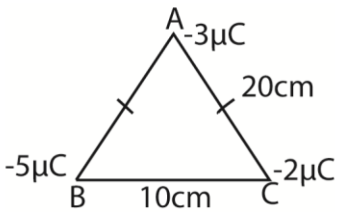
tanθ = $\frac{F_y}{F_x}$ = $\frac{111.135}{60.52}$

θ = 61.4^0

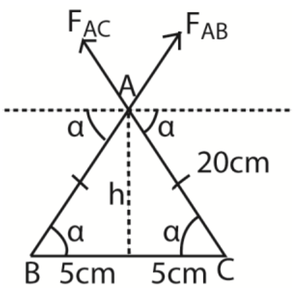
Hence the resultant force at C = 126.54N at 61.4^0 to the horizontal

Example 12

Find the resultant force at A



Solution



$h^2 + 5^2 = 20^2$

$h = \sqrt{375} = 19.36cm$

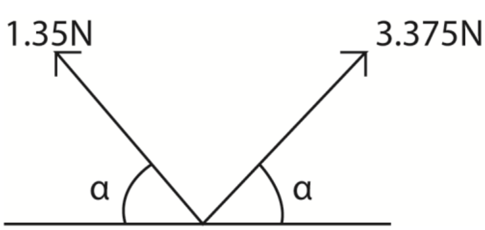
$\cos\alpha = \frac{5}{20}$; $\sin\alpha = \frac{19.36}{20}$

Force on A due to charge at B

$F_{AB} = \frac{9.0 \, x \, 10^9 \, x \, 3 \, x \, 10^{-6} \, x \, 5 \, x \, 10^{-6}}{(20 \, x \, 10^{-2})^2}$ = 3.375N

Force on A due to charge at C

$F_{AB} = \frac{9.0 \, x \, 10^9 \, x \, 3 \, x \, 10^{-6} \, x \, 2 \, x \, 10^{-6}}{(20 \, x \, 10^{-2})^2}$ = 1.35N



Resolving vertically

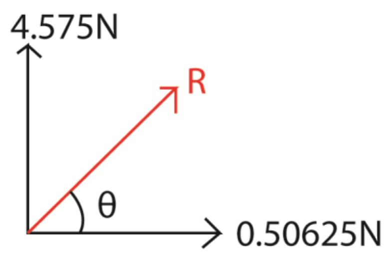
$F_y = 1.35\sin\alpha + 3.375\sin\alpha$

$= 1.35sin\frac{19.36}{20} + 3.375\sin\frac{19.36}{20} = 4.575N$

Resolving horizontally

$F_x = 1.35\cos\alpha + 3.375\cos\alpha$

$= 1.35cos\frac{5}{20} + 3.375\cos\frac{5}{20} = 0.50625N$



$R^2 = F_x^2 + F_y^2$

$0.50625^2 + 4.575^2$

R = 4.603N

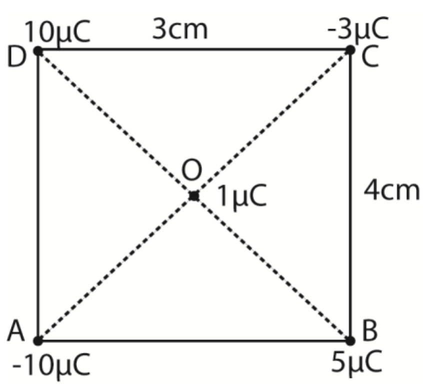
$\tan\theta = \frac{F_y}{F_x} = \frac{4.575}{0.50625}$

$\theta = 83.7^0$

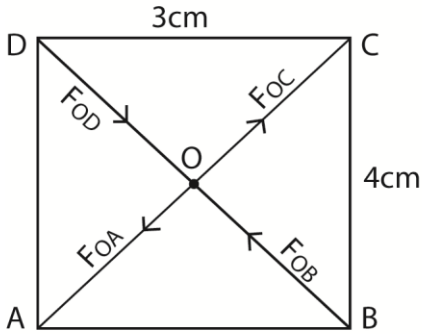
Hence the resultant force at C is 4.603N at 83.7^0 to the horizontal

Example 13

Find the force at O



Solution



Force on O due to charge at A

$F_{OA} = \frac{9.0\ x\ 10^9\ x\ 1\ x\ 10^{-6}\ x\ 10\ x\ 10^{-6}}{(2.5\ x\ 10^{-2})^2}$ = 144N

Force on O due to charge at B

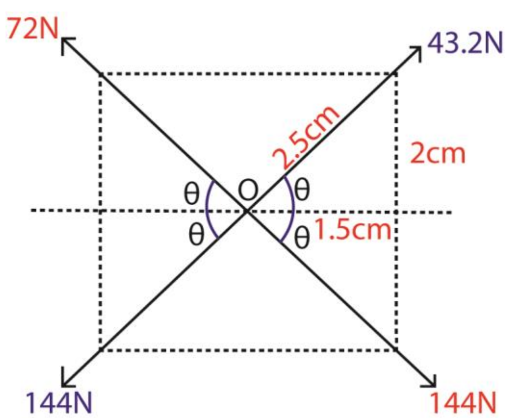
$F_{OB} = \frac{9.0\ x\ 10^9\ x\ 1\ x\ 10^{-6}\ x\ 5\ x\ 10^{-6}}{(2.5\ x\ 10^{-2})^2}$ = 72N

Force on O due to charge at C

$F_{OC} = \frac{9.0\ x\ 10^9\ x\ 1\ x\ 10^{-6}\ x\ 3\ x\ 10^{-6}}{(2.5\ x\ 10^{-2})^2}$ = 43.2N

Force on O due to charge at D

$F_{OD} = \frac{9.0\ x\ 10^9\ x\ 1\ x\ 10^{-6}\ x\ 10\ x\ 10^{-6}}{(2.5\ x\ 10^{-2})^2}$ = 144N



sin θ = $\frac{2}{2.5} = 0.8$, cosθ = $\frac{1.5}{2.5}$ = 0.6

Resolving vertically;

F_y = 72sinθ + 43.2sinθ -144sinθ-144sinθ

= 172.8 sinθ

= 172.8 x 0.8

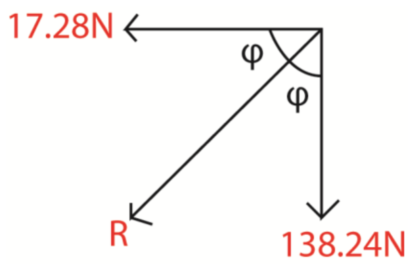
= 138.24N

Fx = 43.2cosθ + 144cosθ – 144cosθ – 72cosθ

= -28.8cosθ

= -28.8 x 0.6

= -17.2N



R[2] = $F_y^2 + F_x^2$ =17.28[2] + 138.24[2]

R = $\sqrt{19408.896}$ = 139.3N

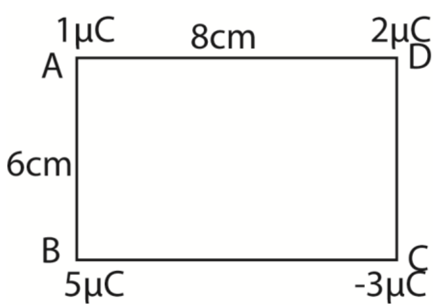
tanϕ = $\frac{F_y}{F_x} = \frac{138.24}{17.24}$

$\varphi = 82.9^0$

Hence the resultant force at O is 139.3N at 82.9[0] to horizontal

Example 14

Find the resultant force on B



Force B due to charge at D

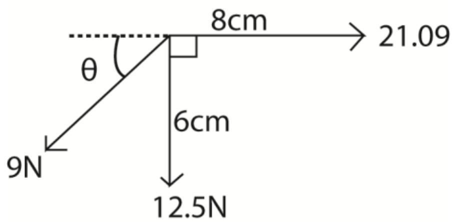
$F_{BD} = \frac{9.0\ x\ 10^9\ x\ 5\ x\ 10^{-6}\ x\ 2\ x\ 10^{-6}}{(10\ x\ 10^{-2})^2}$ = 9N

Force B due to charge at C

$F_{BC} = \frac{9.0\ x\ 10^9\ x\ 5\ x\ 10^{-6}\ x\ 3\ x\ 10^{-6}}{(8\ x\ 10^{-2})^2}$ = 21.094N

Force B due to charge at A

$F_{BA} = \frac{9.0\ x\ 10^9\ x\ 5\ x\ 10^{-6}\ x\ 1\ x\ 10^{-6}}{(6\ x\ 10^{-2})^2}$ = 12.5N

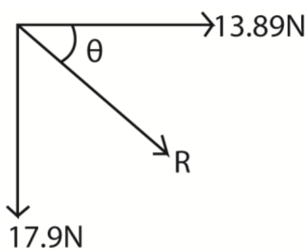


Sin θ =$\frac{6}{10} = 0.6$, cosθ =$\frac{8}{10} = 0.8$

Resolving vertically

$F_y = -12.5 - 9sin\theta$ = -12.5 – 9 x 0.7 = 17.9N

$F_x = 21.09 - 9cos\theta$ = 21.09- 9 x0.8 = 13.89N



$R^2 = 13.89^2 + 17.9^2$

R =$\sqrt{513.5}$ = 22.7N

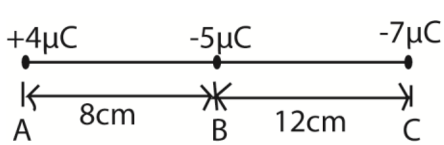
tan θ = $\frac{17.9}{13.89}$

θ = 52.1^0

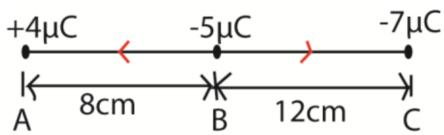
The resultant force at C is 22.7N at 52.1^0 to the horizontal

Example 15

Find the resultant force on B



Solution



Force on B due to charge A

$F_{BA} = \frac{9.0\ x\ 10^9\ x\ 5\ x\ 10^{-6}\ x\ 4\ x\ 10^{-6}}{(8\ x\ 10^{-2})^2}$ = 28.125N

Force on B due to charge C

$F_{BC} = \frac{9.0\ x\ 10^9\ x\ 5\ x\ 10^{-6}\ x\ 7\ x\ 10^{-6}}{(8\ x\ 10^{-2})^2}$ = 21.875N

Resultant force, R = 28.125 – 21.875 = 6.25N towards A

Electric field

Al electric field is a region around an electric charge where an electric force is experienced. This can be mapped out by electrostatic lines of force.

Lines of force or electric field lines show the **direction and strength** of the electric field at different points in space. They are drawn such that the **tangent at any point** gives the direction of the electric field vector at that point. A collection of lines of force is called an **electric flux.**

Properties of Electric Field Lines

(i) **Start and End**:
- They begin on **positive charges** and end on **negative charges**.
- For an isolated positive charge, lines radiate outward; for a negative charge, they converge inward.

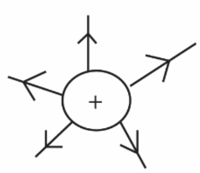
(ii) **Density of Lines**:
- The closer the lines, the stronger the field.
- Sparse lines indicate weaker fields.

(iii) **Never Intersect**:
- Two field lines can’t cross, because the electric field at a point has only one direction.

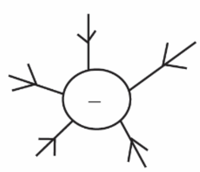
(iv) **Perpendicular to Conductors**:
- At the surface of a charged conductor, field lines are always drawn perpendicular to the surface.

Electric field pattern

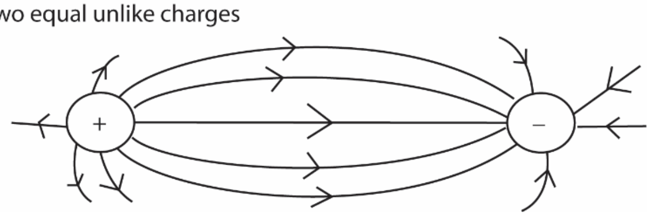
1. Isolated positive charge



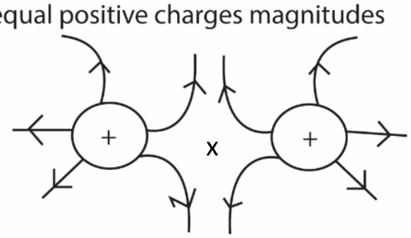
2. Isolated negative charge



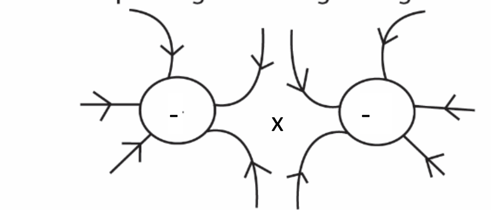
3. Two equal unlike charges



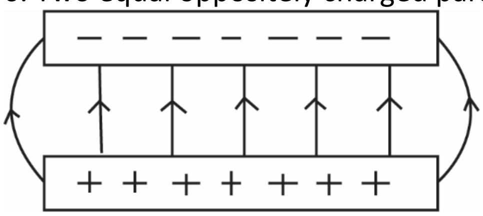
4. Two equal positive charges magnitudes



5. Two equal negative charges magnitudes



6. Two equal oppositely charged parallel plates



Definition

1. A neutral point, x, is a point where the resultant electric field intensity is zero.

   The force exerted on the charged body in an electric field depends on the charge on the body and the intensity or strength of the field.

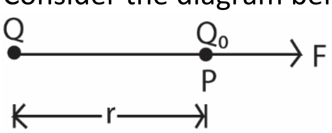
2. Electric field intensity or strength is defined as the force exerted on a positive charge of 1 coulomb placed at a point in electric field

   Therefore, electric field intensity, E is a vector quantity

   $E = \frac{F}{Q}$

   The S.I. unit of E is NC^{-1}

Consider the diagram below



Placing a small charge Q_0 at point P, in an electric field, the force at P due to Q is given by

$F = \frac{1}{4\pi\varepsilon_0} x \frac{QQ_0}{r^2}$

Electric field intensity, E, at P = $\frac{F}{Q_0}$

$= \frac{Q}{4\pi\varepsilon_0 r^2}$

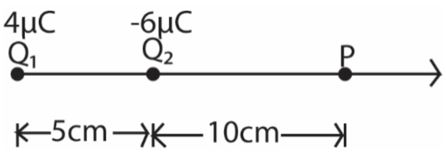
Therefore electric field intensity, E, at a point due to charge Q is given by

$E = \frac{Q}{4\pi\varepsilon_0 r^2}$

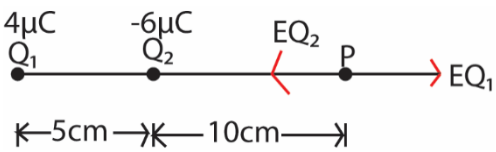
Note E is directed away from a positive charge and directed towards the negative charge.

Example 10

Find the electric field intensity at P



Solution



Solution

E at P due to Q_1

$EQ_1 = \frac{9.0 \, x \, 10^9 \, x \, 4 \, x \, 10^{-6}}{(15 \, x \, 10^{-2})^2}$ = 16 x 10^5 NC^{-1}(→)

E at P due to Q_2

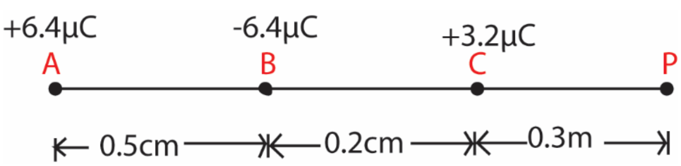
$EQ_2 = \frac{9.0 \, x \, 10^9 \, x \, 4 \, x \, 10^{-6}}{(10 \, x \, 10^{-2})^2}$ = 54 x 10^5 NC^{-1}(←)

EP = EQ_1- EQ_2

= (16 x10^5 – 54 x 10^5) NC^{-1}

=38 x 10^5 NC^{-1}(←)

Example 11



Three point charges of +6.4μC, -6.4μC and +3.2μC are arranged in line at points A, B, and C respectively as shown in diagram above. Find the electric field intensity at P.

Solution

E at P due to A (+6.4μC)

$E_{AP} = \frac{9.0 \ x\ 10^9\ x\ 6.4\ x\ 10^{-6}}{(1)^2}$ = 5.76 x $10^4 NC^{-1}$(→)(repulsive)

E at P due to A (- 6.4μC)

$E_{BP} = \frac{9.0 \ x\ 10^9\ x\ 6.4\ x\ 10^{-6}}{(0.5)^2}$ = 2.304 x $10^5 NC^{-1}$(←)(attractive)

E at P due to A (+3.2μC)

$E_{CP} = \frac{9.0 \ x\ 10^9\ x\ 3.2\ x\ 10^{-6}}{(0.3)^2}$ = 3.2 x $10^5 NC^{-1}$(→)(repulsive)

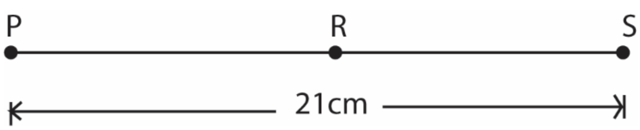
$E_P = E_{AP} - E_{BP} + E_{CP}$

= (5.76 x 10^4 -2.30 x 10^5 + 3.2 x 10^5) NC^{-1}

= 1.472x $0^5 NC^{-1}$(→)

Hence the electric field at P is 1.472x $0^5 NC^{-1}$ to the right

Example 12



Two point Charges P and S of -17.6μC and -9.0μC respectively are placed in a vacuum at a distance of 21cm apart. When a 3rd charge R is placed midway between P and S as shown in figure above, then, the net force at S is zero

(i) Determine the charge on R
(ii) Calculate the electric potential at position R.
(iii) Sketch the electric field lines corresponding to charge distribution

Solution

Let the charge on R be Q

From

$F = \frac{1}{4\pi\varepsilon_0}\ x\ \frac{QQ_0}{r^2}$ where $\frac{1}{4\pi\varepsilon_0} = 9.0 \text{ x } 10^9$

Force on S due to R

$F_{SR} = \frac{9.0\ x\ 10^9\ x\ 9\ x\ 10^{-6}\ x\ Q}{(10.5\ x\ 10^{-2})^2}$

Force on S due to P

$F_{SP} = \frac{9.0\ x\ 10^9\ x\ 9\ x\ 10^{-6}\ x\ 17.6\ x\ 10^{-6}}{(21\ x\ 10^{-2})^2}$

Resultant force on S = $F_{SP} - F_{SR} = 0$

⇨ $F_{SP} = F_{SR}$

$$\frac{9.0\,x\,10^{9}\,x\,9\,x\,10^{-6}\,x\,17.6\,x\,10^{-6}}{(21\,x\,10^{-2})^2} = \frac{9.0\,x\,10^{9}\,x\,9\,x\,10^{-6}\,x\,Q}{(10.5\,x\,10^{-2})^2}$$

$$Q = \frac{17.6\,\,x\,10^{-6}\,x\,(10.5\,x\,10^{-2})^2}{(21\,x\,10^{-2})^2} = 4.4 \text{ x } 10^{-6}\text{C}$$

Hence the charge on R = +4.4x 10^{-6}C

(ii) Electric field potential, at a distance r fro charge D is given by

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Electric potential at R due to S

$$V_{RS} = \frac{9\,x10^{9}\,x{-9}\,x\,10^{-6}}{10.5\,x\,10^{-2}} = -\,7.71 \text{ x } 10^{5}\text{V}$$

Electric potential at R due to P

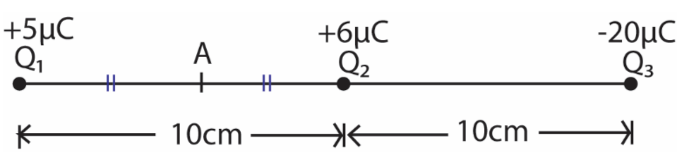
$$V_{RP} = \frac{9\,x10^{9}\,x{-17.6}\,x\,10^{-6}}{x\,10.5\,x\,10^{-2}} = -1.51 \text{ x } 10^{6}\text{V}$$

$V_R = V_{RS} + V_{RP}$

= - 7.71 x 10^5 +-1.51 x 10^6

= -2.28 x 10^6V

Example 13



Calculate the electric field intensity midway between Q1 and Q2

Solution

From $E = \frac{Q}{4\pi\varepsilon_0 r^2}$

E_{Q_1} at A due to Q_1 (= $\frac{9\,x10^{9}\,x5\,x\,10^{-6}}{(5\,x\,10^{-2})^2}$ = 1.8 x 10^7NC^{-1}(→)

E_{Q_2} at A due to Q_2 = $\frac{9\,x10^{9}\,x\,6\,x\,10^{-6}}{(5\,x\,10^{-2})^2}$ = 2.16 x 10^7NC^{-1}(←)

E_{Q_3} at A due to Q_3 = $\frac{9\,x10^{9}\,x\,20\,x\,10^{-6}}{(15\,x\,10^{-2})^2}$ = 8.0 x 10^6NC^{-1}(→)

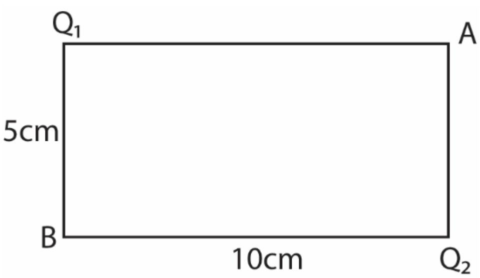
$E_A = E_{Q_1} - E_{Q_2} + E_{Q_3}$

$= 1.8 \times 10^7 - 2.16 \times 10^7 + 8.0 \times 10^6$

$= 4.4 \times 10^6 NC^{-1}$ (→)

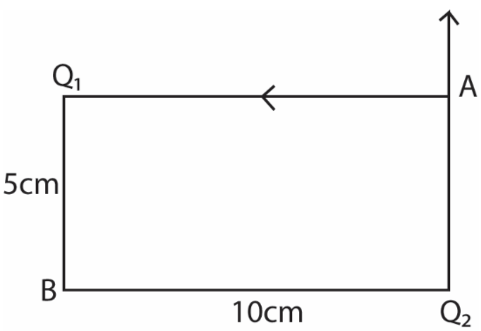
Example 14

Charges Q_1 and Q_2 of -5μC and +2.0μC respectively are placed at two opposite corners of a rectangle of sides 5.0cm and 10.0cm as shown below



Calculate the electric field at A

Solution



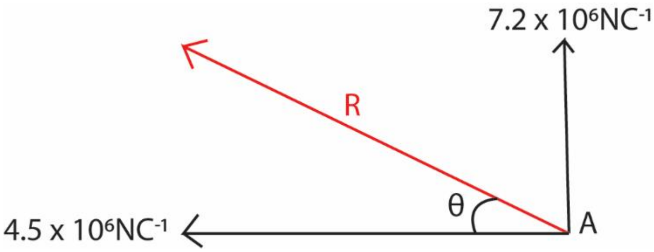
From $= \frac{Q}{4\pi\varepsilon_0 r^2}$, and $E = \frac{1}{4\pi\varepsilon_0} = 9.0 \, x \, 10^9$

E at A due to Q1

$E_{Q_1} = \frac{9 \, x 10^9 \, x \, 5 \, x \, 10^{-6}}{(10 \, x \, 10^{-2})^2}$ = $4.5 \times 10^6 NC^{-1}$

E at A due to Q_2

$E_{Q_2} = \frac{9 \, x 10^9 \, x \, 2 \, x \, 10^{-6}}{(5 \, x \, 10^{-2})^2}$ = $7.2 \times 10^6 NC^{-1}$



$R^2 = (4.5 \times 10^6)^2 + (7.2 \times 10^6)^2$

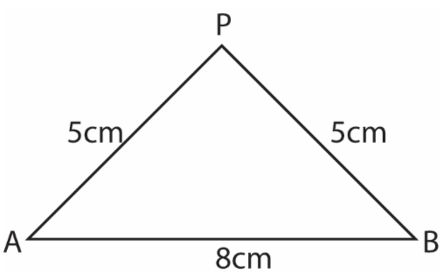
$R = 8.49 \times 10^6\ NC^{-1}$

$\tan \theta = \frac{7.2 \times 10^6}{4.5 \times 10^6}$

$\theta = 58^0$

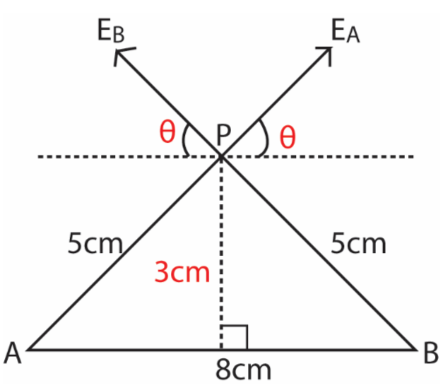
Hence the electric filed intensity at A is $8.49 \times 10^6\ NC^{-1}$ at 58^0 to the horizontal

Example 15



Two point charges A and B of +10µC and +0.05µC are separated by a distance of 8cm along the horizontal as shown above. Find the electric field intensity at P.

Solution



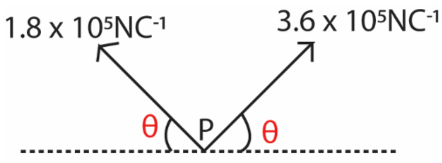
$\sin \theta^0 = \frac{3}{5} = 0.6;\ \cos \theta^0 = \frac{4}{5} = 0.8$

E at P due to charge at A

$E_{Q_1} = \frac{9 x 10^9 \ x \ 0.1 \ x \ 10^{-6}}{(5 \ x \ 10^{-2})^2}$ = 3.6 x 10^5NC^{-1}

E at A due to charge at B

$E_{Q_2} = \frac{9 x 10^9 \ x \ 0.05 \ x \ 10^{-6}}{(5 \ x \ 10^{-2})^2}$ = 1.8 x 10^5NC^{-1}



Resolving vertically

E_y = 3.6 x 10^5sin θ + 1.8 x 10^5sin θ

= 5.4 x 10^5sinθ

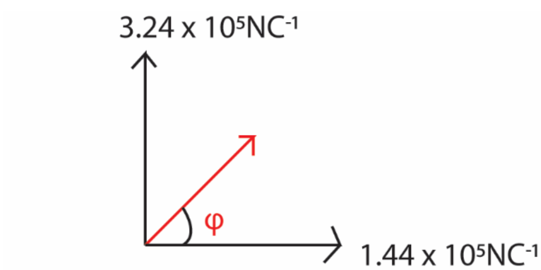
= 5.4 x 0.6 = 3.24 x 10^5 NC^{-1}

Resolving horizontally

E_x = 3.6 x 10^5cos θ - 1.8 x 10^5cos θ

= 1.8 x 10^5 x 0.8

= 1.44 x 10^5 NC^{-1}



R^2 = (1.44 x 10^5)^2 + (3.24 x 10^5)^2

R = 3.55 x 10^5 NC^{-1}

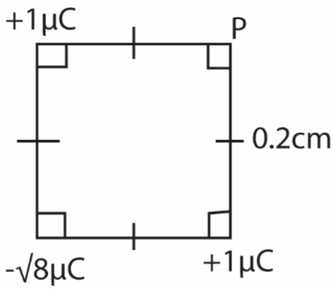
tan θ = $\frac{3.24 \ x \ 10^6}{1.44 \ x \ 10^6}$

θ = 66^0

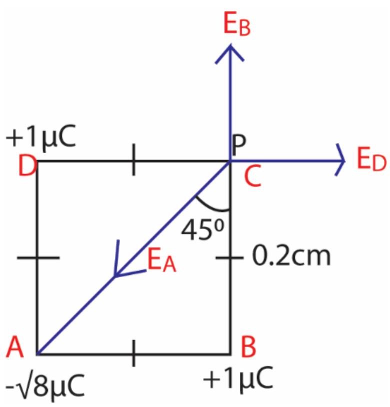
Hence the electric field intensity at A is 3.55 x 10^5NC^{-1} at 66^0 to the horizontal

Example 16

Find the electric field intensity at P.



Solution



$AP^2 = 0.2^2 + 0.2^2$

$AP = \sqrt{8} \times 10^{-1}$ m

E at P due to charge at A

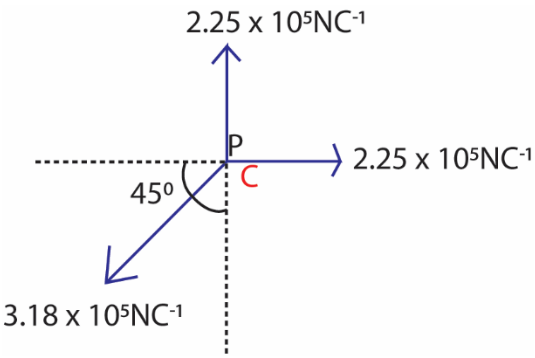
$$E_A = \frac{9 \, x 10^9 \, x \, \sqrt{8} \, x \, 10^{-6}}{\left(\sqrt{8} \, x \, 10^{-1}\right)^2} = 3.18 \times 10^5 NC^{-1}$$

E at P due to charge at B

$$E_{Q_2} = \frac{9 \, x 10^9 \, x \, 1 \, x \, 10^{-6}}{(0.2)^2} = 2.25 \times 10^5 NC^{-1}$$

E at P due to charge at D

$$E_{Q_2} = \frac{9 \, x 10^9 \, x \, 1 \, x \, 10^{-6}}{(0.2)^2} = 2.25 \times 10^5 NC^{-1}$$

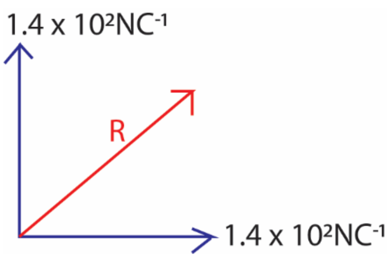


Resolving vertically

$E_y = 2.25 \times 10^5 - 3.18 \times 10^5 \sin 45^0 = 1.4 \times 10^2 NC^{-1}$

Resolving horizontally

$E_x = 2.25 \times 10^5 - 3.18 \times 10^5 \sin 45^0 = 1.4 \times 10^2 NC^{-1}$



$R^2 = (1.4 \times 10^2)^2 + (1.4 \times 10^2)^2$

$R = 1.98 \times 10^2\ NC^{-1}$

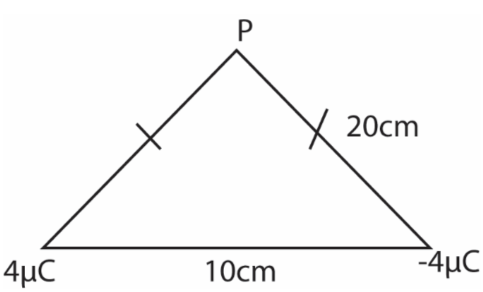
$\tan\theta = \frac{1.4 \, x \, 10^2}{1.4 \, x \, 10^2}$

$\theta = 45^0$

Hence the electric field intensity at P is $1.98 \times 10^2\ NC^{-1}$ at 450 to horizontal

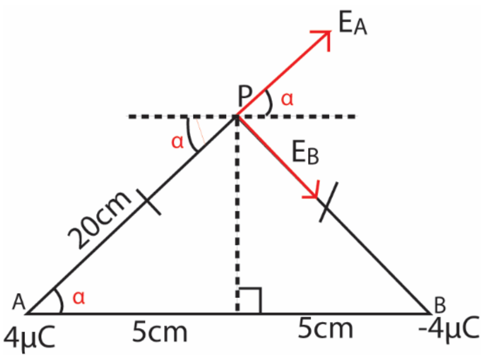
Example 17

Two point charges are separated by 10cm in air as shown



Find the electric field intensity at P.

Solution

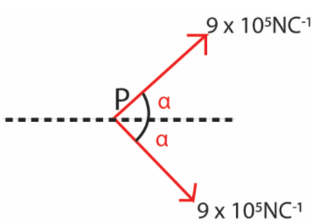


E at P due to charge at A

$E_A = \frac{9 \, x 10^9 \, x \, 4 \, x \, 10^{-6}}{(20 \, x \, 10^{-2})^2}$ = 9 x 10^5NC^{-1}

E at P due to charge at B

$E_B = \frac{9 \, x 10^9 \, x \, 4 \, x \, 10^{-6}}{(20 \, x \, 10^{-2})^2}$ = 9 x 10^5NC^{-1}



Sinα =$\frac{\sqrt{375}}{20}$; cos α = $\frac{5}{20} = 0.25$

Resolving vertically

E_y = 9 x 10^5NC^{-1}sinα - 9 x 10^5NC^{-1}sinα = 0

Resolving horizontally

E_x = 9 x 10^5NC^{-1}cosα + 9 x 10^5NC^{-1}cosα

= 18 x 10^5 x .25 = 4.5 x 10^5NC^{-1}

R^2 = (4.5 x 10^5)^2 + (0)^2

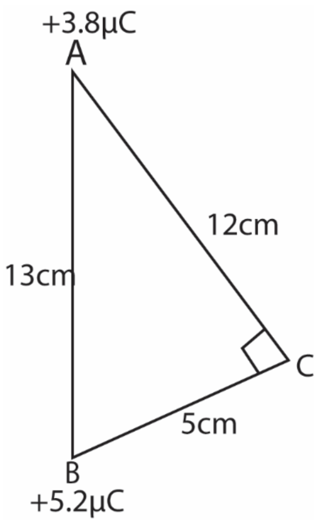
R = 4.5 x 10^5 NC^{-1}

tan θ = $\frac{0}{4.5 \text{ x } 105}$

$\theta = 0^0$

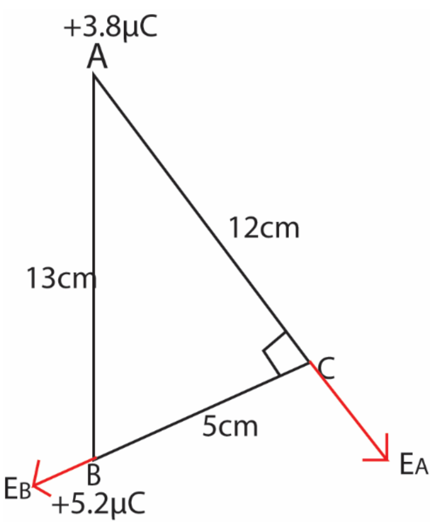
Hence the electric field intensity at P is 4.5×10^5 NC^{-1} at 0^0 to the horizontal

Example 18



Two points +3.8µC and -5.2µC are place in air at points A and B as shown. Determine the electric field intensity at C

Solution

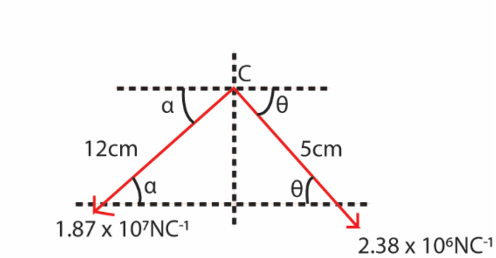


E at C due to charge at A

$$E_A = \frac{9\,x10^9\ x\ 3.8\ x\ 10^{-6}}{(12\ x\ 10^{-2})^2} = 2.38 \times 10^5 NC^{-1}$$

E at C due to charge at B

$$E_B = \frac{9\,x10^9\ x\ 5.2\ x\ 10^{-6}}{(5\ x\ 10^{-2})^2} = 1.87 \times 10^7 NC^{-1}$$



tan α = $\frac{5}{12}$; α = 22.6   tan θ =$\frac{12}{5}$; θ =67.4

Resolving vertically;

E_y = 1.87 x 10^7sin 22.6 + 2.38 x 10^6sin 67.4

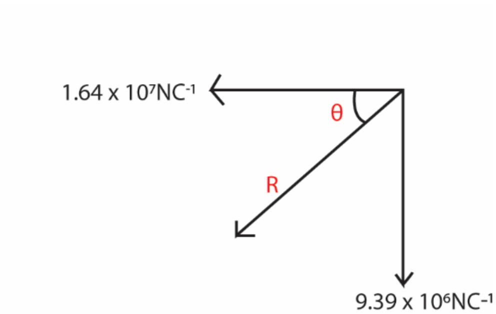
= 9.39 x $10^6$$NC^{-1}$

Resolving horizontally

E_x = 2.38 x 10^6cos67.4 - 1.87 x 10^7cos 22.6

= 9.16 x 106 – 1.73x107

= -1.64 x $10^7$$NC^{-1}$



R^2 = $(1.64 \text{ x } 10^7)^2$ + $(9.39 \text{ x } 10^6)^2$

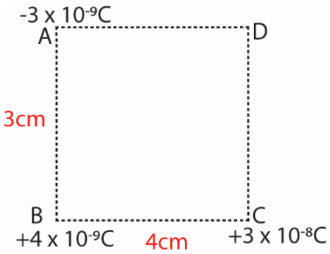
R = 1.89 x $10^7$$NC^{-1}$

tan θ = $\frac{9.39\ x\ 10^6}{1.64\ x\ 10^7}$

θ = 29.79^0

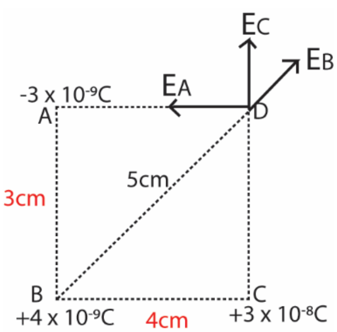
Hence the electric field intensity at C is 1.89 x $10^7$$NC^{-1}$ acting at 29.7^0 to horizontal.

Example 19



Three charges of -3 x 10^{-9}C, +4 x 10^{-9}C and +3 x 10^{-9}C are placed at vertices A, B, C respectively of a rectangle ABCD of side 3cm x 4cm as shown. Calculate the resultant electric field intensity at D

Solution



E at D due to charge at A

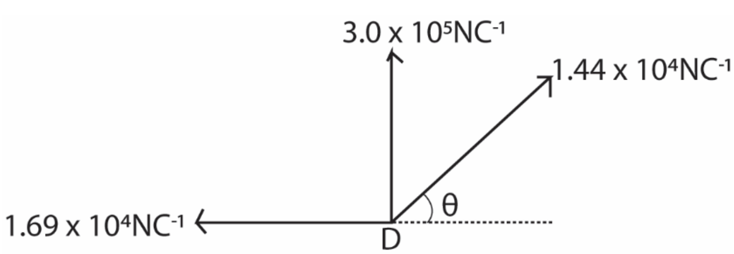
$E_A = \frac{9\,x10^9\;x\;3\;x\;10^{-9}}{(4\;x\;10^{-2})^2}$ = 1.69 x 10^4NC^{-1}

E at D due to charge at B

$E_B = \frac{9\,x10^9\;x\;4\;x\;10^{-9}}{(5\;x\;10^{-2})^2}$ = 1.44 x 10^4NC^{-1}

E at D due to charge at C

$E_C = \frac{9\,x10^9\;x\;3\;x\;10^{-8}}{(3\;x\;10^{-2})^2}$ = 3.0 x 10^5NC^{-1}



sin θ = $\frac{3}{5} = 0.6$; cos θ = $\frac{4}{5} = 0.8$

Resolving vertically;

$E_y = 3.0 \times 10^5 + 1.44 \times 10^4 \sin\theta$

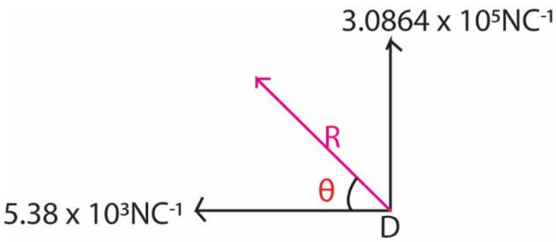
$= 3.0864 \times 10^5\ NC^{-1}$

Resolving horizontally

$E_x = -1.69 \times 10^4 + 1.44 \times 10^4 \cos\theta$

$= -1.69 \times 10^4 + 1.44 \times 10^4 \times 0.8$

$= -5380 NC^{-1}$



$R^2 = (3.0864 \times 10^5)^2 + (5380)^2$

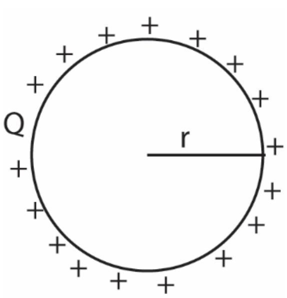
$R = 3.09 \times 10^5 NC^{-1}$

$\tan\theta = \frac{3.0864 \times 10^5}{5380}$

$\theta = 89^0$

Hence electric field intensity at D = $3.09 \times 10^5 NC^{-1}$ at 89^0 to horizontal

Field strength and charge density of a hollow sphere



Let Q be the charge on the surface of the sphere and δ be the charge density (charge per unit area)

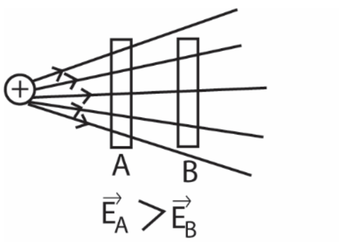
Charge on the sphere = charge density x area

$Q = \delta \times 4\pi r^2$

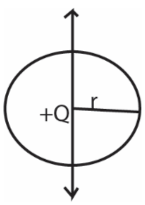
Now E = $\frac{Q}{\varepsilon_0 \, x \, 4\pi r^2} = \frac{\delta \, x \, 4\pi r^2}{\varepsilon_0 \, x \, 4\pi r^2} = \frac{\boldsymbol{\delta}}{\boldsymbol{\varepsilon_0}}$

Flux from a point charge

The density of lines of force increase near the charge where the intensity is high. The electric field intensity; E, at a point can be represented by the number of lines per unit area through the perpendicular unit area through the perpendicular surface to the line of force at a point. The product E x area is called the electric flux through a perpendicular area to a line of force.



Flux across a sphere



For a sphere of radius, r, drawn concentric with positive +Q in free space, then

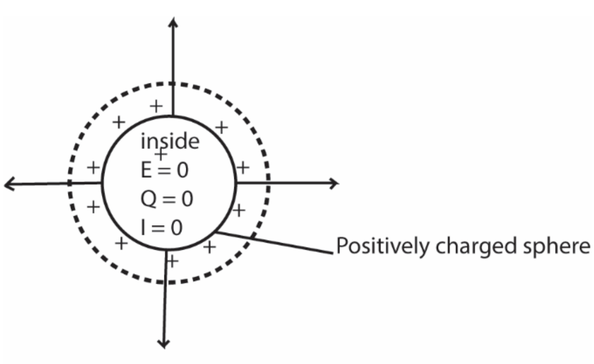
Electric flux, Φ = $\vec{E}$ x area

$$= \frac{Q}{\varepsilon_0 \, x \, 4\pi r^2} \text{ x } 4\pi r^2 = \frac{\boldsymbol{Q}}{\boldsymbol{\varepsilon_0}}$$

Electric field intensity of a hollow conductor

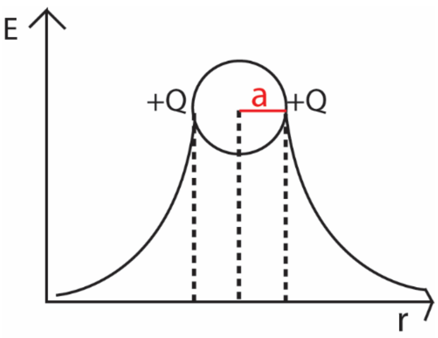
Inside a charged hollow sphere, there is no charge so

Q = 0 and I = 0, hence E = 0



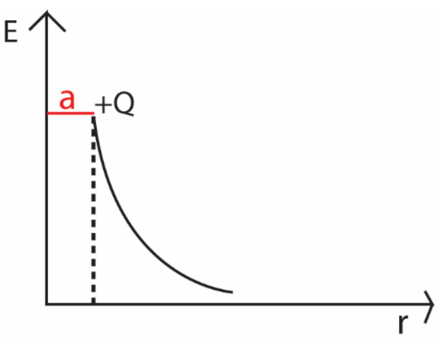
Therefore, at all points inside the sphere, the field strength is zero. There is no work done when a charge is moved between any two points inside a sphere.

A graph showing electric intensity of a hollow conductor of radius, a



Or

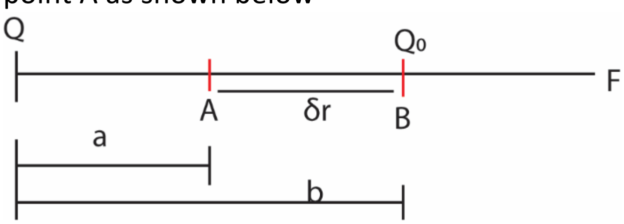
A graph showing electric intensity from the center of a hollow conductor of radius, a



Electrode potential

Electric potential, the amount of work needed to move a unit charge from a reference point to a specific point against an electric field without producing an acceleration.

Consider a charge Q_0 moved distance **δr** against an electric field of charge Q from point B to point A as shown below



Work done = force x distance

$\delta W = F\delta r$

$= \frac{Q \cdot Q_0}{4\pi\varepsilon_0 r^2}\delta r$

Total work done, W is given by

$$W = \frac{Q \cdot Q_0}{4\pi\varepsilon_0} \int_a^b \frac{1}{r^2} \delta r$$
$$= \frac{Q \cdot Q_0}{4\pi\varepsilon_0} \left[\frac{1}{r}\right]_a^b$$
$$= \frac{Q \cdot Q_0}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right)$$

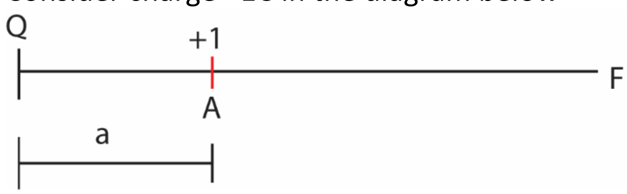
Note: the zero of potential is at an infinity distance from an electric charge.

Electric potential at a point

Electric potential at a point is the work done in moving a positive charge of one coulomb from infinity to that point in an electric field against electrostatic force.

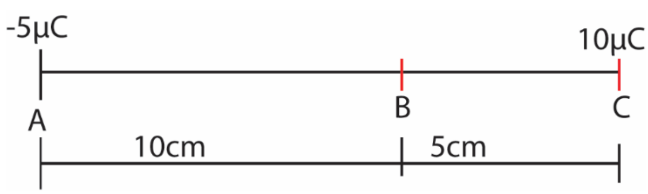
Potential is a property of a point in a field and therefore it is a scalar quantity since it defines work done or potential per unit charge. It is measured in volts.

Consider charge +1C in the diagram below



The electric field at A = $\frac{Q \cdot 1}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{\infty}\right) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{a}$

Example 20



Find electric field at B

Electric potential at B due to charge at A

Solution

$V_A = \frac{9 \times 10^9 \times -5 \times 10^{-6}}{10 \times 10^{-2}}$ = -4.5 x 10^5V

Electric potential at B due to charge at C

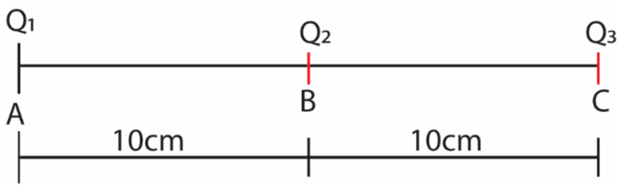
$V_C = \frac{9 \times 10^9 \times 10 \times 10^{-6}}{5 \times 10^{-2}}$ = 1.8 x 10^6V

Potential at B = $V_A + V_C$

$= -4.5 \times 10^5 V + 1.8 \times 10^6 V$

$= 1.35 \times 10^6 V$

Example 21



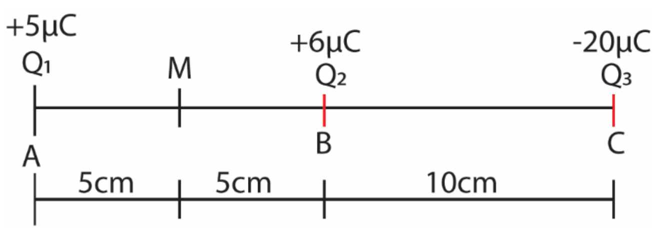
Three point charges Q_1, Q_2 and Q_3 of magnitude +5µC, +6µC and -20µC respectively are situated along a straight line as shown in figure above. Calculate the electric field

(i) Intensity mid-way between Q_1 and Q_2

(ii) Potential midway between Q_1 and Q_2

Solution

Let M be midpoint between Q_1 and Q_2



(i) E at M due to Q_1

$$E_{Q_1} = \frac{9 \times 10^9 \times 5 \times 10^{-6}}{(5 \times 10^{-2})^2} = 1.8 \times 10^7 NC^{-1} (\rightarrow)$$

E at M due to Q_2

$$E_{Q_2} = \frac{9 \times 10^9 \times 6 \times 10^{-6}}{(5 \times 10^{-2})^2} = 2.16 \times 10^7 NC^{-1} (\leftarrow)$$

E at M due to Q_3

$$E_{Q_3} = \frac{9 \times 10^9 \times 20 \times 10^{-6}}{(15 \times 10^{-2})^2} = 8 \times 10^6 NC^{-1} (\rightarrow)$$

$$E_M = E_{Q_1} + E_{Q_2} + E_{Q_3}$$

$= 1.8 \times 10^7 - 2.16 \times 10^7 + 8 \times 10^6 NC^{-1} = 4.40 \times 10^6 NC^{-1}$

Therefore the electric field intensity midway between Q_1 and Q_2 is $4.40 \times 10^6 NC^{-1}$ towards right.

(ii) Electric potential at M midway between Q_1 and Q_2

Electric potential at M due to charge Q_1

$V_{Q_1} = -\frac{9 \, x \, 10^9 \, x \, 5 \, x \, 10^{-6}}{5 \, x \, 10^{-2}}$ = - 9.0 x 10^5V

Electric potential at M due to charge Q_2

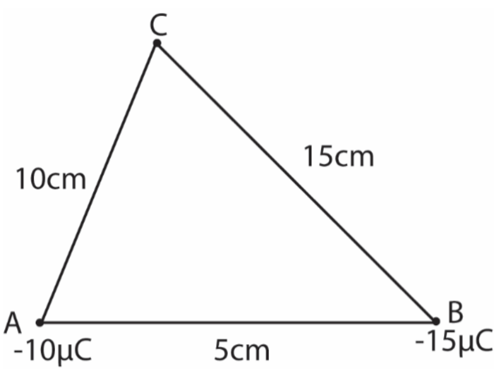
$V_{Q_2} = +\frac{9 \, x \, 10^9 \, x \, 6 \, x \, 10^{-6}}{5 \, x \, 10^{-2}}$ = 1.08 x 10^6V

Electric potential at M due to charge Q_3

$V_{Q_2} = -\frac{9 \, x \, 10^9 \, x \, 20 \, x \, 10^{-6}}{15 \, x \, 10^{-2}}$ = -1.20 x 10^6V

V_M = V_{Q_1}+ V_{Q_2}+V_{Q_3}
= - 9.0 x 10^5 + 1.08 x 10^6 +-1.20 x 10^6
= 1.02 x 10^6V

Example 22



Two point charge A and B are situated along a straight line as shown in figure above. Calculate the electric potential at C.

Solution

Electric potential at C due to A

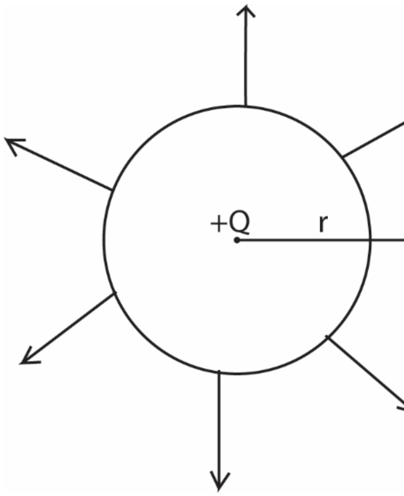
$V_A = \frac{Q}{4\pi\varepsilon_0 r_A} = \frac{9 \, x \, 10^9 \, x \, -10 \, x \, 10^{-6}}{10 \, x \, 10^{-2}}$ = -9.0 x 10^5V

Electric potential at C due to B

$V_A = \frac{Q}{4\pi\varepsilon_0 r_B} = \frac{9 \, x \, 10^9 \, x \, -15 \, x \, 10^{-6}}{15 \, x \, 10^{-2}}$ = -9.0 x 10^5V

Potential at C, V_c = $V_A + V_B$ = -9.0 x 10^5 +-9.0 x 10^5 = -1.8.0 x 10^6V

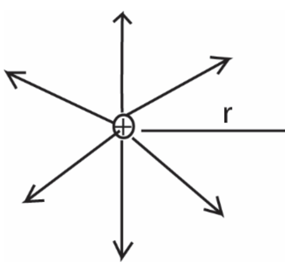
Potential due to a conducting sphere



$V_A = \frac{Q}{4\pi\varepsilon_0 r}$ Where r is the distance of A from center of sphere

Note that potential at the surface and any point inside a sphere of radius r_0 = $\frac{Q}{4\pi\varepsilon_0 r_0}$

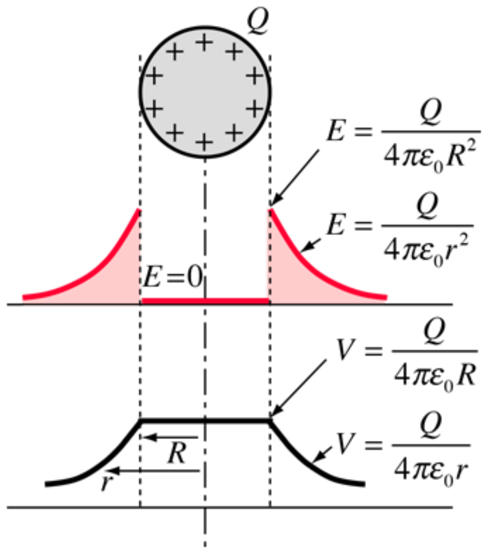
Potential due to a point charge



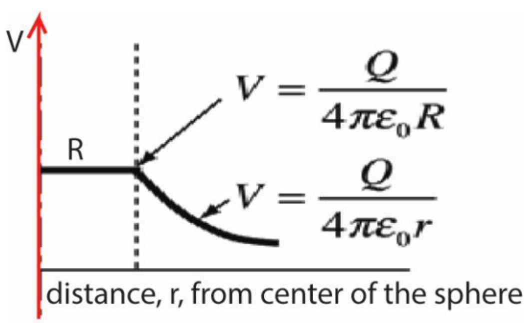
$V_A = \frac{Q}{4\pi\varepsilon_0 r}$ Where r is the distance of A from center of charge

Variation of potential with distance from the center of hollow conductor.

The potential inside a hollow sphere is constant equal to the potential on the surface of the sphere. From the surface of the sphere the potential varies with the inverse of the distance, r, of a point from the center of the sphere.



Drawn from the center

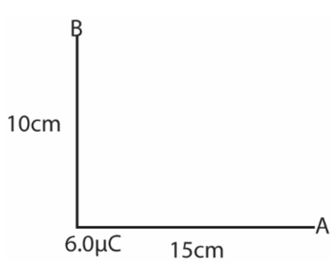


Potential difference (p.d)

P.d between two points is the work done per unit charge (or the energy per unit charge) passing from one point to another

Example 23

Consider two points A and B at a distance of 15cm and 20cm respectively from a point charge of 6.0μC as shown below



(i) Find electric potential difference between A and B

(ii) Calculate the energy required to bring a charge 1μV from infinity to point A

(iii) What is the significance of the sign of the energy above

Solution

(i) Potential at A

$V_A = \frac{Q}{4\pi\varepsilon_0 r_A}$ = $\frac{9\,x\,10^9\,x\,6\,x\,10^{-6}}{15\,x\,10^{-2}}$ = 3.60 x 10^5V

Potential at B

$V_B = \frac{Q}{4\pi\varepsilon_0 r_B}$ = $\frac{9\,x\,10^9\,x\,6\,x\,10^{-6}}{20\,x\,10^{-2}}$ = 2.70 x 10^5V

$V_{AB} = V_A - V_B$

3.60 x 10^5 - 2.70 x 10^5V = 9.0 x10^4V

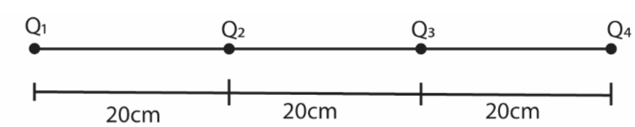
(ii) Work done = Q x V_A

= 1.0 x 10^{-6} x 3.60 x 10^5 = 0.36J

(iii) Positive energy implies that work was done by external force on the charge to bring it from infinity of A

Example 24

A figure below shows charges Q_1, Q_2, Q_3 and Q_4 of -1, +2, -3, and +4μC arranged in a straight line as shown



(i) Calculate the potential energy of Q_2

(ii) What is the significance of the sign of the potential energy.

Solution

Electric potential at Q_2 due to Q_3

V_{Q_1} = $\frac{9\,x\,10^9\,x\,-1\,x\,10^{-6}}{20\,x\,10^{-2}}$ = -4,5 x 10^4V

Electric potential at Q_2 due to Q_3

V_{Q_3} = $\frac{9\,x\,10^9\,x\,-3\,x\,10^{-6}}{20\,x\,10^{-2}}$ = 1.35 x 10^5V

Electric potential at Q_2 due to Q_4

$V_{Q_4} = \frac{9 x 10^9 x 4 x 10^{-6}}{40 x 10^{-2}}$ = 9.0 x 10^4V

$Potential\ at\ Q2, V_{Q_2}$ = $V_{Q_1} + V_{Q_3} + V_{Q_4}$

= -4,5 x 10^4 + 1.35 x 10^5 + 9.0 x 10^4

= -9.0 x 10^4V

Potential energy at Q_2 = Q_2 x V_{Q_2} = 2.0 x 10^{-6} x -9.0 x 10^4 = -0.18J

(ii) Negative sign on energy shows that work is done by electric field on the charge from infinity to that point.

Thank you
Dr. Bbosa Science