SENIOR SIX TERM 2

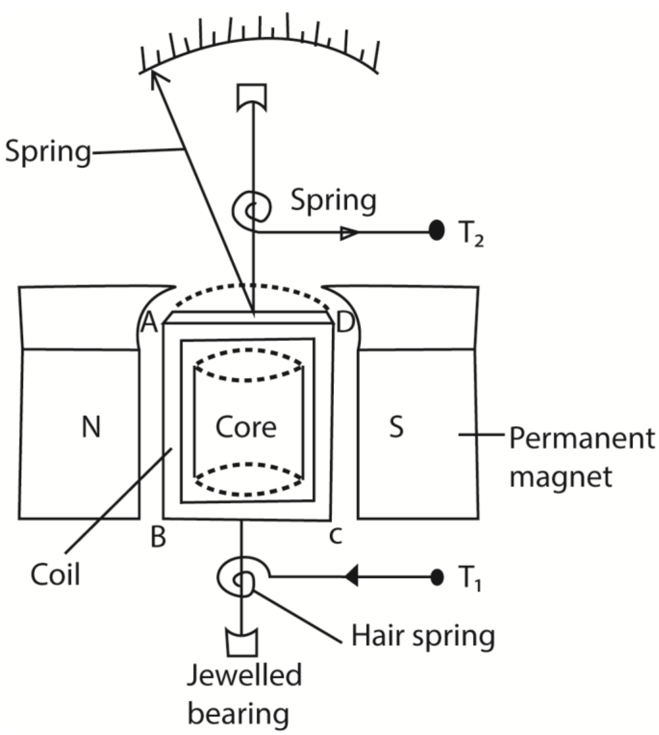
TOPIC 5/5: A.C CIRCUITS

Competency: The learner measures a.c and investigates its behaviour in different devices.

Difference between a d.c generator and d.c motor

A d.c. generator converts mechanical energy into electrical energy while a d.c motor converts electrical energy into mechanical energy

Moving coil galvanometer.



Structure

- It consist of a rectangular coil of fine insulated copper wire wound on an aluminium frame to provide electromagnetic damping.
- The coil together with the frame of aluminium are mounted over a soft iron cylindrical core and freely pivoted on jeweled bearing to minimize friction at contact.
- The suspension torsion wire suspending the coil is attached to a pair of control hair springs T_1 and T_2 for feeding current in and out of the coil and control rotation of the coil.
- The coil is then suspended between concave pieces of a strong magnet to provide magnetic field.

Mode of action

- Current I to be measured is passed into the coil via hair spring T_1.
- The current then causes the coil to experience a deflection torque, τ = NABI due to a couple force causing rotation in a radial magnetic field.
- The coil turns with the pointer through angle θ until stopped by a restoring torque, $\tau = k\theta$ provided by a pair of hair springs T_1 and T_2.
- At equilibrium, NABI = Kθ
- $\therefore$ current I = $\left(\frac{k}{NAB}\right)\theta$
- I $\propto$ θ, hence the instrument has a linear scale

  Where B = magnetic field strength between the poles of the magnets
  A = area of the plane of the coil
  N = number of turns of the coil
  k = torsion constant of suspension spring

The factors which affect the current sensitivity of a moving coil galvanometer

- Strength of magnet, current sensitivity is proportional to the strength of the magnets
- Number of turns, current sensitivity is proportional to the number of turns
- Nature of suspension torsion wire, current sensitivity is inversely proportional to the torsion constant of the suspension torsion wire.
- Area A of the plane of the coil; current sensitivity is proportional to the area of the plane of the coil
- Size of copper wire making the coil; current sensitivity is proportional to the size of copper wires making the coil since the bigger the wires the lower the resistance.

Example 1

Explain why a moving coil ammeter cannot be used to measure alternating current from the mains. (03marks)

The coil rotates about a vertical axis between the north and south concave poles of strong magnets. Thes provides a radial magnetic field. When an alternating current is passed through the coil, the torque on the coil reverses direction at the same frequency as current. The pointer vibrates with very small amplitude about the mean position hence a steady current reading cannot be taken

Example 2

Explain why a moving coil galvanometer should have a radial magnetic field, fine springs and many turns. (06marks)

Radial magnetic field ensures that the Force, F, remains normal to the plane of the coil when it turns through an angle i.e.

Torque on the coil τ = BANIsinα, where α is the angle between the normal to the coil and magnetic field this is balanced by restoring torque = kθ due to current, where θ is the angle turned through

Thus, current sensitivity given by $\frac{\theta}{I} = \frac{BANIsin\alpha}{k}$

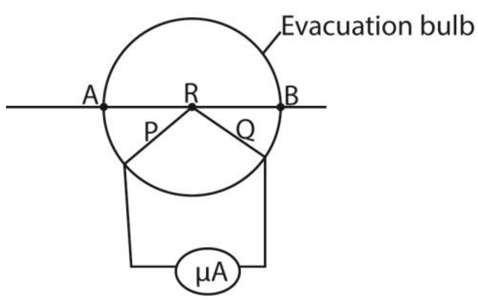
For a linear scale, sin α = 1 => radial field

For current sensitivity, k must be small; i.e. the springs must be fine

For current sensitivity, N, must be large

a.c measuring instruments

(a) Thermocouple ammeter

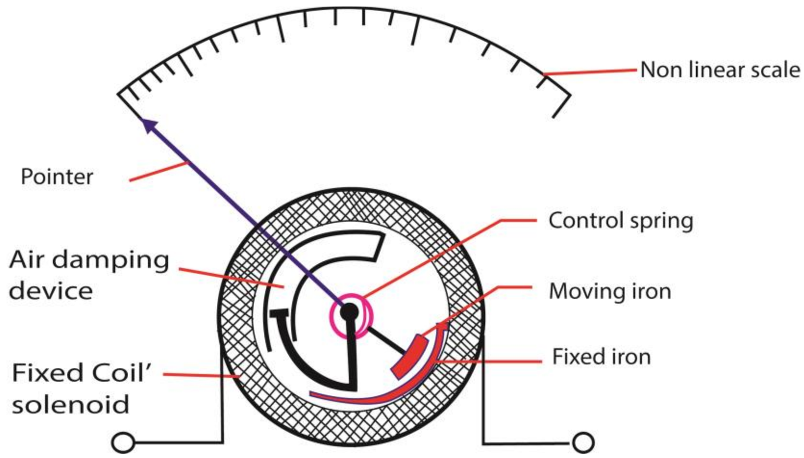


P and Q are dissimilar wires
Current to be measured is passed through the wire AB and heats the junction R of the thermocouple. The thermoelectric effect generated at R causes a direct current to flow through the micrometer calibrated to measure the r.m.s value of current.

Example 3
Explain any precautionary measure taken in the design of thermocouple meter (02mark)

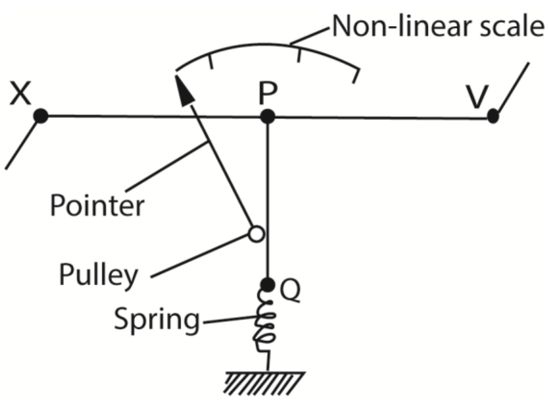
The fine wire is enclosed in an evacuated glass bulb to shield it from draughts. If the wire was in the open, some heat would be lost to the surrounding so that the temperature difference between the hot and cold junctions would not be proportional to the power dissipated in the wire.

The repulsion type moving iron meter.



- When a current is passed through the coil, the iron rods magnetize in the same poles adjacent to each other in whatever the direction of current. Hence they repel and the pointer moves in the same direction until it is stopped by the restoring spring.
- Since the magnetic force is proportional to the square of the average current, hence, the deflection is proportional to the square of average current.
- **Advantage:** it measured both direct and alternating current.
- **Disadvantage:** it has nonlinear scale

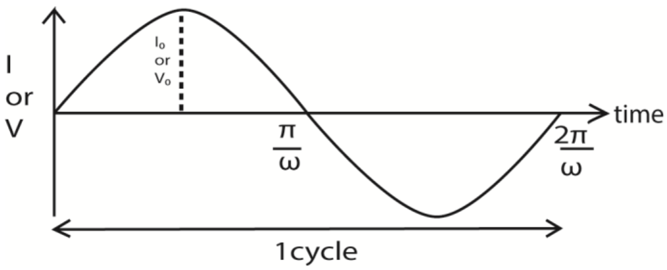
A hot wire ammeter.



- The current flows through a fine resistance-wire XY, which it heats.
- The wire warms up to such a temperature that it loses heat-mainly by convection-at a rate equal to the average rate at which heat is developed in the wire.
- The rise in temperature of the wire makes it expand and sag; the sag is taken up by a second fine wire PQ, which is held taut by a spring.
- The wire PQ passes round a pulley attached to the pointer of the instrument, which rotates as the wire XY sags.

The deflection of the pointer is roughly proportional to the average rate at which heat is developed in the wire XY; it is therefore roughly proportional to the average value of the square of the alternating current, and the scale is a square-law one.

Sinusoidal a.c Currents



Terminologies

An alternating current or voltage

It is the voltage or current whose magnitude and direction varies periodically with time.

NB. Its variations with time can be represented by sine curve or wave and it is sometimes referred to as a sinusoidal current or voltage.

Example 4

What is meant by a sinusoidal voltage?

A sinusoidal voltage is a voltage whose variation with time can be represented by a sine curve.

Peak value/ amplitude

It is the maximum current or voltage of an alternating current.

NB: peak values are denoted by V_0, V_m, I_0. or I_m.

Root mean squared value (r.m.s)

It is a steady current or voltage that would dissipate heat in a given resistor at the same rate as the alternative current or voltage.

$I_{r.m.s} = \frac{I_0}{\sqrt{2}}$ and $V_{r.m.s} = \frac{V_0}{\sqrt{2}}$

Period, T

It is the time taken to complete one cycle

Frequency, f

It is the number of cycles per second

f = $\frac{1}{T}$

a.c wave equation

The sinusoidal wave equation for a.c are represented by

$V = V_0 \sin \omega t$ and,

$I = I_0 \sin\omega t$

Where I_0 and V_0 are the peak alternating current or voltage

But $\omega = \frac{2\pi}{T}$ and $T = \frac{1}{f}$

$\therefore\ \omega = 2\pi f$

Thus the wave equations can be written as

$V = V_0 \sin 2\pi f\, t$ and,

$I = I_0 \sin 2\pi f\, t$

From Ohm's law

$V = IR$

$R = \frac{V_{r.m.s}}{I_{r.m.s}}$

Example 5

Show that the r.ms value of an alternating voltage is $V_{r.ms} = \frac{V_0}{\sqrt{2}}$, where V_0 is the peak voltage (03marks)

Instantaneous power $= \frac{V^2}{R}$, where $V = V_0 sin\omega t$

$P_{inst} = \frac{V_0^2 sin^2 \omega t}{R}$

Average power, $P = \frac{V_0^2 sin^2 \omega t}{R}$

But $\sin^2\omega t = \frac{1}{2}$

For steady voltage, $P = \frac{V_{r.m.s}^2}{2R}$

$=> \frac{V^2}{R} = \frac{V_{r.m.s}^2}{2R}$

Hence $V_{r.m.s} = \frac{V_0}{\sqrt{2}}$

Example 6

A sinusoidal alternating voltage $V = 170\sin 120\pi t$, voltage, is applied across a resistor of resistance 100Ω Determine

(i) The r.m.s value of current which flows. (03marks)

$I_{r.m.s} = \frac{V_{r.m.s}}{R}$ but $\frac{V_0}{\sqrt{2}}$

$\therefore\ I_{r.m.s} = \frac{V_0}{R\sqrt{2}} = \frac{170}{100\sqrt{2}} = 1.2\ A$

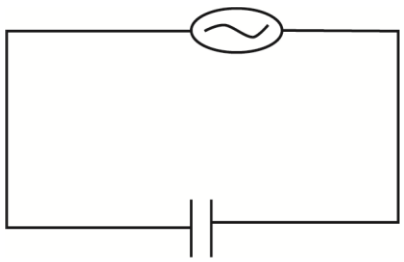
(ii) The frequency of the current through the resistor. (02marks)

$\omega = 2\pi f$; $\omega = 120\pi$

$\therefore f = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi} = 60Hz$

A capacitor in an a.c circuit

Consider a capacitor C farads in an a.c circuit as shown below



Assume a sinusoidal voltage

$V = V_0 \sin\omega t$ ……………………….. (i)

Across the capacitor plates from definition of capacitor.

The instantaneous charge Q on plates is

$Q = CV$……………………………. (ii)

Putting (i) into (ii)

$Q = C V_0 \sin\omega t$ …………………. (iii)

From definition of current

$I = \frac{dQ}{dt}$ (rate of charge flow)

$I = \frac{d(CV_0 \sin \omega t)}{dt}$…………………. (iv)

$I = \omega CV_0 \cos \omega t$ …………………. (v)

Putting $\omega CV_0 = I_0$

$I = I_0 \cos\omega t$

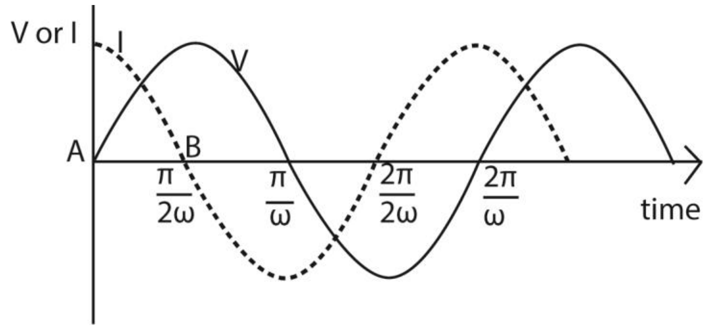
From trigonometry, $\cos \theta = \sin (\theta + \frac{\pi}{2})$

$\therefore I = I_0 \sin (\omega t+\frac{\pi}{2})$

$= \omega CV_0 \sin (\omega t+\frac{\pi}{2})$

Hence the current leads the voltage by a phase angle of $\frac{\pi}{2}$

Variation of V and I against time



Explanation of the curve

- The current I through (not across) a capacitor is $C\frac{dV}{dt}$. This is the slope of the voltage.
- When a capacitor is connected to an AC voltage (at the zero crossing) the maximum current flow will occur immediately, i.e. t = 0, $\frac{dV}{dt}$ will approach infinity instantly and then repeat as a sinusoidal waveform.
- There will be a peak current at every zero crossing of the voltage waveform. The current waveform will lead the voltage waveform 90 degrees.

Reactance

This is the non-resistive opposition to the flow of alternating current in either a capacitor or an inductor.

Greater reactance leads to smaller currents for the same applied voltage. Reactance is similar to electric resistance, although it differs in several respects

Reactance of a capacitor (capacitive reactance)

It's denoted by Xc.

Definition, Xc = $\frac{V_0}{I_0}$ or Xc = $\frac{V_{r.m.s}}{I_{r.m.s}}$

Derivation of an expression for reactance of a capacitor

Assume a sinusoidal voltage;

$V = V_0\sin\omega t$ ……………………… (i)

Across capacitor's plates; the instantaneous charge Q is

Q =CV ……………………………. (ii)

$Q = CV_0\sin\omega t$ …………………. (iii)

From definition of current,

I = $\frac{dQ}{dt}$ (rate of charge flow) ……. (iv)

I = $\frac{d(CV_0 \sin \omega t)}{dt}$ ……………………… (v)

I = $\omega CV_0 \cos \omega t$ …………………… (vi)

But $\omega CV_0 = I_0$ ……………………… (vii)

From definition

Capacitance reactance, Xc = $\frac{V_0}{I_0}$

$= \frac{V_0}{\omega CV_0}$

$= \frac{1}{\omega C}$

But ω = 2πf

$\therefore X_c = \frac{1}{2\pi fC}$

Example 7

A sinusoidal voltage of r.m.s value 13.2V is connected across a 50µF capacitor.

(i) Find peak value of the charge on the capacitor (02marks)

$Q_0 = CV_0$ but $V_0 = V_{r.m.s} x \sqrt{2}$

Hence $Q_0 = CV_{r.m.s}\ x \sqrt{2}$

$= 50 \times 10^{-6} \times 13.2 \times \sqrt{2}$

$= 9.333 \times 10^{-4}$C

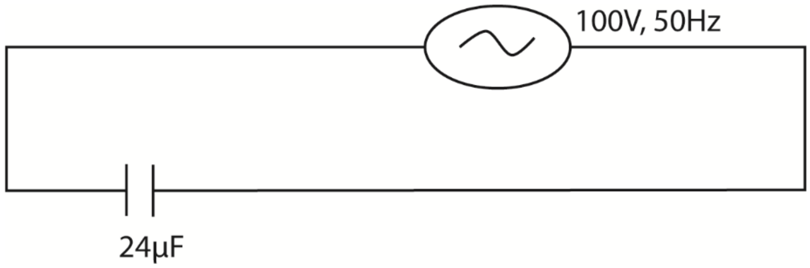
(ii) If the frequency of the alternating current is 49.6Hz, calculate the r.m.s value of current through the capacitor. (03marks)

$\frac{V_{r.m.s}}{I_{r.m.s}} = \frac{1}{2\pi fC} = X_C$

$I_{r.m.s} = V_{r.m.s}\ x\ 2\pi fC$

$= 13.2 \times 2\pi \times 49.6 \times 50 \times 10^{-6}$ =0.206A

Example 8



A 100V, 50Hz a.c. supply is connected across a capacitor of 24µF as shown in figure above. Calculate the reactance of the circuit (03marks)

Capacitive reactance, Xc = $\frac{1}{2\pi f C} = \frac{1}{2\pi \; x \; 50 \; x \; 24 \; x \; 10^{-6}}$ = 132.6Ω

Example 9

A capacitor of capacitance 60µF is connected to an a.c. voltage supply of frequency 40Hz. An a.c ammeter connected in series with the capacitor reads 2.2A. Find the p.d across the capacitor. (03marks).

$X_c = \frac{I}{2\pi f C} = \frac{1}{2\pi \; x \; 40 \; x \; 60 \; x \; 10^{-6}}$

Voltage, $V_{r.m.s} = I_{r.m.s}$ x Xc

$= \frac{2.2 \; x \; 1}{2\pi \; x \; 40 \; x \; 60 \; x \; 10^{-6}}$

= 146V

Example 10

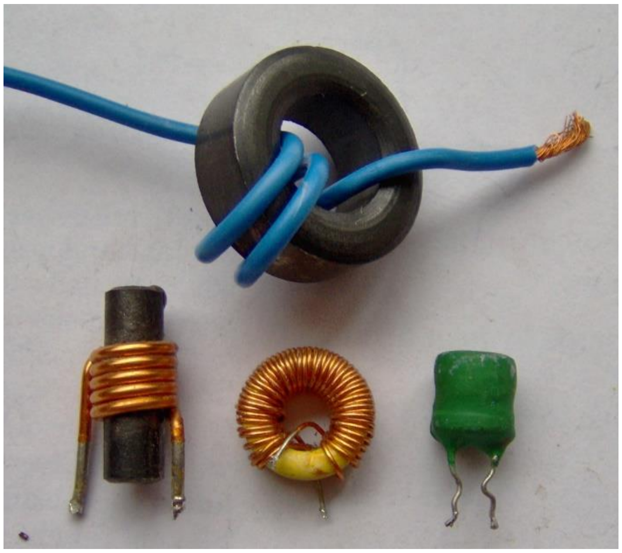
A sinusoidal p.d of r.m.s value of 20V and frequency50Hz is applied across a 100µF capacitor. Calculate the capacitive reactance of the circuit. (02 marks)

Xc = $\frac{1}{2\pi f c} = \frac{1}{2\pi \; x \; 50 \; x \; 100 \; x \; 10^{-6}}$ = 31.8Ω

a.c through an inductor

An inductor, also called a coil, choke, or reactor, is a passive two-terminal electrical component that stores energy in a magnetic field when electric current flows through it. An inductor typically consists of an insulated wire wound into a coil

Example of inductors



Example 11

(i) Show that in a coil placed in an a.c circuit, the voltage across the coil leads the current by a phase angle of $^{\pi}/_{2}$ radian or 90^0

Instantaneous voltage, $V = V_0\sin 2\pi ft$,

$E_B = -L\frac{dI}{dt}$

But for finite current, $V = -E_b$

$V = \frac{LdI}{dt}$; but $V = V_0\sin 2\pi ft$,

$\frac{dI}{dt} = \frac{-V_0}{L} sin2\pi ft$

$\int dI = \frac{V_0}{L}\int \sin 2\pi ftdt$

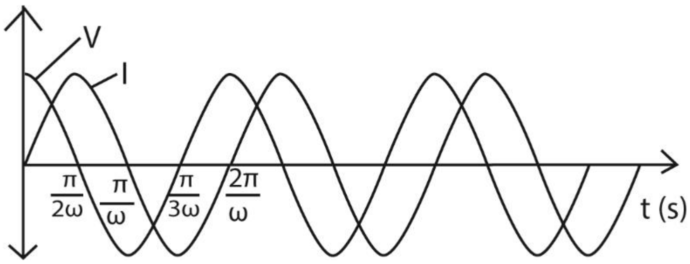
$I = \frac{-V_0}{2\pi ftL} cos2\pi ft$

From trigonometry, $\cos\theta = \sin(\theta + \frac{\pi}{2})$

$I = \frac{-V_0}{2\pi ftL}\sin(2\pi ft + \frac{\pi}{2})$

⇨ I lags V by $\frac{\pi}{2}$ or 90^0

(ii) Using the axes, sketch graphs to show the relative phases of the current and voltage across the inductor. (02marks)



Note that the current starts at zero and rises to its peak *after* the voltage that drives it, i.e., the voltage across an inductor leads the current because the Lenz's Law behavior resists the buildup of current and it takes a finite time for an imposed voltage to force the buildup of current to its maximum.

Derivation of an expression for reactance of an inductor

Method I

Induced e.m.f = $-L\frac{dI}{dt}$

$E = -L\frac{d(I_{)}sin\omega t)}{dt} = -\omega LI_0\cos\omega t$

Current flow in a pure inductors, $V = -E = \omega LI_0\cos\omega t$

Hence $V = V_0\cos\omega t$ where $V_0 = \omega LI_0$

Inductance = $X_L = \frac{V_0}{I_0} = \frac{\omega LI_0}{I_0} = \omega L$

Method II

$V = V_0\cos\omega t$

Induced e.m.f $E = -L\frac{dI}{dt}$

Since the inductor is a coil of zero resistance (for finite current) V = -E

$V_0\cos\omega t = L\frac{dI}{dt}$

$dI = \frac{V_0}{L}\cos\omega t\,.\,dt$

$\int dt = \frac{V_0}{L}\int cos\omega t.\,dt$

$I = \frac{V_0}{\omega L}sin\omega t$

But $\frac{V_0}{\omega L} = I_0$

$X_L = \frac{V_0}{I_0} = \frac{V_0}{V_0/_{\omega L}} = \omega L$

Ballistic galvanometer

It is a moving coil galvanometer used for estimating the quantity of charge flow whose deflection of the coil is directly proportional to the charge that passes through it.

Thank you
Dr. Bbosa Science