Subsidiary Mathematics

SENIOR FIVE term 1

TOPIC 1/3: Matrices

Sub math matrices

Matrices

A matrix is a regular arrangement of array of numbers called elements or entries. The plural of matrix is matrices.

Matrices are named by capital letters and the elements y small letters. For example

P = $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$

The order of matrix

The of a matrix is stated by the number of rows and the number of columns. A matrix having p rows and q column is called a p x q matrix written as p x q. For example

O = $\begin{pmatrix} a \\ b \end{pmatrix}$ has order 1 x 2

P = $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ has order 2 x 2

R = $\begin{pmatrix} 1 & 3 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ has order 2 x 3

S = $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ hasorder3 x3

Types of matrices

(a) **Row matrix** has only one row e.g. $(a \quad b \quad c)$, $(2 \quad 1)$, etc.

(b) **Column matrix** has only one column e.g. $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} x \\ y \end{pmatrix}$ etc.

(c) **Square matrix** has the same number of rows as columns $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$, etc

(d) Equal matrices have equal corresponding elements, e.g. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

(e) A Zero (or null) matrix has all zero elements e.g. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(f) An identity matrix or unity matrix is a square matrix whose every element leading (principal) diagonal is and whose every other element is zero. It is denoted by I, e.g. I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, I = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Operation of matrices

Given A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, B = $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$, I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(a) Addition of matrices

A + B = $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} p & q \\ r & s \end{pmatrix}$

$= \begin{pmatrix} a+p & b+q \\ c+r & d+s \end{pmatrix}$

B + A = $\begin{pmatrix} p & q \\ r & s \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$= \begin{pmatrix} p+b & q+b \\ r+c & s+d \end{pmatrix}$

Note that A+B = B + A

(b) Subtraction of matrices

A - B = $\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} p & q \\ r & s \end{pmatrix}$

$= \begin{pmatrix} a-b & b-q \\ c-r & d-s \end{pmatrix}$

B – A = $\begin{pmatrix} p & q \\ r & s \end{pmatrix} - \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

= $\begin{pmatrix} p-a & q-b \\ r-c & s-d \end{pmatrix}$

Note A – B ≠ B – A

(c) Multiplication of matrices

AB = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} p & q \\ r & s \end{pmatrix}$

= $\begin{pmatrix} ap+br & bq+ds \\ cp+dr & cq+ds \end{pmatrix}$

BA = $\begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

= $\begin{pmatrix} pa+qc & pb+qd \\ ra+sc & rb+sd \end{pmatrix}$

Note that AB ≠ BA

Hence the commutative property does not apply for matrix

(d) Multiplication of a matrix with and identity

AI = $\begin{pmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{pmatrix}\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

= $\begin{pmatrix} \text{a x 1 + b x 0} & a\ x\ 0 + bx1 \\ cx1 + d\ x0 & cx0 + dx1 \end{pmatrix}$

= $\begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix}$

IA = $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{pmatrix}$

= $\begin{pmatrix} \text{1xa + 0xc} & 1xb + 0xd \\ 0xa + 1xb & 0xb + 1xd \end{pmatrix}$

= $\begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix}$

Note that AI = IA = A

(e) Determinant of a matrix

Given that A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Determinant of A is denoted by det(A) or $|A|$=ad – bc

(f) Inverse of matrix

Given that A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Adjunct of A = $\begin{pmatrix} a & -b \\ -c & d \end{pmatrix}$

Inverse of A is denoted by A^{-1}

$A^{-1} = \frac{Adjuctof\ A}{\det(A)}$

Example 1

Given that A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, B = $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Find

(i) A + B = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

= $\begin{pmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{pmatrix}$

= $\begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$

(ii) B + A = $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

= $\begin{pmatrix} 5+1 & 6+2 \\ 7+3 & 8+4 \end{pmatrix}$

= $\begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$

(iii) A – B = = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

$$= \begin{pmatrix} 1-5 & 2-6 \\ 3-7 & 4-8 \end{pmatrix}$$
$$= \begin{pmatrix} -4 & -4 \\ -4 & -4 \end{pmatrix}$$

(iv) B - A $= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$
$$= \begin{pmatrix} 5-1 & 6-2 \\ 7-3 & 8-4 \end{pmatrix}$$
$$= \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$$

(v) AB $= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$
$$= \begin{pmatrix} 1x5+2x7 & 1x6+2x8 \\ 3x5+4x7 & 3\,x6+4x8 \end{pmatrix}$$
$$= \begin{pmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{pmatrix}$$
$$= \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$$

(vi) BA $= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$
$$= \begin{pmatrix} 5x1+6x3 & 5x2+6x4 \\ 7x1+8x3 & 7x2+8x4 \end{pmatrix}$$
$$= \begin{pmatrix} 5+18 & 10+24 \\ 7+24 & 14+32 \end{pmatrix}$$
$$= \begin{pmatrix} 23 & 34 \\ 31 & 46 \end{pmatrix}$$

(vii) BI $= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$$= \begin{pmatrix} 5x1+6x0 & 5x0+6x1 \\ 7x1+8x0 & 7x0+8x1 \end{pmatrix}$$
$$= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

(viii) IB $= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$
$$= \begin{pmatrix} 1x5+0x7 & 1x6+0x8 \\ 0x5+1x7 & 0x6+1x8 \end{pmatrix}$$
$$= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

From (vii) and (vii), BI = IB = B

(ix) Det(A)

A $= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

Det(A) = 1 x 4 – (2x3)

= -2

(x) Det(B) given B $= \begin{pmatrix} 2 & 2 \\ 1 & 4 \end{pmatrix}$

Det (B)= 2 x 4 – 1 x 2

= 6

(xi) 2A

2 A $= 2\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$

Example 2

Given that A $= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, B $= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, find

(i) A^{-1}

Adjunct of A $= \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$

Det(A) = 1 x 4 – 2 x 3 = -2

$A^{-1} = \frac{Adjuctof\ A}{\det(A)}$

$$= -\frac{1}{2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

(ii) B^{-1}

Adjunct of B $= \begin{pmatrix} 8 & -7 \\ -6 & 5 \end{pmatrix}$

Det(B) = 5x8 – 6 x 7

= -2

$B^{-1} = \frac{Adjuctof\ B}{\det(B)}$

$B^{-1} = -\frac{1}{2}\begin{pmatrix} 8 & -6 \\ -7 & 5 \end{pmatrix}$
$$= \begin{pmatrix} -4 & 3 \\ \frac{7}{3} & -\frac{5}{2} \end{pmatrix}$$

(iii) $(BA)^{-1}$

BA $= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$
$$= \begin{pmatrix} 5x1+6x3 & 5x2+6x4 \\ 7x1+8x3 & 7x2+8x4 \end{pmatrix}$$
$$= \begin{pmatrix} 5+18 & 10+24 \\ 7+24 & 14+32 \end{pmatrix}$$
$$= \begin{pmatrix} 23 & 34 \\ 31 & 46 \end{pmatrix}$$

Adjunct of BA $= \begin{pmatrix} 46 & -34 \\ -31 & 23 \end{pmatrix}$

Det(BA) = 23 x 46 – 34 x 31 = 4

$(BA)^{-1} = \frac{1}{4}\begin{pmatrix} 46 & -34 \\ -31 & 23 \end{pmatrix}$
$$= \begin{pmatrix} \frac{23}{2} & -\frac{17}{2} \\ -\frac{31}{4} & \frac{23}{4} \end{pmatrix}$$

Comment on your result (05marks)

AB ≠BA; hence the commutative property does not apply for matrix

Example 3

Given the matrices

A = $\begin{pmatrix}1 & 3\\4 & 2\end{pmatrix}$, B =$\begin{pmatrix}2 & 3\\5 & 1\end{pmatrix}$and C = $\begin{pmatrix}1 & 1\\2 & 2\end{pmatrix}$, find $(ABC)^{-1}$

Solution

ABC = $\begin{pmatrix}1 & 3\\4 & 2\end{pmatrix}\begin{pmatrix}2 & 3\\5 & 1\end{pmatrix}\begin{pmatrix}1 & 3\\4 & 2\end{pmatrix}$

=$\begin{pmatrix}1x2+3x5 & 1x3+3x1\\4x2+2x5 & 4x3+2x1\end{pmatrix}\begin{pmatrix}1 & 3\\4 & 2\end{pmatrix}$

=$\begin{pmatrix}17 & 6\\18 & 14\end{pmatrix}\begin{pmatrix}1 & 3\\4 & 2\end{pmatrix}$

= $\begin{pmatrix}17+24 & 51+12\\18+56 & 54+28\end{pmatrix}$

= $\begin{pmatrix}41 & 63\\74 & 82\end{pmatrix}$

Det(ABC) = 82 x 41 – 74x63 = -1300

Adjunct of (ABC) = $\begin{pmatrix}82 & -63\\-74 & 41\end{pmatrix}$

$(ABC)^{-1}$ = $-\frac{1}{1300}\begin{pmatrix}82 & -63\\-74 & 41\end{pmatrix}$

=$\begin{pmatrix}\frac{-82}{1300} & \frac{63}{1300}\\ \frac{74}{1300} & \frac{41}{1300}\end{pmatrix}$

Example 4

Given that A = $\begin{pmatrix}2 & -3\\1 & 1\end{pmatrix}$ and B = $\begin{pmatrix}4 & 1\\0 & -2\end{pmatrix}$.

Find

(i) AB = $\begin{pmatrix}2 & -3\\1 & 1\end{pmatrix}\begin{pmatrix}4 & 1\\0 & -2\end{pmatrix}$

$=\begin{pmatrix}8+0 & 2+6\\4+0 & 1-2\end{pmatrix}$

$=\begin{pmatrix}8 & 8\\4 & -1\end{pmatrix}$

(ii) BA = $\begin{pmatrix}4 & 1\\0 & -2\end{pmatrix}\begin{pmatrix}2 & -3\\1 & 1\end{pmatrix}$

$=\begin{pmatrix}8+1 & -12+1\\0-2 & 0-2\end{pmatrix}$

$=\begin{pmatrix}9 & -11\\-2 & -2\end{pmatrix}$

Example 5

The matrix A = $\begin{pmatrix}2 & 1\\-3 & 0\end{pmatrix}$ and **I** is a 2 x 2 Identity matrix.

Determine the matrix B such A^2 + ½ B = **I** (05marks)

$\begin{pmatrix}2 & 1\\-3 & 0\end{pmatrix}^2 + \frac{1}{2}B = I$

$\begin{pmatrix}2 & 1\\-3 & 0\end{pmatrix}\begin{pmatrix}2 & 1\\-3 & 0\end{pmatrix} + \frac{1}{2}B = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$\begin{pmatrix}1 & 2\\-9 & -3\end{pmatrix} + \frac{1}{2}B = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$\frac{1}{2}B = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} - \begin{pmatrix}1 & 2\\-9 & -3\end{pmatrix} = \begin{pmatrix}0 & -2\\9 & 4\end{pmatrix}$

$B = 2\begin{pmatrix}0 & -2\\9 & 4\end{pmatrix} = \begin{pmatrix}0 & -4\\18 & 8\end{pmatrix}$

Hence matrix of B =$\begin{pmatrix}0 & -4\\18 & 8\end{pmatrix}$

Example 6

Given the matrices A = $\begin{pmatrix}3 & 5\\-2 & 4\end{pmatrix}$ and

B = $\begin{pmatrix}8 & -3\\-4 & 7\end{pmatrix}$, find

(a) matrix C such that 3A – 2C + B = I, where I is a 2 x 2 identity matrix.(03marks)

$3\begin{pmatrix}3 & 5\\-2 & 4\end{pmatrix} - 2C + \begin{pmatrix}8 & -3\\-4 & 7\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

2C = $\begin{pmatrix}9 & 15\\-6 & 12\end{pmatrix} + \begin{pmatrix}8 & -3\\-4 & 7\end{pmatrix} - \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

= $\begin{pmatrix}16 & 12\\-10 & 18\end{pmatrix}$

C = $\frac{1}{2}\begin{pmatrix}16 & 12\\-10 & 18\end{pmatrix} = \begin{pmatrix}8 & 6\\-5 & 9\end{pmatrix}$

(b) the determinant of C. (02 marks)

Det (C) = 8 x 9 –(-5 x 5) = 72 + 30 = 102

Example 6

The determinant of the matrix

P = $\begin{pmatrix}3a & -1\\ 7 & a\end{pmatrix}$ = 55

(a) Find the value of a. (03 marks)

$|P|$= 3a x a - (7 x -1) = 55

3a^2 + 7 = 55

3a2 = 48

a2 = 16

a = 4

Using one of the value of a, determine the inverse of P.

P =$\begin{pmatrix}12 & -1\\ 7 & 4\end{pmatrix}$

Adjunct of P = $\begin{pmatrix}4 & 1\\ -7 & 12\end{pmatrix}$

$|P|$ = 48 – (7x-1) = 55

Inverse of P^{-1} = $\frac{1}{55}\begin{pmatrix}4 & 1\\ -7 & 12\end{pmatrix}$

= $\begin{pmatrix}\frac{4}{55} & \frac{1}{55}\\ \frac{-7}{55} & \frac{12}{55}\end{pmatrix}$

Revision Exercise1

(Answers are given in square rackets at the end of each question)

1. Given that A = $\begin{pmatrix}0 & 1\\ 3 & 0\end{pmatrix}$ and B =$\begin{pmatrix}2 & 1\\ 3 & 2\end{pmatrix}$
   (a) N^2 [$\begin{pmatrix}7 & 4\\ 12 & 7\end{pmatrix}$]
   (b) MN [$\begin{pmatrix}3 & 2\\ 6 & 3\end{pmatrix}$]
   (c) The value of scalar k if N^2 + kN MN [-2]
2. Given that P = $\begin{pmatrix}1 & 2\\ 4 & 6\end{pmatrix}$, Q = $\begin{pmatrix}9 & 9\\ 1 & 0\end{pmatrix}$ and
   R = $\begin{pmatrix}5 & 0\\ 1 & 4\end{pmatrix}$
   Find (PQR)-1 [$\frac{1}{360}\begin{pmatrix}144 & -36\\ -246 & 64\end{pmatrix}$]
3. If $\begin{pmatrix}4 & 1\\ x & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}4\\ 8\end{pmatrix}$, determine x and y
   [x = -6 or 2 and y =28 or -4]
4. Given A = $\begin{pmatrix}1 & 3\\ 4 & 11\end{pmatrix}$ and B = $\begin{pmatrix}-1 & 2\\ 1 & 3\end{pmatrix}$
   Find
   (i) AB [$\begin{pmatrix}2 & 11\\ 7 & 41\end{pmatrix}$]
   (ii) (AB)$^{-1}$ [$\begin{pmatrix}\frac{41}{5} & \frac{-11}{5}\\ \frac{-7}{5} & \frac{2}{5}\end{pmatrix}$]
5. If matrix P and Q are such that
   P = $\begin{pmatrix}2 & 1\\ 2 & 3\end{pmatrix}$ and PQ = $\begin{pmatrix}12 & 0\\ 0 & 12\end{pmatrix}$
   Find Q [$\begin{pmatrix}9 & -3\\ -6 & 6\end{pmatrix}$]
6. Given the matrix P = $\begin{pmatrix}4.5 & 1\\ 0 & 7\end{pmatrix}$ and
   Q = $\begin{pmatrix}2 & -1\\ 3 & 1\end{pmatrix}$
   Find matrix R such that 3R -1I= 2A – B,
   where I is an identity of order 2. [$\begin{pmatrix}3 & 3\\ -1 & 5\end{pmatrix}$]
7. Given that P = $\begin{pmatrix}1 & 2\\ -2 & 1\end{pmatrix}$, Q = $\begin{pmatrix}2 & -1\\ 1 & 2\end{pmatrix}$, find
   PQ- QP. [$\begin{pmatrix}-4 & 0\\ 0 & 4\end{pmatrix}$]
8. If $\begin{pmatrix}2 & 4\\ -3 & 3\end{pmatrix} + k\begin{pmatrix}3 & 1\\ 0 & n\end{pmatrix} = \begin{pmatrix}8 & 6\\ -3 & -1\end{pmatrix}$, find
   the values of k and n. [k=2, n = -4]

Using matrices to solve simultaneous equation

Example 7

Using the matric method, solve the simultaneous equations.

3x – y = 16

x + 2y = 3 (05marks)

Solution

$\begin{pmatrix}3 & -1\\ 1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}16\\ 3\end{pmatrix}$

Let A = $\begin{pmatrix}3 & -1\\ 1 & 2\end{pmatrix}$

Adjunct A =$\begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix}$

Det(A) = (3 x 2) – (-1 x 1) = 6 + 1 = 7

$A^{-1} = \frac{1}{7}\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$

Pre-multiply both sides with the inverse

$$\frac{1}{7}\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 16 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 32+3 \\ -16+9 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 35 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

Hence x = 5 and y = -1

Example 8

Using the matric method, solve the simultaneous equations.

2x -3y = 12

x + 2y = -1

Forming matrices

$$\begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ -1 \end{pmatrix}$$

Let P = $\begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}$

Det(P) = (2 x 2) – (1x -3) = 7

Adjunct of P = $\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$

$P^{-1} = \frac{1}{7}\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$

Pre-multiply both sides with the inverse

$$\frac{1}{7}\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 12 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

Hence x = 3, y = -2

Example 9

A family bought the following items for three successive days. The first day it bought three bunches of matooke, two kilogram of rice, five kilograms meat and two kilogram of sugar. The second day it bought one kilogram of sugar. The third the family bought a bunch of matooke and two kilogram of rice. A bunch of matooke costs shs. 15,000. A kilogram of rice, meat and sugar cost 3,300, shs 8,000 and shs 3,000 respectively.

(i) represent the family's requirements in a 3 x 4 matrix

Purchase

$$\begin{array}{c} D \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{matrix} B & R & M & S \end{matrix} \\ \begin{pmatrix} 3 & 2 & 5 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{pmatrix} \end{array} = \begin{pmatrix} 3 & 2 & 5 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{pmatrix}$$

(ii) write down the cost of each item as column matrix

Item costs

$$\begin{array}{c} B \\ R \\ M \\ S \end{array}\begin{pmatrix} 15000 \\ 3300 \\ 8000 \\ 3000 \end{pmatrix} = \begin{pmatrix} 15000 \\ 3300 \\ 8000 \\ 3000 \end{pmatrix}$$

(iii) Use the matrices in b(i) and b(ii) to find the family's total expenditure for the three days. (10 marks)

$$\begin{pmatrix} 3 & 2 & 5 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{pmatrix}\begin{pmatrix} 15000 \\ 3300 \\ 8000 \\ 3000 \end{pmatrix}$$

= 3x 15000+2 x 3300 + 5 x 8000 + 2 x 3000 + 1 x 3000 + 1 x 15000 + 2 x 3300

= 45000 + 6600 + 40,000 +6000 + 3000 + 15000 + 6600

= 122,200

Example 10

Given that M = $\begin{pmatrix} 4x & 6 \\ -5 & -2x \end{pmatrix}$, N = $\begin{pmatrix} -1 & -2 \\ 3 & 3y \end{pmatrix}$. K = $\begin{pmatrix} y & 4 \\ -2 & 12 \end{pmatrix}$ and K = M + N,

find the value of x and y. (07marks)

Solution

$$\begin{pmatrix} y & 4 \\ -2 & 12 \end{pmatrix} = \begin{pmatrix} 4x & 6 \\ -5 & -2x \end{pmatrix} + \begin{pmatrix} -1 & -2 \\ 3 & 3y \end{pmatrix}$$

y = 4x – 1 ……….(i)

12 = -2x + 3y…….(ii)

Substitution of (i) in (ii)

12 = -2x + 3(4x -1)

12 = -2x + 12x – 3

10x = 15

x = 1.5

Using equation (i)

y = 4 x 1.5 – 1 = 5

Therefore x = 1.5 and y = 5

(b) In a football tournament, three teams Arsenal, Chelsea and Liverpool gad the following results

- Arsenal won two matches, drew once and lost one match
- Chelsea won two matches and lost two matches
- Liverpool won 1 match, drew twice and lost one match.

The teams are awarded 3 points of a win, 1 point for a draw and no point for a loss.

(i) Write a 3 x 3 matrix for the results and a column matrix for points. (04marks)

$$\begin{matrix} & W & D & l \\ A & & & \\ C & & & \\ L & & & \end{matrix}\begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix} \text{ and } \begin{matrix} w \\ D \\ L \end{matrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

(ii) By matric multiplication, determine the winner of the tournament. (04 marks)

$$\begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3\,x3 + 1x1 + 1x0 \\ 2x3 + 0x1 + 2x0 \\ 1x3 + 2\,x1 + 1x0 \end{pmatrix} = \begin{pmatrix} 6 + 1 + 0 \\ 6 + 0 + 0 \\ 3 + 2 + 0 \end{pmatrix} = (5)$$

Asernal won with 7 points

Revision Exercise 2

(Answers are given in the square, [ ] brackets at the end of each question)

1. Use matrix method to find the values of x and y
   (a) 2x – 3y = 12
   x + 2y + 1= 0 [x =3, y = -2]
   (b) 2y – 4x + 2 =0
   3x – 2y = 5 [x = -3, y = -7]
   (c) 2x – 3y + 5 = 0
   x + 2y - 8= 0 [x =2, y = 3]
   (d) 4x + 5y = 13
   3x – 2y = 4 [x =2, y = 1]
   (e) 4y – 3x = 9
   x – 2y = -5 [x =1, y = 3]
   (f) 3x + 2y = 8
   3y + 4x = 11 [x = 2, y=1]
2. Given that matrix A = $\begin{pmatrix} 3 & -2 \\ -4 & 5 \end{pmatrix}$ fin a matrix B such that AB = $\begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$. Hence or otherwise find the inverse of matrix A $\left[B = \begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix},\ A^{-1} = \begin{pmatrix} \frac{5}{7} & \frac{2}{7} \\ \frac{4}{7} & \frac{3}{7} \end{pmatrix}\right]$
3. Given that matrix, P = $\begin{pmatrix} 1 & 3 & 7 \\ -1 & 3 & 2 \end{pmatrix}$ and Q = $\begin{pmatrix} 1 & -1 \\ 0 & 3 \\ 5 & 3 \end{pmatrix}$, find matrix R such that R = PQ. $\left[\begin{pmatrix} 8 & 14 \\ 9 & 14 \end{pmatrix}\right]$

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Thank You

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