Subsidiary Mathematics

SENIOR Six term 2

TOPIC 2/2: Random Variables

Competency: The learner analyses concepts of random variables to model real-world situations in various contexts for informed decision making.

Discrete probability distribution

A probability density function (p.d.f) if it takes on specific values

Properties of discrete probability density functions

- **(i)** **$P(X=x) \geq 0$**
- **(ii)** $\sum f(x) = 1 \ or \sum P(X = x) = 1$
- **(iii)** **The expected value of x is given by E(X) =** $\sum xP(X \ or \ x)$=$\sum xf(x)$
- **(iv)** **Var(X) = $E(X^2)$ - $[E(x)]^2$ where $E(X^2)$ =** $\sum x^2 P(X = x)$ **or** $\sum x^2 f(x) - (\sum xf(x))^2$
- **(v)** **Standard deviation =**$\sqrt{Var(X)}$ **=**$\sqrt{\sum x^2 f(x) - (\sum xf(x))^2}$

Examples 1

A discrete random variable has a probability distribution

y	-3	-2	-1	0	1
P(Y=y)	0.1	0.25	0.3	0.15	a

Find

(a) value of a

$\sum P(Y = y) = 1$

0.1 + 0.25 + 0.3 + 0.15 + a = 1;

a + 0.8 = 1

a = 0.2

(b) mode

Mode is the value y with the highest probability, mode = -1

(c) P(-3$\leq Y < 0$)

P(-3$\leq Y < 0$) = P(Y= -3) + P(Y = -2) + P(Y = -1)
= 0.1 + 0.25 + 0.3
= 0.65

(d) P(Y>-1)

P(Y> -1) = P(Y = 0) + P(Y = 1)
= 0.15 + 0.2
= 0.35

(e) P(-1<Y<1)

P(-1<Y<1) = P(Y = 0)

= 0.15

Example 2

A discrete random variable X has a probability distribution

X	1	2	3	4	5
P(X = x)	0.15	0.20	0.15	c	0.1

Find

(i) the value of c

$\sum P(X = x) = 1$

0.15 + 0.20 + 0.15 + c + 0.1 = 1
c +0.6 = 1
c = 0.4

(ii) mode
The mode is a value with highest probability = 4

(iii) P(X<4)
P(X<4) = P(X=1) + P(X= 2) + P(X = 3)
= 0.15 + 0.20 + 0.15
= 0.5

(iv) P(X$\leq$4
P(X<4) = P(X=1) + P(X= 2) + P(X = 3) + P(X= 4)
= 0.15 + 0.20 + 0.15 + 0.4
= 0.9

(v) P(2$\leq X \leq 4$)
P(2$\leq X \leq 4$) = P(X = 2) + P(X = 3) + P(X = 4)
= 0.20 + 0.15 + 0.4
= 0.75

(vi) P$\left(\frac{X>2}{X\leq 4}\right)$

P$\left(X > 2/_{X \leq 4}\right)$ = $\frac{P(X>2,\ X\leq 4)}{P(X\leq 4)}$

$= \frac{P(X=3)+P(X=4)}{P(X=1)+P(X=2)+P(X=3)+P(X=4)}$

$= \frac{0.15 + 0.4}{0.9}$

= 0.6111

Example 3

The table below shows the probability distribution of the number of Compact Discs (CDs) sold.

Number of CDs (x)	0	1	2	3	4
Probability, P(X = x)	0.05	0.28	c	0.22	0.09

Determine the:

(a) Value of c (03 marks)

$\sum P(X = x) = 1$

⇨ 0.05 + 0.28 + c + 0.22+ 0,09 = 1

c =0.36

(b) Probability that at least 2 CD's are sold. (03 marks)

P(x ≥2) = P(x =2) + P(x = 3) + P(x = 4)

= 0.36 + 0.22 + 0.09

= 0.67

(c) Expectation, E(X) (03 marks)

E(X) = ∑xP(X= x)

= 0.05 x 0 + 0.28 x 1 + 0.36 x 2 + 0.22 x 3 + 0.09 x 4

= 2.02

(d) Standard deviation. (06marks)

Var(X) =E(X^2) = ∑x^2P(X= x)

= 0.05 x 0^2 + 0.28 x 1^2 + 0.36 x 2^2 + 0.22 x 3^2 + 0.09 x 4^2

=5.14

S.D =$\sqrt{E(X^2) - (E(x))^2} = \sqrt{5.14 - (2.02)^2}$ = 1.0294

Example 4

A discrete random variable W has a probability distribution shown below

w	-3	-2	-1	0	1
P(W =w)	0.1	0.25	0.3	0.15	d

Find

(a) The value of d (02 marks)

Solution

Total probability = 1 = 0.1 + 0.25 + 0.3 + 0.15 + d

d + 0.8 = 1

d = 0.2

(b) P(-3 ≤ W ≤ -1) (03marks)

P(-3 ≤ W ≤ -1) = P(W = -3) + P (W = -3) + P(W = -1)
= 0.1 + 0.25 + 0.3
= 0.65

(c) P(W > -1) (02 marks)

P(W > -1) = P(W = 0) + P(W = 1)
= 0.15+ 0.2
= 0.35

(d) (i) the mode (01 mark)

-1

(ii) the mean (01mark)

(iii) the variance of the distribution (05marks)

w	-3	-2	-1	0	1
w^2	9	4	1	0	1
P(W =w)	0.1	0.25	0.3	0.15	0.2
w. P(W =w)	-0.3	-0.5	-0.3	0	0.2
w^2. P(W =w)	0.9	1.0	0.3	0	0.2

Mean E(W) = $\sum_{all} wP(W = w)$ = -0.3 – 0.5 – 0.3+ 0.2 = -0.9

Var(W) = $\sum_{all} w^2P(W = w) - (\sum_{all} wP(W = w))^2$
= 0.9 + 0.1 + 0.3 + 0.2 – (-0.9)2
= 1.5 – 0.81
= 0.69

Example 5

A random variable X has a probability distribution given by

$$P(X = x) = \begin{cases} \frac{x}{10}, & x = 1, 2, 3,4 \\ 0 & elsewher \end{cases}$$

Calculate:

(a) P(1 ≤ X < 3). (03marks

P(1 ≤ X < 3) = P(X=1) + P(X = 2) = $\frac{1}{10} + \frac{2}{10} = \frac{3}{10}$ = 0.3

(b) the mean of X, E(X) (02 marks)

X	1	2	3	4
X =x	0.1	0.2	0.3	0.4
x(P(X=x)	0.1	0.4	0.9	1.6

E(X) = 0.1 + 0.4 + 0.9 + 1.6 = 3

Example 6

A discrete random variable has a probability function $P(X = x) = \begin{cases} cx^2 & x = 0, 1, 2, 3, 4 \\ 0 & otherwise \end{cases}$

Find the value of c and draw the graph of P(X = x)

Solution

$\sum P(X = x) = 1$

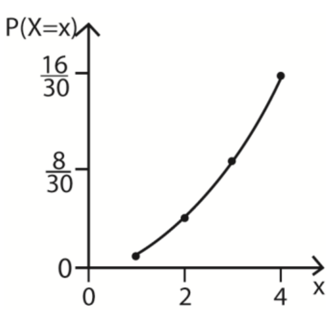
$c(0^2) + c(1^2) + c(2^2) + c(3^2) + c(4^2) = 1$

c + 4c + 9c + 16c = 2

c = $\frac{1}{30}$

X	**0**	**1**	**2**	**3**	**4**
P(X =x)	**0**	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{16}{30} =$

Graph



Example 7

A discrete random variable has probability function

$f(x) = \begin{cases} kx, & x = 1, 2,3,4 \\ 0, & otherwise \end{cases}$, find the value of k and draw the graph of f(x)

Solution

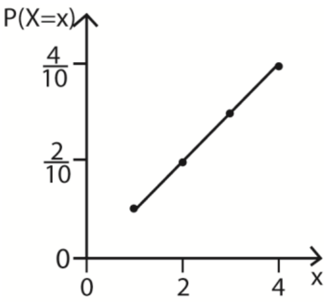
$\sum f(x) = 1$

k + 2k + 3k + 4k = 1

k = $\frac{1}{10}$

X	0	1	2	3	4
P(X =x)	**0**	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

Graph



Example 8

A random variable X of a discrete probability distribution given by

P(X=1) = 0.2, P(X=2) = P(X=3) = 0.1, P(X=4) = P(X=5) = c

Find the value of the constant c and draw the graph of P(X = x)

Solution

$\sum P(X = x) = 1$

0.2 + 0.1 + 0.1 + c + c = 2; c = 0.3

Example 9

A discrete random variable has a probability function

$$P(X = x) = \begin{cases} k\left(\frac{2}{3}\right)^x, & x = 1, 2,3, \dots \\ 0, & otherwise \end{cases}$$

Find the value of k

Solution

$$k\left(\frac{2}{3}\right)^0 + k\left(\frac{2}{3}\right)^1 + k\left(\frac{2}{3}\right)^2 + k\left(\frac{2}{3}\right)^3 + \cdots . = 1$$

$$k\left(1 + \left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots .\right) = 1$$

Sum to infinity = $S_\infty = \frac{a}{1-r}$

⇨ $k\left(\frac{1}{1-\frac{2}{3}}\right)$ = 1; k = $\frac{1}{3}$

Example 10

A discrete random variable X has a probability function

$$f(x) = \begin{cases} kx, & x = 1, 2,3,4,5 \\ 0, & otherwise \end{cases}$$

Find

(i) the value of k

$\sum P(X = x) = 1$

k + 2k + 3k + 4k + 5k = 1

15k = 1

k = $\frac{1}{15}$

(ii) P(X = 3)

P(X=3) = 3k = $\frac{3}{15} = \frac{1}{5}$

(iii) P(X≥3)

P(X≥3) = P(X=3) + P(X=4) + P(X = 5)

= 3k + 4k + 5k

= 12k

= 12 $x\frac{1}{15}$

$= \frac{4}{5}$

(iv) P(X$\leq$ 3)

P(X$\leq$ 3)= P(X= 1) + P(X=2) + P(X = 3)

= k + 2k + 3k

=6 $x\frac{1}{15}$

$= \frac{2}{5}$

(v) (v) P(1$<X\leq$3)

P(1$<X\leq$3) = P(X=2) + P(X = 3)

= 2k + 3k

=5$x\frac{15}{15}$

$= \frac{1}{3}$

(vi) P$\left(\frac{X\geq1}{X<4}\right)$

P$\left(X \geq 2/_{X} < 4\right) = \frac{P(X\geq2, X<4)}{P(X<4)}$

$$= \frac{P(X=2)+P(X=3)}{P(X=1)+P(X=2)+P(X=3)}$$

$$= \frac{2k+3k}{k+2k+3k}$$

$$= \frac{5k}{6k}$$

$$= \frac{5}{6}$$

Example 11

A discrete random variable X can take on values 0, 1, 2, and 3 only. If E(X) = 1.4, P(X$\leq$ 2) = 0.9 and P(X$\leq$1) = 0.5. Find (i) P(X=1) (ii) P(X =0)

Let P(X = 0) = a, P(X=1)=b, P(X=2)= c p(X = 3) = d
a + b + c + d = 1 ……………. (i)
P(X$\leq$ 2) = a + b+ c = 0.9 … (ii)
Eqn. (i) and eqn. (ii)
d = 0.1
P(X$\leq$1) = a + b = 0.5 ……. (iii)
Eqn. (i) and eqn. (iii)
0.5 + c + 0.1 = 1
c = 0.4
E(X) = 0 x a + 1 x b + 2 x c + 3 x 0.1 = 1.4
= b + 2c + 0.3 = 1.4
b + 2c = 1.1
b + 2 x 0.4 = 1.1
b = 0.3
a = 0.5 – 0.3 = 0.2
Hence, (i) P(X=1) = 0.3 (ii) P(X =0) = 0.2

Revision exercise 1

(Answers are given in the square brackets, [ ], at end of each question)

1. The discrete random variable Y has a probability distribution is given by
   P(Y=y) = cy, y = 1, 2, 3, 4
   Find (i) value of c = 0.1 (ii) E(X) = $\frac{11}{3}$
2. A discrete random variable has p.d.f
   $$P(X=x) = \begin{cases} k2^x, & x = 0,1,2,3,4,5,6 \\ 0, & otherwise \end{cases},$$
   Find
   (i) (i) value of k $\left[\frac{1}{127}\right]$,
   (ii) Mean [5.01]

3. A discrete random variable X has a probability distribution

x	0	1	2	3	4	5
P(X= x)	0.11	0.17	0.2	0.13	p	0.09

Find
(i) the value of p [0.3]
(ii) Expected value of X [2.61]

4. A random variable X of discrete probability distribution is given by

x	-1	0	1	2
P(X= x)	0.25	0.10	0.45	0.20

Find

(i) $P(-1 \leq X < 1)$ [0.35]
(ii) E(X) = [0.6]

5. A discrete random variable X has a probability distribution

x	1	2	3	4	5
P(X = x)	0.2	0.25	0.4	0.1	0.05

Find

(i) Mean [2.55]
(ii) The variance (1.148]
(iii) Standard deviation = [1.0714]

6. A random variable X of a discrete probability distribution given by

x	10	20	30
P(X = x)	0.2	0.3	0.5

Find

(i) E(X) [22]
(ii) Var (X) [36]
(iii) Standard deviation[6]

7. A random variable X of discrete probability distribution is given by

x	1	2	3
P(X = x)	0.2	0.3	0.5

Find (i) E(X) = 2.3 (ii) $E(X^2) = 5.9$

(iii) Var (X) = 0.61

8. A random variable X of discrete probability distribution is given by

x	-1	0	1	2
P(X = x)	0.25	0.1	0.45	0.2

Find: (i) P(-1≤ X < 2) = 0.8 (ii) E(X) = 0.6 (iii) E(2x + 3) = 4.2

9. A random variable X of a discrete probability distribution
   P(X = 0) = 0.05, P(X = 1) = 0.45 P(X = 2) = 0.5
   Find: (i) E(X) = 1.45, (i) E(X^2) = 2.45 (iii) Var (X) = 0.348
10. A random variable X of discrete probability distribution is given by P(X = 1) 0.1, P(X= 2) = 0.2, P(X = 3) = 0.3, P(X =4)= 0.4
    Find (i) E(X) = 3 (ii) Var (X) = 1 (iii) P(X = 2/X≥2) = $\frac{2}{9}$
11. The discrete random variable Y has a probability distribution P(Y = y) = k =1, 2, 3, 4, 5,6
    Find (i) mean, μ = 3.5 (ii) E(3X +4) = 15$\frac{1}{6}$ (iii) E(X^2)=14.5 (iv) standard deviation = 1.708
12. The discrete random variable R has a probability distribution is given by
    P(R = r) = $\frac{3r+1}{22}$; r = 0, 1, 2, 3
    Find (i) mean, μ= $\frac{24}{11}$, E(R2) = $\frac{61}{11}$ (iii) E(3R-2) = $\frac{50}{11}$
13. The discrete random variable R has a probability distribution given by
    $$P(R = r) = \begin{cases} \frac{2r+1}{20}; r = 0, 1, 2,3 \\ \frac{11-r}{20}, \quad r = 4, 5 \end{cases}$$
    Find (i) E(R) = 2.55, (ii) Var(R) = 1.45
14. The discrete random variable X has a probability distribution given by
    $$P(X = x) = \begin{cases} kx, & x = 1, 2, 3, 4, 5 \\ k(10 - x), & x = 6, 7,8, 9 \end{cases}$$
    Find (i) constant, k = 0.04, (i) E(X) = 5 (iii) Var(X) = 4
15. The discrete random variable X has a probability distribution is given by
    P(X =x) = kx, x = 1, 2, 3,.....n ; where k is a constant
    Show that k = $\frac{2}{n(n+1)}$, hence find in terms of n the mean X = $\frac{1}{3}(2n + 1)$
16. A random variable X of a discrete probability distribution given by
    P(X = 0) = P(X=1) = 0.1, P(X = 2) = 0.2, P(X= 3) = P(X = 4) = 0.3. Find Var (X) = 1.64
17. A random variable X of a discrete probability distribution given by
    P(X = 2) = 0.1; P(X= 4) = 0.3; P(X = 6) = 0.5; P(X = 8) = 0.1. Find Var (X) 2.56

Continuous random variable

A probability density function (p.d.f) is continuous if it takes on values between intervals.

Properties of a continuous probability density functions

(i) **f (x) ≥ 0**
(ii) $\int \boldsymbol{f(x)dx = 1}$
(iii) **E(X)=** $\int \boldsymbol{xf(x)dx}$
(iv) **E(X²)=** $\int \boldsymbol{x^2 f(x)dx}$
(v) **Var(X) = E(X²) - [E(x)]²**
(vi) **Standard deviation =** $\sqrt{\boldsymbol{Var(X)}}$ **=** $\sqrt{\sum \int \boldsymbol{x^2 f(x)} - (\sum \boldsymbol{f(x)})^2}$

Example 12

A continuous random variable X has a probability density function given by,

$$f(x) = \begin{cases} \frac{kx}{6}, & 1 \le x \le 2, \\ 0 & Otherwise \end{cases}$$

Where k is a constant

(a) **Find**

(i) **The value of k (04marks)**

$\int f(x)dx = 1$

$\frac{k}{6}\int_1^2 xdx = \frac{k}{6}\left[\frac{x^2}{2}\right]_1^2 = 1$

$\frac{k}{6}\left(2 - \frac{1}{2}\right) = \frac{k}{6}\left(\frac{3}{2}\right) = 1$

$k = 4$

(ii) **P(X ≥ 1.5) (04marks)**

P(X ≥ 1.5)= $\frac{4}{6}\left[\frac{x^2}{2}\right]_{1.5}^2 = \frac{2}{3}\left(2 - \frac{2.25}{2}\right) = 0.58$

(iii) **The mean of X, E(X) (03 marks)**

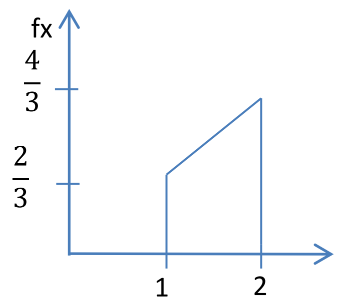
E(X) = $\int xf(x)dx$ **=** $\frac{4}{6}\int_1^2 x^2dx = \frac{4}{6}\left[\frac{x^3}{3}\right]_1^2 = \frac{4}{6}\left(\frac{8}{3} - \frac{1}{3}\right) = \frac{2}{3}.\frac{7}{3} = \frac{14}{9}$

(b) **Sketch the graph of f(x) (04 marks)**

f(x) = $\frac{2}{3}x$

At x = 1, fx = $\frac{2}{3}$

At x = 2, fx = $\frac{4}{3}$



Example 12

A random variable X has a probability density function, f(x), defined by

$$f(x) = \begin{cases} kx(x+2), & 0 \leq x \leq 2, \\ 0, & otherwise \end{cases}$$

where k is constant

Determine

(a) **value of k (04marks)**

$\int f(x)dx = 1$

⇨ $k\int_0^2 (x^2 + 2x)\,dx = 1$

$k\left[\frac{x^3}{3} + x^2\right]_0^2$ **= 1**

$k\left(\frac{8}{3} + 4\right) = 1$ **=> k =**$\frac{3}{20}$

(b) **P(1≤ X ≤1.5) (03marks)**

P(1≤ X ≤1.5) =$\frac{3}{20}\left[\frac{x^3}{3} + x^2\right]_1^2$**=** $\frac{3}{20}\left[\left(\frac{8}{3} + 4\right) - \left(\frac{1}{3} + 1\right)\right]$**=0.306**

(c) **Expectation, E(X) (04marks)**

E(X) = $\int xf(x)dx$ **=** $\frac{3}{20}\int_0^2 (x^3 + 2x^2)\,dx$

=$\frac{3}{20}\left[\frac{x^4}{4} + \frac{2x^3}{3}\right]_1^2$

=$\frac{3}{20}\left(\frac{16}{4} + \frac{16}{3}\right)$

= 1.4

(d) **Variance, Var(X) (04marks)**

E(X^2) = $\int x^2 f(x)dx$ **=** $\frac{3}{20}\int_0^2 (x^4 + 2x^3)\,dx$

=$\frac{3}{20}\left[\frac{x^5}{5} + \frac{2x^4}{4}\right]_1^2$

=$\frac{3}{20}\left(\frac{32}{5} + \frac{32}{4}\right)$

= 2.16

Var(X) = E(X^2) – (E(x))^2

= 2.16 – (1.4)^2 = 0.2

Example 14

A cumulative random variable X, has a probability density function (pdf) given by

$$f(x) = \begin{cases} k(x^2+6), & 0 \le x \le 3 \\ 0, & otherwise \end{cases}$$

Where k is a constant

Determine the:

(i) **value of k (04 marks)**

$\int_0^3 f(x)dx = 1$

$k\int_0^3 (x^2+6)dx = 1$

k$\left[\frac{x^3}{3}+6x\right]_0^3$ **= 1**

$k\left(\frac{3^3}{3}+6x3\right)$ **=1**

$27k = 1$

k = $\frac{1}{27}$

(ii) P(X> 1). (04 marks)

$\frac{1}{27}\left[\frac{x^3}{3}+6x\right]_1^3$ **=** $\frac{1}{27}\left[27-6\frac{1}{3}\right]$ **=** $\frac{62}{81}$**= 0.7654**

(iii) expectation, E(X) (03marks)

E(X) **=** $\frac{1}{27}\int_0^3 xf(x)dx$

= $\frac{1}{27}\int_0^3 x(x^2+6)dx$ **=** $\int_0^3 (x^3+6x)$

= $\frac{1}{27}\left[\frac{x^4}{4}+3x^2\right]_0^3$**=** $\frac{1}{27}\left(\frac{81}{4}+27\right)$

= 1.75

(iv) variance, Var (X)(04marks)

E(X^2) **=** $\frac{1}{27}\int_0^3 x^2f(x)dx$

= $\frac{1}{27}\int_0^3 x^2(x^2+6)dx$ **=** $\int_0^3 (x^4+6x^2)$

= $\frac{1}{27}\left[\frac{x^5}{5}+2x^3\right]_0^3$**=** $\frac{1}{27}\left(\frac{243}{5}+54\right)$

= 3.8

Var(X) = E(X^2) – (E(x))^2

= 3.8 – (1.75)^2

= 0.7375

Example 15

A random variable of continuous p.d.f is given by $f(x) = \begin{cases} kx^2 & 0 \le x \le 4 \\ 0, & elsewhere \end{cases}$

Find (i) value of k and sketch f(x)   (ii) P(1 ≤ x ≤ 3)

Solution

(i) $\int_0^4 kx^2 dx = 1$

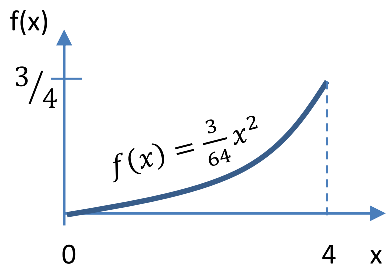
$k\left[\frac{x^3}{3}\right]_0^4 = k\left[\frac{4^3}{3} - \frac{0^3}{3}\right] = 1$

k = $\frac{3}{64}$

When x = 0, f(x) = $\frac{3}{64}0^2 = 0$

When x = 4, f(x) = $\frac{3}{64}4^2 = \frac{3}{4}$

Sketch



(ii) P(1 ≤ x ≤ 3)= $\frac{3}{64}\int_1^3 kx^2 dx = 1$

$= \frac{3}{64}\left[\frac{x^3}{3}\right]_1^3 = \frac{3}{64}\left[\frac{3^3}{3} - \frac{1^3}{3}\right] = 0.4063$

Example 16

A random variable X of a continuous p.d.f is given by $f(x) = \begin{cases} k(x^2+1) & 0 \le x \le 3 \\ 0, & elsewhere \end{cases}$

Find

(i) value of k and sketch f(x)

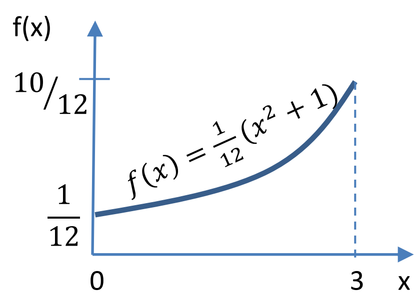
$\int_0^3 k(x^2+1)dx = 1$

$k\left[\frac{x^3}{3} + x\right]_0^3 = k\left[\left(\frac{3^3}{3} + 3\right) - \left(\frac{0^3}{3} + 0\right)\right] = 1$

k = $\frac{1}{12}$

When x = 0, f(x) = $\frac{1}{12}(0^2+1) = \frac{1}{12}$

When x = 3, f(x) = $\frac{1}{12}[3^2+1] = \frac{10}{12}$



(ii) P(1 ≤ x ≤ 3)

$\frac{1}{12}\int_1^3 (x^2+1)dx = \frac{1}{12}\left[\frac{x^3}{3} + x\right]_1^3 = \frac{1}{12}\left[\left(\frac{3^3}{3} + 3\right) - \left(\frac{1^3}{3} + 1\right)\right] = 0.8889$

Example 17

A random variable X of continuous p.d.f is given by $f(x) = \begin{cases} k, & 0 \le x \le 2 \\ k(2x-3), & 2 \le x \le 3 \\ 0, & elsewhere \end{cases}$

Find (i) value of k and sketch f(x) (ii) P(X<1) (ii) P(X > 2.5) (iv) $\left(0 \le X \le 2 / X \ge 1\right)$

Solution

$\int_0^2 kdx + \int_2^3 k(2x-3)dx = 1$

$k[x]_0^2 + k[x^2 - 3x]_2^3 = 1$

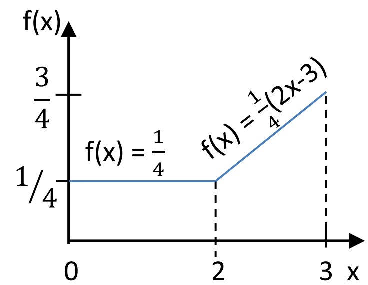
k = $\frac{1}{4}$

When x = 0, f(x) = k = $\frac{1}{4}$

When x = 2, f(x) = k = $\frac{1}{4}$

When x = 3, f(x) = $\frac{1}{4}$(2 x3 - 3)

= $\frac{3}{4}$



(ii) P(X<1) $= \frac{1}{4}\int_0^1 dx = \frac{1}{4}[x]_0^1 = \frac{1}{4}$

(iii) P(X > 2.5) $= \frac{1}{4}\int_{2.5}^3 (2x-3)dx = \frac{1}{4}[x^2 - 3x]_{2.5}^3 = 0.3125$

(iv) $P\left(0 \le X \le 2 / X \ge 1\right) = \frac{P(0 \le X \le 2)}{P(X \ge 1)} = \frac{P((0 \le X \le 2) \cap (X \ge 1))}{P(X \ge 1)} = \frac{\frac{1}{4}\int_1^2 dx}{\frac{1}{4}\int_1^2 dx + \frac{1}{4}\int_2^3 (2x-3)dx} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2}} = \frac{1}{3}$

Revision exercise 2

(Answers are given in the square brackets, [ ], at end of each question)

1. A random variable x of a continuous p.d.f is given by $f(x) = \begin{cases} kx^2, & 0 \le x \le 4 \\ 0, & elsewhere \end{cases}$
   (a) Sketch f(x)
   (b) Find (i) value of k (=$\frac{3}{64}$) (ii) E(X) = 3 and var (X) = 0.6 (iii) P(1<X<2) = $\frac{7}{64}$
2. A random variable x of a continuous p.d.f is given by $f(x) = \begin{cases} kx, & 0 \le x \le 1 \\ k(2-x) & 0 \le x \le 1 \\ 0, & elsewhere \end{cases}$

   Find (i) constant k = 1 (ii) E(X) = 1 (iii) var(X) =$\frac{1}{6}$ (iv)P(0.75 < X < 1.5) = $\frac{19}{32}$ (v) mode =1
3. A random variable x of a continuous p.d.f is given by $f(x) = \begin{cases} \frac{1}{27}x^2, & 0 \le x \le 3 \\ \frac{1}{3} & 3 \le x \le 5 \\ 0, & elsewhere \end{cases}$
   (a) Sketch f(x)
   (b) Find (i) E(X) = 3417 (ii) standard deviation = 1.008
4. A random variable x of a continuous p.d.f is given by $f(x) = \begin{cases} \frac{k}{x(4-x)}, & 1 \le x \le 3 \\ 0, & elsewhere \end{cases}$
   (i) Show that k = $\frac{3}{Inx}$
   (ii) Find (i) E(X) = 2 (ii) Var(X) = 4-$\frac{4}{Inx}$
5. A random variable x of a continuous p.d.f is given by $f(x) = \begin{cases} k(ax - x^2), & 0 \le x \le 2 \\ 0, & elsewhere \end{cases}$
   (i) Show that k = $\frac{8}{6a-8}$
   (ii) Given that E(X) = 1, find the values of a (=2) and k(=0.75)
   (iii) For the above values of a and k, find Var(X) = 0.2
6. A random variable x of a continuous p.d.f is given by $f(x) = \begin{cases} 12(x^2 - x^3), & 0 \le x \le 1 \\ 0, & elsewhere \end{cases}$

   Find the (i) mean = 0.6 (ii) standard deviation = 0.2
7. A random variable x of a continuous p.d.f is given by $f(x) = \begin{cases} \frac{k}{\beta}, & 0 \le x \le \beta \\ 0, & elsewhere \end{cases}$

   Find (i) value of k (=1) (ii) mean = $\frac{\beta}{2}$ (iii) standard deviation = $\frac{\beta}{2\sqrt{3}}$
8. A random variable x of a continuous p.d.f is given by $f(x) = \begin{cases} \frac{1}{8}(x+1), & 2 \le x \le 4 \\ 0, & elsewhere \end{cases}$

   Find (i) mean = $\frac{37}{12}$ (ii) var(X) = $\frac{47}{144}$ (iii) P(2.5 < x< 3) = 0.234
9. A random variable x of a continuous p.d.f is given by $f(x) = \begin{cases} k(1-x)^2, & 2 \le x \le 4 \\ 0, & elsewhere \end{cases}$

   Find (i) constant k = $\frac{3}{26}$ (ii) mean = $\frac{1}{4}$ (iii) standard deviation = 0.94
10. A random variable x of a continuous p.d.f is given by $f(x) = \begin{cases} kx, & 0 \le x \le 2 \\ k(4-x) & 2 \le x \le 4 \\ 0, & elsewhere \end{cases}$

Find (i) value of k = $\frac{1}{4}$ (ii) E(X) = 2 (iii) Var(X) = $\frac{2}{3}$ (iv) P(X< 1) = $\frac{1}{8}$ (iv) P(X<X<3) = $\frac{3}{8}$

Thank You

Dr. Bbosa Science