Subsidiary Mathematics

SENIOR Six term 3

TOPIC 1/2: Probability Distributions

Competency: The learner interprets binomial and normal distributions for analysis of data patterns and trends in real-world contexts.

Binomial distribution

A binomial distribution is a special type of a discrete random variable in which an experiment gives rise to only two outcomes either success or failure.

Conditions for binomial distribution

(i) The experiment has a finite (repeated) number of trials, n
(ii) The trials are independent
(iii) The outcome of each trial is either a success or a failure
(iv) The probability, p of successful outcome is constant for all trials

If a discrete random variable X is the number of **successful** outcomes in n trials and satisfies the above conditions, then X follows a binomial distribution written as X~B(n, p) or X~Bin (n, p)

Formula for Binomial distribution

(a) If X~B(n, p), the probability of obtaining, r success in n trials P(X = r) where

$P(X = r) = {}^{n}C_{r}p^{r}q^{n-r}$ for r = 0, 1, 2, 3, …,n where q = 1-p

$= \frac{n!}{(n-r)!r!}p^{r}q^{n-r}$

(b) E(X) = np
(c) Var(X) = npq where q = 1- p
(d) s.d = $\sqrt{npq}$

Example 1

The random variable X is distributed B(7, 0.2). find

(i) P(X=3) (ii) $P(1<X \leq 4)$ (iii) P(X>1)

Solution

(i) n = 7, p= 0.2, q= 1- 0.2 = 0.8

$P(X=3) = {}^7C_3 \times 0.2^3 \times 0.8^4$

$= \frac{7!}{(7-3)!3!} x 0.2^3\ x\ 0.8^4$

$= \frac{7\ x\ 6\ x\ 5\ x\ 4\ x\ 3\ x\ 2\ x\ 1}{(4\ x\ 3\ x\ 2\ x\ 1)(3\ x\ 2\ x\ 1)} x 0.2^3\ x\ 0.8^4 = 0.115$

(ii) $P(1<X \leq 4) = P(x=2) + P(X=3) + P(X=4)$

$= {}^7C_2 \times 0.2^2 \times 0.8^5 + {}^7C_3 \times 0.2^3 \times 0.8^4 + {}^7C_4 \times 0.2^4 \times 0.8^3$

= 0.275 + 0.115 + 0.029 = 0.419

(iii) $P(X>1) = 1- (P(X\leq 1) = \left[1 - P(X=0) + P(X=1)\right]$

$= 1 - [{}^7C_0 \times 0.2^0 \times 0.8^7 + {}^7C_1 \times 0.2^1 \times 0.8^6]$

=[1 – (0.210 + 0.367)

= 0.423

Example 2

At freedom city super market, 60% of the customers shop on Saturday. Find the probability that in a randomly selected sample of 10 customers

(i) Exactly 2 shop on Saturday

n= 10, p = 0.6 q = 1- 0.6 = 0.4

$P(X = 2) = {}^{10}C_2 \times 0.6^2 \times 0.4^8 = 0.011$

(ii) More than 7 shop on Saturday

P(X > 7) = P(X = 8) + P(X = 9) + P(X = 10)

$= {}^{10}C_8 \times 0.6^8 \times 0.4^2 + {}^{10}C_9 \times 0.6^9 \times 0.4^1 + {}^{10}C_{10} \times 0.6^{10} \times 0.4^2$

= 0.121 + 0.040 + 0.006 = 0.167

Example 3

The probability that a marble drawn from a box is red is 0.4. If a sample of 6 marbles is taken, find the probability that it will contain;

(i) No red marble

n = 6, p= 0.4, q= 1 – 0.4 = 0.6

$P(X = 0) = {}^6C_0 \times 0.4^0 \times 0.6^6 = 0.047$

(ii) $P(X = 5 \text{ or } 6) = P(X= 5) + P(X = 6) = {}^6C_5 \times 0.4^5 \times 0.6^1 + {}^6C_6 \times 0.4^6 \times 0.6^0$

= 0.037 + 0.004 = 0.041

(iii) Less than half red marbles

P(X<3) = P(X = 0) + P(X = 1) + P(X = 2)

$= {}^6C_0 \times 0.4^0 \times 0.6^6 + {}^6C_1 \times 0.4^1 \times 0.6^5 + {}^6C_2 \times 0.4^2 \times 0.6^4$

= 0.047 + 0.187 + 0.331 = 0.545

Example 4

A biased con is such that the chance of a head appearing upper most when tossed is twice of the tail appearing uppermost. If the coin is tossed 10 times. Find the probability that

(i) Exactly 6 heads will appear

P(H) + P(T) = 1

2x + x = 1

$x = \frac{1}{3}$

$n = 10$, $p = \frac{2}{3}$; $q = \frac{1}{3}$

$P(X = 6) = {}^{10}C_6 \times \left(\frac{2}{3}\right)^6 x \left(\frac{2}{3}\right)^4 = 0.228$

(ii) $P(5<X<8) = P(X = 6) + P(X = 7) = {}^{10}C_6 \times \left(\frac{2}{3}\right)^6 x \left(\frac{2}{3}\right)^4 + {}^{10}C_7 \times \left(\frac{2}{3}\right)^7 x \left(\frac{2}{3}\right)^3$

$= 0.228 + 0.260 = 0.488$

Example 5

A box contains a large number of pens. The probability that a pen is faulty is 0.1. How many pens would you need to select to be more than 95% certain of picking one faulty pen?

Solution

n? p= 0.1, q = 0.9

$P(X \geq 1) = 1 - P(X = 0) > 0.95$

$= 1 - {}^nC_0 \times 0.1^0 \times 0.9^n > 0.95$

$= 0.05 > 0.9^n$

$n > \frac{\log_{10} 0.05}{\log_{10} 0.9}$

n >29

The least value n = 29

Example 6

In Binomial experiment, the probability of a success for n trials is 0.6. If the mean is 7.2, find the

(a) value of n. (02 marks)

Mean = np

7.2 = 0.6n

n = 12

(b) probability of obtaining 7 success. (03marks)

$P(X = x) = {}^nC_x p^x q^{(n-x)}$

$= {}^{12}C_7(0.6)^7(0.4)^5$

= 0.227

Example 7

It was observed that 3 seeds in every four seeds planted germinate. If 16 seeds were planted, calculate

(i) expected number of seeds that will germinate (03marks)

Given $p = \frac{3}{4}$, $q = \frac{1}{4}$, n = 16

Let X be the random variable that the seed germinates

$X \sim B(16, \frac{3}{4})$

$E(x) = np = 16 \times \frac{3}{4} = 12$ seeds

(ii) probability that exactly 14 seeds will germinate (02marks)

$P(X = 14) = {}^{16}C_{14}\left(\frac{3}{4}\right)^{14}\left(\frac{1}{4}\right)^{2} = 0.1336$

Using Cumulative binomial probability table

The table give values of $P(X \geq x)$ for values of n and p.

(i) $P(X \leq X) = 1- (P(X \geq (X+1)$

(ii) $P(X = x) + P(X \geq x) – (PX \geq x+1)$

Example 8

The random variable is distributed B(5, 0.3). Find

(i) $P(X \geq 3)$ (ii) $P(X > 1)$ (iii) $P(X \leq 4)$ (iv) $P (X < 3)$ (v) $P(X = 2)$

Solution

n = 5, p = 0.3

(i) $P(X \geq 3) = 0.1631$

(ii) $P(X > 1) = P(X \geq 2) = 0.4718$

(iii) $P(X \leq 4) = 1 – (P(X \geq 5) = 1 – 0.0024 = 0.9976$

(iv) $P(X < 3) = P(X \geq 2) = 1 – P(X \geq 3) = 1 – 0.1631) = 0.8369$

(v) $P(X = 2) = P(X \geq 2) – P(X \geq 3)\ 0.4718 – 0.1631 = 0.3087$

Example 9

An unbiased coin is tossed 15 times. Find th probability that

(i) Exactly eight heads will appear upper most

(ii) Between 6 and 10 heads will appear

(iii) Between 6 and 10 heads inclusive will appear

Solution

n = 15, p = 0.5

(i) $P(X = 8) = P(X \geq 8) – P(X \geq 9) = 0.5000 – 0.3036 = 0.1964$

(ii) $P(6< X < 10) = P(X \leq X \leq 9) = P(X \geq 7) – P(X \geq 10) = 0.6964 – 0.1509 = 0.5455$

(iii) $P(X \leq X \leq 10) = P(X \geq 6) – P(X \geq 11) = 0.8491 – 0.0592 = 0.7899$

Example 10

A student attempts 20 objective questions by guest work. Each questions has got four possible alternatives out of which one is correct. Find the probability that he gets

(i) Exactly 9 correct answers

(ii) At least 12 correct answers

(iii) At most 6 correct answers

(iv) Between 6 and 14 correct answers inclusive

(v) Exactly 7 correct answers.

Solution

For correct answers, n = 20, p = 0.25, q = 0.75

(i) $P(X = 9) = P(X \geq 9) – P(X \geq 10) = 0.0409 – 0.0139 = 0.027$

(ii) $P(X \geq 12) = 0.0009$

(iii) $P(X \leq 6) = 1 – P(X \geq 7) = 1 – 0.2142 = 0.7858$

(iv) $P(X \leq X \leq 14) = P(X \geq 6) – P(X \geq 15) = 0.3828 – 0.0000 = 0.3828$

For incorrect answers, n = 20, P = 0.75, q = 0.25

Revision exercise 1

1. A biased coin is such that a head is three times as likely to occur as a tail. The coin is tossed 5 times. Find the probability that at most two tails occur = 0.8965
2. Tom's chance of passing an examination is $\frac{2}{3}$. If he sits for four examinations, calculate the probability he passes

(i) Only two examinations = 0.2963

(ii) More than half of the examinations = 0.5926

3. A fair die is rolled 6 times, calculate the probability that
   (i) A 2 or 4 appears on the first throw = $\frac{1}{3}$
   (ii) Four 5's will appear in the six throws= 0.0080
4. Usain Bolt makes 5 practice ran in the 100m sprint. A run is successful if he runs it in less than 11 seconds. There are 8 chances out of 10 that he is successful. Find the probability that;
   (i) He records at least no success at all= 0.0003

(ii) Exactly 5 games= 0.3277

5. The probability that Alex wins a chess game is $\frac{2}{3}$. He plays 8 games, what is the probability that he wins
   (i) At least 7 games= 0.1951
   (ii) Exactly 5 games= 0.2731
6. Usain Bolt makes 5 practice ran in the 100m sprint. A run is successful if he runs it in less than 11 seconds. There are 8 chances out of 10 that he is successful. Find the probability that;
   (i) He records at least no success at all= 0.0003
   (ii) He record at least 2 success= 0.9933
   (iii) If he is successful in 5 practice runs, he makes two additional runs. The probability of success in either of the additional runs is 0.7. Determine the probability that Bolt will make 7 successful runs = 0.1606
7. In a test there are 10 objective questions each with a choice of five possible alternatives out of which only one is correct. If a student guesses each of the answers, find the probability that he gets at least two answers correct. = 0.6242
8. 30% of the students in the school are day scholars. From a sample of 10 students chosen at random, find the probability that
   (i) Only 3 are day scholars= 0.267
   (ii) Less than half are day scholars=0.850

9. The probability that a shopper buys a cake is 0.25. Find the probability that in a random sample of 9 shoppers
   (i) Exactly 3 buys a cake=0.2334
   (ii) More than 7 buy a cake= 0.0001
10. A bag contains counter books of which 40% are blue and the rest are black. A counter book is taken from the bag, its colour is noted then replaced. This is performed 8 times in all. Calculate the probability that
    (i) Exactly 3 will be blue= 0.279
    (ii) At least one will be blue= 0.983
    (iii) More than half will be black= 0.594

11. At a certain school, the records taken from admission's office the ratio of male to female s. 5 applicants is 4:6. Basing on this experience, what is the probability that will be more female applicants in a random collection of a dozen applicants? = 0.665
12. The random variable X is B(6, 0.42). find
    (i) $P(X = 6) = 0.00549$ (ii) $P(X = 4) = 0.157$ (iii) $P(X \leq 2) = 0.503$
13. An unbiased die is thrown seven times. find the probability of throwing at least 5 sixes.= 0.002
14. In a family a couple is likely to produce a girl or a boy. Find the probability that in a sample of 5 children there will be more boys than girls = 0.5
15. The probability that it will rain on any given day during examination period is 0.3. calculate the probability that in a given week during examination period, it will rain on;
    (i) Exactly 2 days = 0.318
    (ii) At most two days = 0.647
    (iii) Exactly three days that are consecutive = 0.0324

16. A fair coin is tossed 6 times. find the probability of throwing at least four heads = 0.344
17. The random variable x is B(4, p) and P(X = 4) = 0.0256. find P(X = 2) = 0.3456
18. In agriculture lab, Silvia plants bean seeds and the probability that they germinate successfully is $\frac{1}{3}$.
    (a) She takes 9 seeds. Find the probability that
        (i) More than five seeds germinate = 0.0424
        (ii) At least three seeds germinate = = 0.623
    (b) Find the number of seeds that she needs to take in order to 99% certain at least one germinate = 12
19. In a shooting competition, the probability of hitting the target with a single shot is 0.6, if 7 shots are taken; find the probability that the target is hit more than twice. = 0.9037
20. In mass production of shirts, it is found that 5% are defective. Shirts are selected at random of put into packets of 10.
    (a) A packet is selected at random. Find the probability that it contains
        (i) Three defective shirt = 0.0105
        (ii) Less than three defective shirts = 0.9885
    (b) Two packets are selected at random. Find the probability that there are no defective shirts in either packet = $({}^{10}C_0(0.05)^0(0.95)^{10}$ x $({}^{10}C_0(0.05)^0(0.95)^{10}$ = 0.358
21. A biased coin is such that it is twice as likely to show a head as a tail. If the coin is tossed 5 times. find the probability that
    (i) Exactly three heads are obtained = 0.329
    (ii) More than three heads are obtained = = 0.3333
22. The probability that a target is hit 0.3. Find the probability the least number of times shots should be fired if the probability that the target is hit is at least once is greater than 0.95. =9
23. 1% of the light bulbs in a box are faulty. Find the largest sample size which can be taken if it required that the probability that there is no faulty bulb in the sample is greater than 0.5
    (0.99)n > 0.5
    n = 68
24. In a test there are 10 multiple choice questions. Each question has got four possible alternatives out of which only one is correct. If a student guesses each of the answers, find the probability that he gets
    (i) More than 7 correct answers = 0.0004
    (ii) More than half correct answers = 0.0197
25. $X \sim B(n, o.3)$. find the value of n such that P(X ≥ 1) = 0.8 :
26. The random variable X is B(n, 0.6). find the value of n such that P(X<1) = 0.0256

Solutions to revision exercise 1

1. A biased coin is such that a head is three times as likely to occur as a tail. The coin is tossed 5 times. Find the probability that at most two tails occur
   Solution
   n = 5
   p + q = 1
   x + 3x = 1, x = 0.25

p = 0.25, q = 0.73

P(X ≤ 2) = P(X = 0) + P(X =1) + P(X = 2)

= $^5C_0$ x 0.25^0 x 0.75^5 +$^5C_1$ x 0.25^1 x 0.75^4+ $^5C_2$ x 0.25^2 x 0.75^3

=0.2373 + 0.3955 + 0.2637 = 0.8965

2. Tom's chance of passing an examination is $\frac{2}{3}$. If he sits for four examinations, calculate the probability he passes

(i) Only two examinations

n = 4, p = $\frac{2}{3}$, q =$\frac{1}{3}$

P(X = 2) = $^4C_2$ x $\left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2$ = 0.2963

(ii) More than half of the examinations

P(X < 2) = P(X= 3) + P(X =4) = $^4C_3$ x $\left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^1$ + $^4C_4$ x $\left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^0$ = 0.5926

3. A fair die is rolled 6 times, calculate the probability that

(i) A 2 or 4 appears on the first throw

P(X = 2 or 4) = P(X =2) + (X = 4) = $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

(ii) Four 5's will appear in the six throws

n = 6, P= $\frac{1}{6}$, q = $\frac{5}{6}$

P(X = 4) = $^6C_4$ x $\left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^2$ = 0.0080

4. Usain Bolt makes 5 practice ran in the 100m sprint. A run is successful if he runs it in less than 11 seconds. There are 8 chances out of 10 that he is successful. Find the probability that;

(i) He records at least no success at all

n = 5, p = 0.8, q = 0.2

P(X = 0) = $^5C_0$ x $(0.8)^0(0.2)^5$ = 0.0003

(ii) Exactly 5 games

P(X = 5) = $^5C_5$ x $(0.8)^5(0.2)^0$ = 0.3277

5. The probability that Alex wins a chess game is $\frac{2}{3}$. He plays 8 games, what is the probability that he wins

(i) At least 7 games

n = 8, p = $\frac{2}{3}$, q = $\frac{1}{3}$

P(X ≤ 7) = P(X = 7) + P(X= 8) = $^8C_7$ x $\left(\frac{2}{3}\right)^7 \left(\frac{1}{3}\right)^1$ + $^8C_8$ x $\left(\frac{2}{3}\right)^8 \left(\frac{1}{3}\right)^0$

=0.1561+ 0.390 = 0.1951

(ii) Exactly 5 games

P(X =5) = $^8C_5$ x $\left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^3$ = 0.2731

6. Usain Bolt makes 5 practice ran in the 100m sprint. A run is successful if he runs it in less than 11 seconds. There are 8 chances out of 10 that he is successful. Find the probability that;

(i) He records at least no success at all

n = 5, p = 0.8, q = 0.2

P(X = 0) = $^5C_0$ x $(0.8)^0(0.2)^5$ = 0.0003

(ii) He record at least 2 success

$P(X \geq 2) = 1 - P(X < 2) = 1 - [P(X=0) + P(X = 1)$

$= 1 - [{}^5C_0 \times (0.8)^0 (0.2)^5 + {}^5C_1 \times (0.8)^1 (0.2)^4) = 1 - (0.0003 + 0.0064) = 0.9933$

(iii) If he is successful in 5 practice runs, he makes two additional runs. The probability of success in either of the additional runs is 0.7. Determine the probability that Bolt will make 7 successful runs

Probability = ${}^5C_5 \times 0.8^5 \times 0.2^0$ and ${}^2C^2 \times 0.7^2 (0.3)^0$

$= 0.32768 \times 0.49 = 0.1606$

7. In a test there are 10 objective questions each with a choice of five possible alternatives out of which only one is correct. If a student guesses each of the answers, find the probability that he gets at least two answers correct.

Solution

$n = 10, p = 0.2, q = 0.8$

$P(X \geq 2) = 1 - P(X<2)$

$= 1 - [P(X = 0) + P(X = 1)$

$= 1 - [\ {}^{10}C_0 \times 0.2^0 \times 0.8^{10} \times {}^{10}C_1 \times 0.2^1 \times 0.8^9)$

$= 1 - (0.1074 + 0.2684) = (1 - 0.\ 3758) = 0.6242$

8. 30% of the students in the school are day scholars. From a sample of 10 students chosen at random, find the probability that

(i) Only 3 are day scholars

$n = 10, p = 0.3, q = 0.7$

$P(X = 3) = {}^{10}C_3(0.3)^3(0.7)^7 = 0.267$

(ii) Less than half are day scholars

$P(X < 5) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$= {}^{10}C_0(0.3)^0(0.7)^{10} + {}^{10}C_1(0.3)^1(0.7)^9 + {}^{10}C_2(0.3)^2(0.7)^8 + {}^{10}C_3(0.3)^3(0.7)^7$

$+ {}^{10}C_4(0.3)^4(0.7)^6$

$= 0.850$

9. The probability that a shopper buys a cake is 0.25. Find the probability that in a random sample of 9 shoppers

(i) Exactly 3 buys a cake

$n = 9, p = 0.25, q = 0.75$

$P(X = 3) = {}^9C_3(0.25)^3(0.75)^6 = 0.2334$

(ii) More than 7 buy a cake

$P(X > 7) = P(X = 8) + P(X = 9)$

$= {}^9C_8(0.25)^8(0.75)^1 + {}^9C_9(0.25)^9(0.75)^0 = 0.0001$

10. A bag contains counter books of which 40% are blue and the rest are black. A counter book is taken from the bag, its colour is noted then replaced. This is performed 8 times in all. Calculate the probability that

(i) Exactly 3 will be blue

$n = 8, p = 0.4, q = 0.6$

$P(X = 3) = {}^8C_3(0.4)^3(0.6)^5 = 0.279$

(ii) At least one will be blue

$P(X \geq 1) = 1 - P(X<1) = 1 - P(X=0) = 1 - {}^8C_0(0.4)^0\ (0.6)^8 = 0.983$

(iii) More than half will be black

$P(X\leq3) = P(X = 0) + P(X= 1) + P(X = 2) + P(X = 3)$

$= {}^8C_0(0.4)^0 (0.6)^8 + {}^8C_1(0.4)^1 (0.6)^7 + {}^8C_2(0.4)^2 (0.6)^6 + {}^8C_3(0.4)^3 (0.6)^5$

$= 0.594$

11. At a certain school, the records taken from admission's office the ratio of male to female s. 5 applicants is 4:6. Basing on this experience, what is the probability that will be more female applicants in a random collection of a dozen applicants?

Solution

$n = 12, p = 0.4, q = 0.6$

$P(X\leq 5) = P(X = 0) + P(X =1) + P(X =2) + P(X= 3) + P(X = 4) + P(X = 5)$

$= {}^{12}C_0(0.4)^0 (0.6)^{12} + {}^{12}C_1(0.4)^1 (0.6)^{11} + {}^{12}C_2(0.4)^2 (0.6)^{10} + {}^{12}C_3(0.4)^3 (0.6)^9 +$

${}^{12}C_4(0.4)^4 (0.6)^8 + {}^{12}C_5(0.4)^5 (0.6)^7$

$= 0.6652$

12. The random variable X is B(6, 0.42). find
    (i) $P(X = 6)$

    $n = 6, p = 0.42, q= 1- 0.42= 0.58$

    $P(X = 6) = {}^6C_6(0.42)^6(0.58)^0 = 0.00549$

    (ii) $P(X = 4)$

    ${}^6C_4(0.42)^4(0.58)^2= 0.157$

    (iii) $P(X \leq 2)$

    $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X= 2)$

    $= {}^6C_0(0.42)^0(0.58)^6 + {}^6C_1(0.42)^1(0.58)^6 + {}^6C_2(0.42)^2(0.58)^4= 0.503$

13. An unbiased die is thrown seven times. find the probability of throwing at least 5 sixesn = 7, p = $\frac{1}{6}$, q = $\frac{5}{6}$

    $P(X \leq 5) = P(X = 5) + P(X = 6) + P(X = 7)$

14. $= {}^7C_5(\frac{1}{6})^5(\frac{5}{6})^2 + {}^7C_6(\frac{1}{6})^6(0.58)^1 + {}^7C_7(\frac{1}{6})^7(\frac{5}{6})^0 .= 0.002$

15. In a family a couple is likely to produce a girl or a boy. Find the probability that in a sample of 5 children there will be more boys than girls

    $n = 5, p = 0.5, q = 0.5$

    $P(X \geq 3) = P(X = 3) + P(X = 4) + P(X= 5)$

    $= {}^5C_3(0.5)^3(0.5)^2 + {}^5C_4(0.5)^4(0.5)^1 + {}^5C_5(0.5)^5(0.5)^0 = 0.5$

16. The probability that it will rain on any given day during examination period is 0.3. calculate the probability that in a given week during examination period, it will rain on;
    (i) Exactly 2 days

    $n = 7, p = 0.3, q = 0.7$

    $P(X = 2) = {}^7C_2(0.3)^2(0.7)^5= 0.318$

    (ii) At most two days = 0.671

    $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

    $= {}^7C_0(0.3)^0(0.7)^7 + {}^7C_1(0.3)^1(0.7)^6 + {}^7C_2(0.3)^2(0.7)^5 = 0.647$

17. A fair coin is tossed 6 times. find the probability of throwing at least four heads

    $n = 6, p = 0.5, q = 0.5$

    $P(X \leq 4) = P(X = 4) + P(X =5) + P(X =6)$

$= {}^6C_4(0.5)^4(0.5)^2 + {}^6C_5(0.5)^5(0.5)^1 + {}^6C_6(0.5)^6(0.5)^0 = 0.344$

18. The random variable x is B(4, p) and P(X = 4) = 0.0256. find P(X = 2)
    $n = 4, p = p, q = (1-p)$
    $P(X = 4) = {}^4C_4(p)^4(q)^0 = 0.0256$
    $(p)^4 = 0.0256; p = 0.4$
    $P(X = 2) = {}^4C_2(0.4)^2(0.6)^2 = 0.3456$
19. In agriculture lab, Silvia plants bean seeds and the probability that they germinate successfully is $\frac{1}{3}$.
    (a) She takes 9 seeds. Find the probability that
        (i) More than five seeds germinate
        $P(X<5) = P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9)$
        $= {}^9C_6(\frac{1}{3})^6(\frac{2}{3})^3 + {}^9C_7(\frac{1}{3})^7(\frac{2}{3})^2 + {}^9C_8(\frac{1}{3})^8(\frac{2}{3})^1 + {}^9C_9(\frac{1}{3})^9(\frac{2}{3})^0 = 0.0424$
        (ii) At least three seeds germinate
        $P(X \geq) = 1 - [P(X=0) + P(X = 1) + P(X = 2)]$
        $= 1 - [{}^9C_0(\frac{1}{3})^0(\frac{2}{3})^9 + {}^9C_1(\frac{1}{3})^1(\frac{2}{3})^8 + {}^9C_2(\frac{1}{3})^2(\frac{2}{3})^7] = 0.623$
    (b) Find the number of seeds that she needs to take in order to 99% certain at least one germinate
        $P(X \leq 1) = 1 - P(X = 0) = 1 - {}^nC_0(\frac{1}{3})^0(\frac{2}{3})^n = 0.99$
        $(\frac{2}{3})^n = 0.01$
        $n = 11.36$
        therefore the number = 12
20. In a shooting competition, the probability of hitting the target with a single shot is 0.6, if 7 shots are taken; find the probability that the target is hit more than twice.
    $n = 7, p = 0.6, q = 0.4$
    $P(X > 2) = 1 - P(X \leq 2) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$
    $= 1 - [{}^7C_0(0.6)^0(0.4)^7 + {}^7C_1(0.6)^1(0.4)^6 + {}^7C_2(0.6)^2(0.4)^5]$
    $= 0.9037$
21. In mass production of shirts, it is found that 5% are defective. Shirts are selected at random of put into packets of 10.
    (a) A packet is selected at random. Find the probability that it contains
        (i) Three defective shirt
        $P(X= 3) = {}^{10}C_3(0.05)^3(0.95)^7 = 0.0105$
        (ii) Less than three defective shirts
        $P(X < 3) = P(X = 0) + P(X = 1) + P(X=2)$
        $= {}^{10}C_0(0.05)^0(0.95)^{10} \times {}^{10}C_1(0.05)^1(0.95)^9 + {}^{10}C_2(0.05)^2(0.95)^8 = 0.9885$
    (b) Two packets are selected at random. Find the probability that there are no defective shirts in either packet = $({}^{10}C_0(0.05)^0(0.95)^{10} \times ({}^{10}C_0(0.05)^0(0.95)^{10} = 0.358$
22. A biased coin is such that it is twice as likely to show a head as a tail. If the coin is tossed 5 times. find the probability that
    (i) Exactly three heads are obtained
    $n = 5, p = \frac{2}{3}, q = \frac{1}{3}$

$P(X = 3) = {}^5C_3(\frac{2}{3})^3(\frac{1}{3})^2 = 0.3292$

(ii) More than three heads are obtained = 0.3333

$P(X > 3) = P(X = 4) + P(X = 5)$

$= {}^5C_4(\frac{2}{3})^4(\frac{1}{3})^1 + {}^5C_5(\frac{2}{3})^0(\frac{1}{3})^5 =$

23. The probability that a target is hit 0.3. Find the probability the least number of times shots should be fired if the probability that the target is hit is at least once is greater than 0.95. =9

    $n = ?$ $p = 0.3$, $q = 0.7$

    $P(X \geq 1) = 1 - P(X = 0) = 1 - {}^nC_0(0.3)^0(0.7)^n > 0.95$

    $(0.7)^n > 0.05$

    $n = \frac{\log 0.05}{\log 0.7} = 8.399$

    therefore n =9
24. 1% of the light bulbs in a box are faulty. Find the largest sample size which can be taken if it required that the probability that there is no faulty bulb in the sample is greater than 0.5

    (0.99)n > 0.5

    n = 68
25. In a test there are 10 multiple choice questions. Each question has got four possible alternatives out of which only one is correct. If a student guesses each of the answers, find the probability that he gets

    (i) More than 7 correct answers = 0.0004

    (ii) More than half correct answers = 0.0197
26. $X \sim B(n, o.3)$. find the value of n such that $P(X \geq 1) = 0.8 : 0.7^n = 0.2$. = 5
27. The random variable X is B(n, 0.6). find the value of n such that $P(X<1) = 0.0256$

    $0.4^n = 0.0256$, n = 4

Expectation, variance and standard deviation

If $X \sim B(n, p)$

$E(X) = np$

$Var(X) = npq$ where $q = 1- p$

$s.d = \sqrt{npq}$

Example 9

The random variable x is B(4, 0.8). the mean, variance and standard deviation

Mean = np = 4 x 0.8 = 3.2

Variance = npq = 4 x0.8 x 0.2 = 0.64

$s.d = \sqrt{npq} = \sqrt{0.64} = 0.8$

Example 11

The probability that, it will be a sunny day is 0.4. Find the expected number if sunny days in a week and also find the standard deviation

Solution

E(X) = np = 7 x 0.4 = 2.8

s.d = $\sqrt{npq} = \sqrt{(7 \; x \; 0.4 \; x \; 0.6)}$ = 1.296

Example 12

X is B(n, p) with mean 5 and standard deviation2. Find the value of n and p.

E(X) = np

np = 5 …………………. (i)

s.d = $\sqrt{npq}$

$\sqrt{npq}$ =2

npq = 4 ………………..(ii)

Eqn. (ii) ÷ eqn. (i)

$\frac{npq}{np} = \frac{4}{5} = 0.8$

q = 0.8

p = 0.2

n = $\frac{5}{0.2}$ = 25

Mode of the binomial distribution

The mode is the value of X that is most likely to occur. The value of X with the highest probability and its close to the mean gives the mode.

Example 13

The probability that a student is awarded a distinction in mathematics examination is 0.15. In a randomly selected group of 15 students, what is the most likely number of students awarded a distinction

Solution

E(X) = np = 15 x 0.15 = 2.25

P(X = 2) = $^{15}C_2(0.15)^2(0.85)^{13}$ = 0.286

P(X = 3) = $^{15}C_3(0.15)^3(0.85)^{12}$ = 0.216

The most likely number of students awarded a distinction = 2

Example 14

In a school 80% of the students find difficulties in Physics. If a sample of 12 students is chosen

(i) What is the most likely number of students who find it difficult in physics.
$E(X) = np = 0.8 \times 12 = 9.6$
$P(X = 9) = {}^{12}C_9(0.8)^9(0.2)^3 = 0.236$
$P(X = 10) = {}^{12}C_{10}(0.8)^{10}(0.2)^2 = 0.294$
The most likely number is 10

(ii) Find the probability that fewer than half find difficulty in Physics.
$P(X<6) = 0.0004$

Revision exercise 2

1. 10% of drugs at a certain Pharmacy are expired. A sample of 25 drugs is taken. Find the expected number and standard deviation of expired drugs (25,1.5)
2. The probability that a student scores above 60% in mathematics test is0.5. In a random sample of 15 students, what is the most likely number of students who score above 60%. (7 and 8)
3. The probability that an apple picked at random from a sack is bad is 0.15.
   (a) Find the standard deviation of the number of bad apples in a sample of 15 apples.=1.38
   (b) What is the most likely number of bad apples in a sample of 30 apples = 4
4. The random variable X is B(n, 0.3) and E(X) = 2.4. Find n and standard deviation (8, 1.30)
5. In a group of people, the expected number who wear glasses is 2 and the variance is 1.6, find the probability that;
   (i) A person chosen at random from the group wear glasses = 0.2
   (ii) 6 people in the group wear glasses. =0.00551
6. New vision publishes a governance article in its newspaper each day of the week Sunday. A man is able to read 8 out of ten articles
   (i) Find the expected value and the standard deviation of the number of read articles in a given week= 4.8
   (ii) What is the probability that he will read at least 5 articles in a given week? = 0.98
7. A die is biased and probability, p of throwing a six is known to be less than $\frac{1}{6}$. An experiment consists of recording the number of sixes n 25 throws of the die. The standard deviation of the number of sixes I 1.5. calculate the
   (i) value of p. = 0.1
   (ii) the probability that exactly three sixes are recorded during a particular experiment = 0.23
8. The random variable X is B(10, p) where p < 0.5. The variance of X is 1.875. find
   (i) Value of p. = 0.25
   (ii) E(X) = 2.5
   (iii) P(X = 2) = 0.282

9. In a bag there are 6 red pens, 8 blue pens and 6 black pens. an experiment consists of taking a pen at random from the bag, noting its colour and then replacing it in the bag. This procedure is repeated 10 times in all. find
   (i) Expected number of red pens drawn=3
   (ii) Most likely number of black pens drawn = 3
   (iii) Probability that not more than four blue pens are drawn = 0.633
10. The random variable X is distributed binomially with mean 2 and variance 1.6, Find
   (i) the probability that x is less than 6 = 0.994
   (ii) the most likely value of X = 2
11. Each day a bakery delivers the same number of loaves to a certain shop which sells on average98%of them. Assuming that the number of loaves sold per day has a binomial distribution with standard deviation 7. Find the expected number of loaves the shop would expect to sell per day = 2500
12. On average 20% of the bolts produced by a machine are faulty. Samples of 10 bolts are to be selected at random each day. Each bolt will be selected and replaced in the set of bolts which have been produced on that day.
   (i) Find the probability that in any one sample, two bolts or less will be faulty = 0.68
   (ii) Calculate the expected value and variance of the number of bolts in a sample which will not be faulty. = 8, 1.6
13. An experiment consists of taking 12 shots at a target and counting the number of hits. The expected number of hit was found to be 3. Calculate
   (i) The probability of hitting the target with a single shot. = 0.25
   (ii) Standard deviation of the number of hits in an experiment = 1.5
14. In a certain family, the probability that they will have a baby boy is 0.6. If there are 5 children in a family determine
   (a) Expected number of girls = 2
   (b) The probability that there are at least three girls = 0.317
   (c) The probability they are all boys. = 0.0778
15. The probability of winning a game is 0.8. Ten games are played. What is the;
   (a) Mean number of success and variance. = 8, 1.6
   (b) The probability of at least 8 success in the ten = 0.6778
16. In a test there are 10 multiple choices questions. Each question has four possible alternatives out of which one is correct. If a student guesses each of the answers, find the
   (i) The probability that at least fur answers are correct = 0.2241
   (ii) Most likely number of correct answers = 2
17. A biased coin is such that a head is twice as likely to occur as a tail. the coin is tossed 15 times. Find the
   (i) The expected number of heads = 10
   (ii) Probability that at most two tails occur = 0.0793

Solutions to revision exercise 2

14. In a certain family, the probability that they will have a baby boy is 0.6. If there are 5 children in a family determine
    (a) Expected number of girls
    n = 5, p = 0.4, q = 0.6
    E(X) = np = 5 x 0.4 = 2
    (b) The probability that there are at least three girls
    $P(X\leq 3) = P(X= 3) + P(X = 4) + P(X = 5)$
    $= {}^5C_3(0.4)^3(0.6)^2 + {}^5C_4(0.4)^4(0.6)^1 + {}^5C_5(0.4)^5(0.6)^0 = 0.317$
    (c) The probability they are all boys.
    $P(X = 0) = {}^5C_0(0.4)^0(0.6)^5 = 0.0778$
15. The probability of winning a game is 0.8. Ten games are played. What is the;
    (a) Mean number of success and variance.
    Mean = np = 10 x 0.8 = 8
    Variance = npq = 10 x 0.8 x 0.2 =1.6
    (b) The probability of at least 8 success in the ten
    $P(X \leq 8) = P(X = 8) + P(X = 9) + P(X = 10)$
    $= {}^{10}C_8(0.8)^8(0.2)^2 + {}^{10}C_9(0.8)^9(0.2)^1 + {}^{10}C_{10}(0.8)^{10}(0.2)^0 = 0.6778$
16. In a test there are 10 multiple choices questions. Each question has four possible alternatives out of which one is correct. If a student guesses each of the answers, find the
    (i) The probability that at least four answers are correct
    n = 10, p = 0.25, q = 0.75
    $P(X \geq 4) = 1 - P(X \leq 3) = 1 - [P(X= 0) + P(X= 1) + P(X = 2) + P(X = 3)$
    $= 1 - [{}^{10}C_0(0.25)^0(0.75)^{10} + {}^{10}C_1(0.25)^1(0.75)^9 + {}^{10}C_2(0.25)^2(0.75)^8 + {}^{10}C_3(0.25)^3(0.75)^7]$
    = 0.2241
    (ii) Most likely number of correct answers
    E(X) = np = 0.25 x 10 = 2.5
    $P(X = 2) = {}^{10}C_2(0.25)^2(0.75)^8 = 0.2816$
    $P(X = 3) = {}^{10}C_3(0.25)^3(0.75)^7 = 0.2503$
    ∴ the most likely number of correct answers = 2
17. A biased coin is such that a head is twice as likely to occur as a tail. The coin is tossed 15 times. Find the
    (i) The expected number of heads
    n =15, p = $\frac{2}{3}$, q = $\frac{1}{3}$
    E(X) = np = $\frac{2}{3} x\ 15 = 10$
    (ii) Probability that at most two tails occur
    n = 15, p = $\frac{1}{3}$, q = $\frac{2}{3}$
    $P(X\leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$
    $= {}^{15}C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{15} + {}^{15}C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{14} + {}^{15}C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{13} = 0.0793$

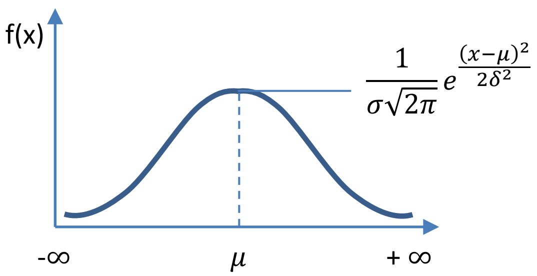
Normal distribution

A continuous random variable X follows a normal distribution with mean, μ and variance, σ^2 if

X~N(μ, σ^2) root

Its p.d.f is given by fx = $\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{(x-\mu)^2}{2\delta^2}}, -\infty < x < \infty$

A sketch of f(x) gives a normal curve



Properties of the curve

- It is bell shaped
- It is symmetrical about μ
- It extends from $-\infty < x < \infty$

The maximum value of f(x) =$\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{(x-\mu)^2}{2\delta^2}}$

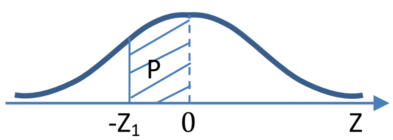
- The total area under the curve = 1

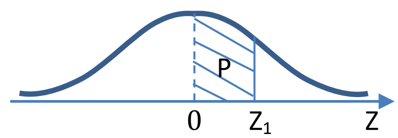
How to read the cumulative normal distribution table

(i) Between 0 and any z value

(a) $P(0 \le Z \le Z_1) = \phi(Z_1)$ = region P   (b) $P(-Z_1 \le Z \le 0) = P(0 \le Z \le Z_1) = \phi(Z_1)$ = region P

By symmetrical



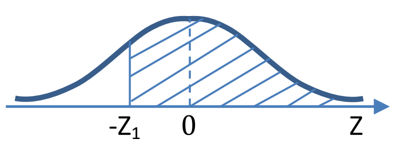


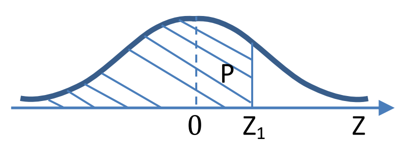
(ii) Less than any positive z value

(a) $P(Z < Z_1) = 0.5 + P(0 \le Z \le Z_1) = \phi(Z_1)$ = region P

(b) $P(Z > -Z_1) = P(Z < Z_1) = 0.5 + P(0 \le Z \le Z_1) = 0.5 + \phi(Z_1)$ = region P

By symmetrical



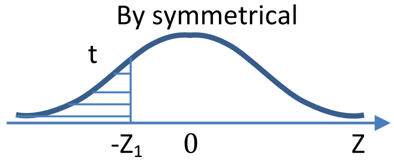


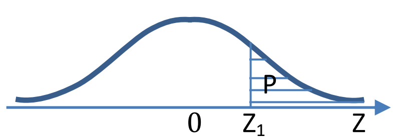
(iii) Greater than any positive z value

$P(Z > Z_1) = 0.5 - P(0 \le Z \le Z_1) = 0.5 - \phi(Z_1)$ = region P

$P(Z < -Z_1) = P(Z > Z_1) = 0.5 - P(0 \le Z \le Z_1) = 0.5 - \phi(Z_1)$ = region P

By symmetrical



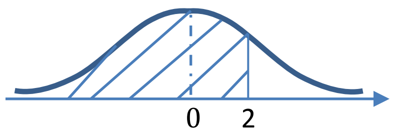


Example 15

Find

(i) P(Z< 2)

By symmetry

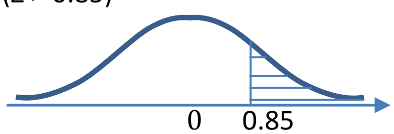


P(Z< 2) = 0.5 + ϕ(2)

= 0.5 + 0.4772

= 0.9772

(ii) P(Z > 0.85)

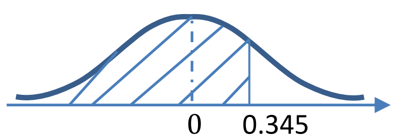


P(Z > 0.85) = 0.5 - ϕ(0.85)

= 0.5- 0.3023

= 0.1977

(iii) P(X< 0.345)



P(X< 0.345) = 0.5 + ϕ(0.345)

= 0.5 + 0.1331 + 0.0019

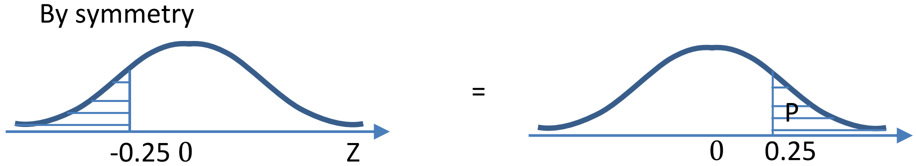
= 0.6350

Example 16

Find

(i) P(Z < -0.25)

By symmetry



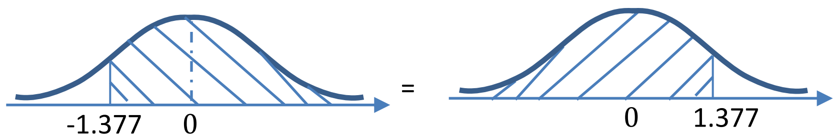
P(Z < -0.25) = P(Z > 0.25) = 0.5 - ϕ(0.25)

= 0.5 – 0.0987

= 0.4013

(ii) P(Z > -1.377)

By symmetry



P(Z > -1.377) = P(Z < 1.377)

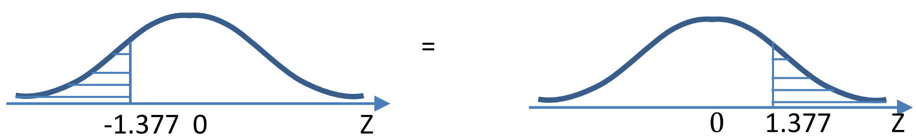
= 0.5 + ϕ(1.377)

= 0.5 + 0.4147 + 0.0011

= 0.9158

(iii) P(Z < -1.377)

By symmetry



P(Z < -1.377) = P(Z > 1.377

= 0.5 - ϕ(1.377)

= 0.5 – (0.4147 + 0.0011)

= 0.0842

Standardizing a random variable X

If a random variable X follows a normal distribution with mean, μ and variance, σ^2, then X ~N(μ, σ^2) and can be standardized using the equation below and read from a cumulative normal table

Z = $\frac{X-\mu}{\sigma}$

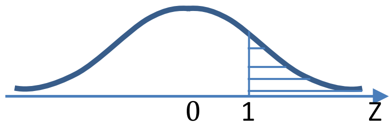
***Example* 3**

Given that the random variable X is X ~N(300, 25). Find

(i) P(X > 305)

P(X> 305) $= P\left(Z < \frac{305-300}{5}\right)$

= P(X > 1)

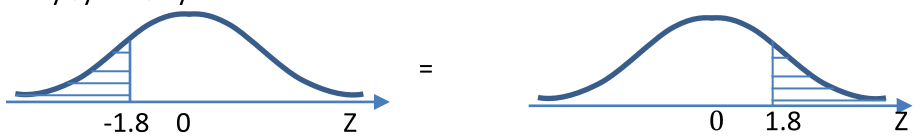


= 0.5 – ϕ(1)
= 0.5 – 0.3413 = 0.1587

(ii) P(X < 291)

P(X < 291) $= P\left(Z < \frac{291-300}{5}\right)$ = P(Z < -1.8)

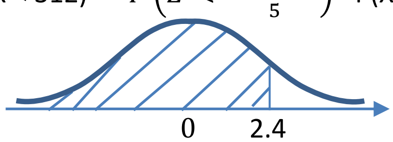
By symmetry



P(Z < -1.8) = P(Z >1.8)
= 0.5 - ϕ(1.8)
= 0.5 – 0.4641
= 0.0359

(iii) P(X < 312)

P(X < 312) $= P\left(Z < \frac{312-300}{5}\right)$= P(X < 2.4)

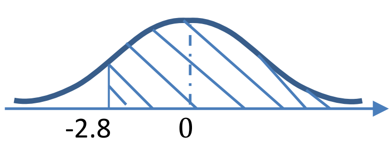


= P(X < 2.4) = 0.5 + ϕ(2.4)

= 0.5 – 0.4918 = 0.9918

(iv) P(X > 286)

P(X > 286) $= P\left(Z < \frac{286-300}{5}\right)$ = P(Z < -2.8



= P(Z < 2.8)
= 0.5 + ϕ(2.8)
= 0.5 + 0.4974= 0.9974

Applications

Example 17

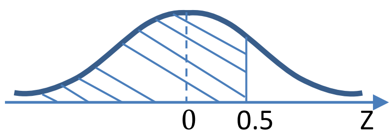
A bakery produces loaves of bread whose weight is normally distributed with mean 1,000g and standard deviation 40g.

(a) Find the probability that a randomly selected loaf has a weight of utmost 1,020g.

$\mu = 1000$, $\sigma = 40$

Let x be the weight of bread

$P(x \leq 1020) = P(z \leq \frac{1020-1000}{40} = P(z \leq 0.5)$



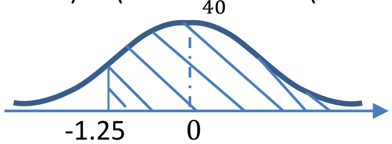
$P(z \leq 0.5) = 0.5 + P(0 \leq x \leq 0.5)$

$= 0.5 + 0.1915$

$= 0.6915$

(b) Assuming that the bakery makes 10,500 loaves, find the approximate number of loaves with weight greater than 950g.

$P(x \leq 950) = P(z \geq \frac{950-1000}{40} = P(z \geq -1.25)$



$P(z \geq -1.25) = P(-1.25 \leq z \leq 0) + 0.5$

$= (0 \leq z \leq 1.25) + 0.5$

$= 0.5 + 0.3944$

$= 0.8944$

Number of loaves = 0.8944 x 10,500 = 9391

Example 18

A factory sells animals food in bags. The weights of the bags are normally distributed with mean weight 50kg and standard deviation 2.8kg.

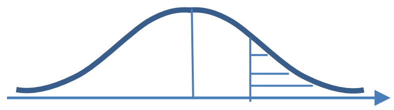
(a) Find the probability that the weight of any bag selected at random;

Let X = random variable for weight of bags

Given $\mu = 50$ and $\sigma = 2.8$kg

(i) is more than 53kg (04marks)

P(X> 52) = P(Z> $\frac{52-50}{2.8}$)= P(Z > 0.714)



P(Z > 0.714) = 0.5 – P(0 < Z < 0.714)

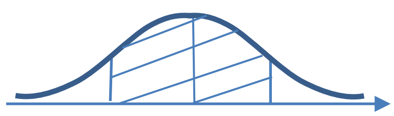
= 0.5 – 0.2623

= 0.2377

(ii) lies between 46 and 55kg (05marks)

P(46 < X < 55) = P$\left(\frac{46-50}{2.8} < Z < \frac{55-50}{2.8}\right)$

= P(-1.429< Z < 1.786)



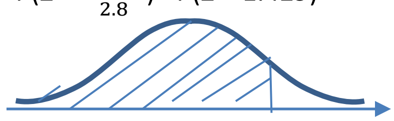
P(-1.429< Z < 1.786) = P(0 <Z< 1.429) + P(0 < Z < 1.789)

= 0.4235 + 0.4630

= 0.8865

(b) Determine the percentage of bags whose weights are less than 54kg. (06marks)

P(X< 54) = P(Z < $\frac{54-50}{2.8}$)= P(Z > 1.429)



P(Z > 1.429) = 0.5 + P(0 < Z < 1.429)

= 0.5 + 0.4235

= 0.9235

Example 19

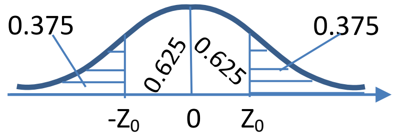
The marks scored by candidates in an examination are normally distributed with a mean score of μ and standard deviation of δ. Given that 37.5% of the candidates scored below 40 and that 12.5% scored above 60, find the;

(i) values of μ and δ. (09marks)

Let X be marks scored

$P(X < 40) = \frac{37.5}{100} = 0.375$

$P(x < 40) = P(Z < Z_0) = 0.375$



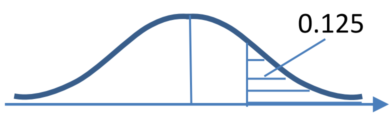
By symmetry, $P(Z < Z_0) = P(Z > Z_0)$

But $Z = \frac{X - \mu}{\sigma}$

$-0.319 = \frac{40 - \mu}{\sigma}$

$\mu - 0.319\sigma = 40$ ……………………(i)

$P(X > 60) = P(Z > Z_1) = \frac{12.5}{100} = 0.125$



$P(0 < Z < Z_1) = 0.875$

$Z_1 = 1.15$

⇨ $1.15 = \frac{60 - \mu}{\sigma}$

$\mu - 0.319\sigma = 60$ ……………………(ii)

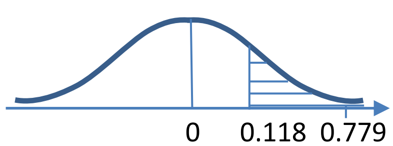
from equation (i) and (ii)

$\sigma = 13.6$ and $\mu = 44.4$

(ii) probability that a candidate score between 46 and 55. (06 marks)

$P(46 < X < 55 = \left[\frac{46-44.4}{13.6} < Z < \frac{55-44.4}{13.6}\right]$

$= P[0.118 < Z < 0.779]$



$= P(0 < Z < 0.779) - P(0 < Z < 0.118)$

$= 0.2822 - 0.0470$

$= 0.2352$

Example 20

The time taken for a bus to make a journey is normally distributed with mean 3 ½ hours and standard deviation ¾ hours.

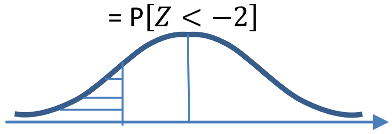
(a) Determine the probability that the bus makes a journey:

(i) in less than 2 hours (05marks)

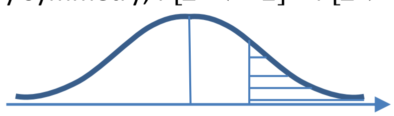
Solution

P(X < 2) = P$\left[Z < \frac{2-3.5}{0.75}\right]$

= P$[Z < -2]$



By symmetry, P$[Z < -2]$ = P$[Z > 2]$



P$[Z > 2]$ = 0.5 – (0< Z< 2)

= 0.5 – 0.4772

= 0.0228

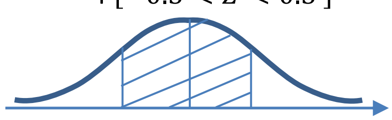
Hence the probability = 0.0228

(ii) between 3¼ and 3¾ hours (07 marks)

Solution

P(3.25<X 3.75) = P$\left[\frac{3.25-3.5}{0.75} < Z < \frac{3.75-3.5}{0.75}\right]$

= P$[-0.3 < Z < 0.3\,]$



P$[-0.3 < Z < 0.3\,]$ = 2P(0 < Z< 0.3)

= 2 x 0.1179

= 0.2358

(b) If the bus made two hundred journeys, how many of these journeys did it take less than 2 hours? (03 marks)

Number of journeys that take less than 2 hours = 0.0228 x 200 = 4

Example 21

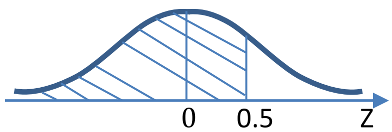
A bakery produces loaves of bread whose weight is normally distributed with mean 1,000g and standard deviation 40g.

(a) Find the probability that a randomly selected loaf has a weight of utmost 1,020g.

μ = 1000, σ = 40

Let x be the weight of bread

P(x≤ 1020) =P(z ≤ $\frac{1020-1000}{40}$ = P(z ≤ 0.5)



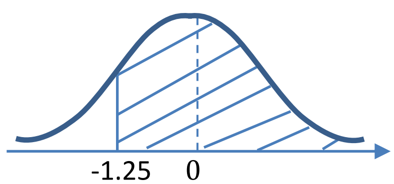
P(z ≤ 0.5)= 0.5 + P(0≤ x≤ 0.5)

= 0.5+0.1915

= 0.6915

(b) Assuming that the bakery makes 10,500 loaves, find the approximate number of loaves with weight greater than 950g.

P(x≤ 950) =P(z ≥ $\frac{950-1000}{40}$ = P(z ≥ -1.25)



P(z ≥ -1.25)= P(-1.25≤ z≤ 0) + 0.5

= (0 ≤ z ≤ 1.25) + 0.5

= 0.5 +0.3944

= 0.8944

Number of loaves = 0.8944 x 10,500 = 9391

Example 22

Given that the random variable X is X ~N(10, 4). Find

Find (i) P(X < 7) (ii) P(X > 12) (iii) P(7 < X < 12) (iv) P(9 < X < 11)

Solution

(i) P(X < 7) $= P\left(Z < \frac{7-10}{2}\right)$ = P(Z < - 0.15) = P(Z > 1.5)
= 0.5 – ϕ(1.5) = 0.5 – 0.4332 0.0668

(ii) P(X > 12) $= P\left(Z > \frac{12-10}{2}\right)$ = P(Z > 1) = 0.5 – ϕ(1) = 0.5 – 0.3413 = 0.1587

(iii) P(7 < X < 12) = $P\left(\frac{7-10}{2} < Z > \frac{12-10}{2}\right)$
= P(-1.5 < Z < 1) = ϕ(1.5) + ϕ(1) = 0.4332 + 0.3413 = 0.7745

(iv) P(9 < X < 11) = $P\left(\frac{9-10}{2} < Z > \frac{11-10}{2}\right)$
= P(-0.5 < Z < 0.5) = ϕ(0.5) + ϕ(0.5) = 2 x 0.1915 = 0.3830

Example 23

Given that the random variable X is X ~N(50, 8). Find

(i) P(48 < X < 54) (ii) P(52 < X < 55) (iii) P(46 < X < 49) (iv) P(|X- 50|< $\sqrt{8}$)

Solution

(i) P(48 < X < 54) = $P\left(\frac{48-50}{\sqrt{8}} < Z > \frac{54-50}{\sqrt{8}}\right)$ = P(-0.707 < Z < 1.414)

= ϕ(1.414) + ϕ(0.707) = 0.4213 + 0.2601 = 0.6814

(ii) P(52 < X < 55) = $P\left(\frac{52-50}{\sqrt{8}} < Z > \frac{55-50}{\sqrt{8}}\right)$ = P(0.707 < Z < 1.768)

= ϕ(1.768) - ϕ(0.707) = 0.4615 -0.2601 = 0.2014

(iii) P(46 < X < 49) = $P\left(\frac{46-50}{\sqrt{8}} < Z > \frac{49-50}{\sqrt{8}}\right)$ = P(-1.414 < Z < -0.354)

=ϕ(1.414) – ϕ(0.354) = 0.4213 – 0.1383 -0.283

(iv) P(|X- 50|< $\sqrt{8}$) = $P\left(\frac{-\sqrt{8}+50-50}{\sqrt{8}} < Z > \frac{\sqrt{8}+50-50}{\sqrt{8}}\right)$ P(-1 < X < 1) = 2 x ϕ(1) = 2 x 0.3413 = 0.6826

Example 24

A random variable X is normally distributed with mean 65 and variance 100, find the probability that X assumes a value between 50 and 90.

P(50 < X < 90) = $P\left(\frac{50-65}{10} < Z < \frac{90-65}{10}\right)$= P(-1.5 < Z < 2.5) = ϕ(1.5) + ϕ(2.5) = 0.4332 + 0.4938 = 0.927

Example 25

Lengths of metal strips produced by a machine are normally distributed with mean length of 150cm and standard deviation of 10cm. find the probability that the length of a randomly selected strip is

(i)shorter than 165 (ii) within 5cm of the mean

Solution

P(X < 165) = $P\left(Z < \frac{165-150}{10}\right)$ =P(Z< 1.5) = 0.5 + ϕ(1.5) = 0.5 + 0.4332 = 0.9332

P(150 – 5 < X < 150 + 5) = $P\left(\frac{-5}{10} < Z < \frac{5}{10}\right)$= P(-0.5 < Z < 0.5) = 2 x ϕ(0.5) = 2 x 0.1915 = 0.383

Example 26

In end of year exams, the marks are normally distributed with a mean mark of 50 and standard deviation 5. If a mark 45 is required to pass the exam, what percentage of the students failed the exam.

P(X < 45) = $P\left(Z < \frac{45-50}{5}\right)$ = P(Z < -1) = P(Z > 1) = 0.5 – ϕ(1) = 0.5 – 0.3413 = 0.1587

Example 9

A bakery supplies bread to a shop every day. The time to deliver bread to the shop is normally distributed with mean 12 minutes and standard deviation of 2 minutes. Estimate the number of days the year when he takes

(i) longer than 17 minutes (ii) less than 10 minutes (iii) between 9 and 13 minutes

Solution

(i) P(X > 17) = $P\left(Z > \frac{17-12}{2}\right)$= P(Z > 2.5) = 0.5 – ϕ(2.5) = 0.5 – 0.4938 = 0.0062

The number of days = 0.0062 x 365 = 2 days

(ii) P(X < 10) = $P\left(Z > \frac{10-12}{2}\right)$= P(Z < -1) = P(Z> 1) = 0.5 – ϕ(1) + 0.5 – 0.3413 0.1587

The number of days = 0.1587 x 365 = 58 days.

(iii) P(9 < X <13) = $P\left(\frac{9-12}{2} < Z < \frac{13-12}{2}\right)$= P(-1.5 < Z < 0.5) = ϕ(1.5) + ϕ(0.5) 0.4332 + 0.1915 = 0.6247

Number of days = 0.6247 x 365 = 228 days.

Example 28

(a) In a certain athletics competition, points are awarded according to level of performance. The average grade was 82 points with standard deviation of 5 points. All competitors whose grades ranged between 88 to 94 points received certificates. If the grades are normally distributed and 8 competitors received certificates. How many participants took part in the competition?

P(88 < X < 94) = $P\left(\frac{88-82}{5} < Z < \frac{94-82}{5}\right)$= P(1.2 < Z < 2.4) =$\frac{8}{n}$

ϕ(2.4) – ϕ(1.2) = 0.4918 – 0.3849 = 0.1069 = $\frac{8}{n}$; n = 74.84

hence 75 participants took part.

(b) If certificates were to be awarded to only those having between 90 and 94 points. What proportion of the participants would acquire certificates.

P(90 < X < 94 = $P\left(\frac{90-82}{5} < Z < \frac{94-82}{5}\right)$=P(1.6 < Z < 2.4 = ϕ(2.4) – ϕ(1.6)

= 0.4918 – 0.4452 = 0.0466

= 0.0466 x 100% = 4.66%

Revision exercise 3

1. The amount of meat sold by a butcher is normally distributed with mean 43kg and standard deviation 4kg. Determine the probability that the amount of meat sold is between 40kg and 50kg. (0.7333)
2. Given that a random variable X is X~N(2, 2.89). Find P(X<0)(0.1198)
3. In a school of 800 students their average weight is 54.5kg and standard deviation 6.8kg. given that the weight of students are normally distributed, find
   (i) Probability that the weight of any student randomly selected is 52.8 kg or less = 0.4014
   (ii) Number of students who weigh over 75kg = 1

(iii) Weight of the middle 56% of the students (49.251< X < 59.750)

4. A sugar factory sells sugar in bags of mean weight 50kg and standard deviation 2.5kg. given that the weight of the bags is normally distributed, find the
   (i) Probability that the weight of any bag of sugar randomly selected lies between 51.5kg and 53kg = 0.1592
   (ii) Percentage of bags whose weight exceeds 54kg = 5.48%
   (iii) Number of bags that will be rejected out of 1000 bags purchased for weighing below 45.0kg = 23
5. A certain maize firm sells maize in bags of mean weight 40kg and standard deviation 2kg, given that the weight of the bags are normally distributed, find
   (i) Probability that the weight of any bag of maize randomly selected lies between 41.0 and 42.5kg = 0.2029
   (ii) Percentage of bags whose weight exceeds 43kg = 6.68%
   (iii) Number of bags that will be rejected out of 500 bags purchased for weighing below 38.5kg = 113
6. Given that the random variable X is X~N(300, 25) Find
   (i) P(X>308) = 0.0548 (ii) P(X> 311.5) = 0.0107 (iii) P(X < 294) = 0.8849
   (iv) P(X < 290.5) = 0.9713 (v) P(X > 302) = 0.6554 (vi) P(X > 312) = 0.9918
7. If X~N(50, 20). Find
   (i) P(X > 60.3) = 0.0106 (ii) P(X < 47.3) = 0.273 (iii) P(X > 48.9) = 0.5972
   (iv) P(X > 53.5) = 0.2831 (v) P(X < 59.8) = 0.9857 (vi) P(X < 62.3) = 0.9970
8. If X~N(-8, 12). Find
   (i) P(X < -9.8) = 0.1587 (ii) P(X > 0) = 0.8413 (iii) P(X < -3.4) =0.9079
   (iv) P(X > -5.7) =0.2533 (v) P(X < 10.8) = 0.2097 (vi) P(X > -1.6) = 0.0323
9. If $X \sim N(\alpha, \alpha^2)$. Find
   (i) P(X < 0) = 0.1587 (ii) P(X > 0) = 0.8413 (iii) P(X < 0.5α) = 0.6915 P(X > 0.5α) = 0.3085
10. If X~N(100, 80) . Find
    (i) P(85 < X < 112) = 0.8634 (ii) P(105 < X < 115) = 0.2413
    (iii) P(85 < X < 92) = 0.1388 (iv) $P(|X| < \sqrt{80}$ = 0.6826
11. If X~N(84, 12) . Find
    (i) P(80 < X < 89)= 0.8014 (ii) P(X <79 or X > 92) = 0.085 (iii) P(76 < X < 82) = 0.2714
    (iv) P(|X- 84|> 2.9) = 0.4028 (v) P(87 < X < 93) = 0.1886
12. The masses of packages from a particular machine are normally distributed with a mean of 200g and standard deviation of 2g, find the probability that a randomly selected package from the machine weighs
    (i) less than 197g = 0.0668
    (ii) more than 200.5g = 0.4013
    (iii) between 198.5g and 199.5g = 0.1747
13. The heights of boys at a certain school follow a normal distribution with mean = 150.3cm and variance 25cm, find the probability that a boy picked at random from the group has a height;
    (i) less than 153cm = 0.7054
    (ii) more than 158cm = 0.018
    (iii) between 150 cm and 158 cm= 0.4621
    (iv) more than 10cm difference from the mean height = 0.0046

14. The masses of a certain type of cabbages are normally distributed with mean of 1000g and standard deviation of 0.15kg. In a batch of 800 cabbages, estimate how many have a mass between 750g and 1290g = 740
15. Cartons of milk from quality super market are advertised as containing 1 litre, but in fact the volume of the content is normally distributed with a mean of 1012ml and standard deviation of 15ml.
    (i) Find the probability that a randomly chosen carton contains more than 1010ml =0.6554
    (ii) In a batch of 1000 cartons, estimate the number of cartons containing less than the advertised volume of milk =8
16. A random variable X is such that X~N(-5, 9). Find the probability that;
    (i) A randomly chosen item from the population will have positive value = 0.0478
    (ii) Out of 10 items chosen randomly, exactly 4 will have a positive value = 0.00082
17. The life of a laptop is normally distributed with a mean of 2000 hours and standard deviation of 120 hours. Estimate the probability that the life of such a laptop will be
    (i) greater than 2150 hours = 0.1056
    (ii) greater than 1910hours = 0.7734
    (iii) within a range 1850 hours to 2090 hours = 0.6678
18. Height of female students at particular college are normally distributed with mean 169cm and standard deviation 9cm.
    If X stands for the height of students in cm. find
    (i) $P(X < X_0) = 0.8$ [176.578]
    (ii) P(X>v1) = 0.6 [166.723]
19. The marks of 500 candidates in an examination are normally distributed with mean 45 marks and standard deviation 20 marks.
    (i) Given that the pass mark is 41, estimate the number of students who passed the examination. [290]
    (ii) If 15% of the candidates obtained a distinction by scoring mark x or more, estimate the value of x. [77.9]
20. The mass of soap powder in certain packets is normally distributed with mean 842 grams and variance 225 grams^2.
    Find the probability that a random sample of 120 packets has mass
    (i) Between 844 grams and 846 [0.0702]
    (ii) Less than 843 grams [0.7673]

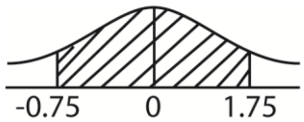
Solutions to revision questions 1

1. The amount of meat sold by a butcher is normally distributed with mean 43kg and standard deviation 4kg. Determine the probability that the amount of meat sold is between 40kg and 50kg.

    X ~N(43,4)

    $P(40<x<50) = P\left(\frac{40-43}{4} < Z < \frac{50-43}{4}\right)$

    $= P(-0.75 < Z < 1.75)$



P(40<x<50) = P(-0.75 < Z < 0) + P(0 < Z < 1.75)

By property of symmetry

P(40<x<50) = P(-0.75 < Z < 0) + P(0 < Z < 1.75)

= 0.2735 + 0.4599

= 0.733

2. Given that a random variable X is X~N(2, 2.89). Find P(X<0)

   μ = 2, σ = $\sqrt{2.89}$ = 1.7

   P(X < 0) = $P(Z < \frac{0-2}{1.7})$ = P(Z < - 1.176) = P(X> 1.176)

   = 0.5 – P(0 < Z < 1.176)

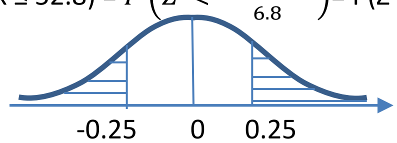
   = 0.5 – 0.3802 = 0.1198

3. In a school of 800 students their average weight is 54.5kg and standard deviation 6.8kg. given that the weight of students are normally distributed, find

   (i) Probability that the weight of any student randomly selected is 52.8 kg or less

   Let x be the weight of the student

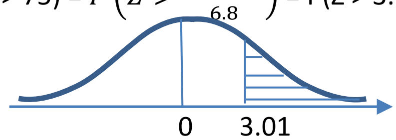
   P(x ≤ 52.8) = $P\left(Z < \frac{52.8-54.5}{6.8}\right)$= P(Z <-0.25)



   = P(Z> 0.25) = 0.5- P(0 < Z < 0.25) = 0.5 -0.0987 = 0.4013

   (ii) Number of students who weigh over 75kg = 1

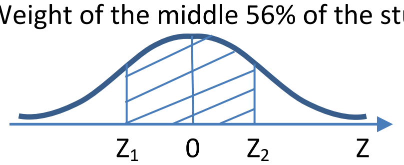
   P(Z > 75) = $P\left(Z > \frac{75-54.5}{6.8}\right)$ = P(Z > 3.01)



   P(Z > 3.01) = 0.5 – P(0 < Z < 3.01) = 0.5 – 0.4990= 0.001

   Number of students who weigh more than 75g = 800 x 0.001 =1

   (iii) Weight of the middle 56% of the students



   $P(X_1 < X < X_2) = P(Z_1 < Z < Z_2) = 0.56$

   But $P(0 < Z < Z_2) = 2.8$; $Z_2 = 0.772$ and $Z_1 = -0.772$

   $Z_1 = \frac{x_1-54.5}{6.8}$

   $-0.772 = \frac{x_1-54.5}{6.8}$; $x_1 = 49.251$

   $Z_2 = \frac{x_1-54.5}{6.8}$

   $0.772 = \frac{x_1-54.5}{6.8}$; $x_2 = 59.750$

   Hence the weight range of the middle 56% of students of the school is 49.251< X < 59.750

4. A sugar factory sells sugar in bags of mean weight 50kg and standard deviation 2.5kg. given that the weight of the bags is normally distributed, find the

(i) Probability that the weight of any bag of sugar randomly selected lies between 51.5kg and 53kg

P(51.5 < X < 53) = $\frac{51.5-50}{2.5} < Z < \frac{53-50}{2.5}$ = P(0.6 < Z 1.2)

= ϕ(1.2) – ϕ(0.6) = 0.3849 – 0.2257== 0.1592

(ii) Percentage of bags whose weight exceeds 54kg

P(X > 54) =P($Z > \frac{54-50}{2.5}) = P(Z > 1.6)$= 0.5 –ϕ(1.6) = 0.5 – 0.4452 = 0.0548

= 0.0548 x 100 = 5.48%

(iii) Number of bags that will be rejected out of 1000 bags purchased for weighing below 45.0kg

P(X < 45) = P($Z < \frac{45-50}{2.5}) = P(Z < -2) = P(Z < 2)$ = 0.5 –ϕ(2)= 0.5- 0.4772 = 0.0228

Number of bags rejected = 0.0228 x 1000 = 22.8 ≈23

5. A certain maize firm sells maize in bags of mean weight 40kg and standard deviation 2kg, given that the weight of the bags are normally distributed, find

(i) Probability that the weight of any bag of maize randomly selected lies between 41.0 and 42.5kg

P(41.0 < X < 42.5) = $\frac{41-40}{2} < Z < \frac{42.5-40}{2}$ = P(0.5 < Z 1.25)

= ϕ(1.25) – ϕ(0.5) = 0.3944 - 0.1915 = 0.2029

(ii) Percentage of bags whose weight exceeds 43kg

P(X > 43) =P($Z > \frac{43-40}{2}) = P(Z > 1.5)$= 0.5 –ϕ(1.5) = 0.5 – 0.4332 = 0.0668

= 0.0668 x 100 = 6.68%

(iii) Number of bags that will be rejected out of 500 bags purchased for weighing below 38.5kg

P(X < 38.5) = P($Z < \frac{38.5-40}{2}) = P(Z < -0.77) = P(Z < 0.75)$ = 0.5 –ϕ(0.75)

= 0.5- 0.2734 = 0.2266

Number of bags rejected = 0.2266 x 500 = 113

7. If X~N(50, 20). Find

(i) P(X > 60.3)

P(X > 60.3) = P(Z > $\frac{60.3-50}{\sqrt{20}}$) = P(Z > 2.303) = 05 –ϕ(2.303)

= 0.5 –(0.4893 + 0.0001) = 0.0106

(ii) P(X < 47.3)

P(X < 47.3) P(Z > $\frac{47.3-50}{\sqrt{20}}$) = P(Z <-0.6037) =P(Z> 0.6037) = 05 –ϕ(0.6037)

= 0.5 –(0.2257 + 0.0013) = 0.273

(iii) P(X > 48.9)

P(X < 48.9) = P(Z > $\frac{48.9-50}{\sqrt{20}}$) = P(Z <-0.246) = P(Z < 0.246) = 0.5 +ϕ(0.246)

= 0.5 + 0.0948 + 0.0022 = 0.597

(iv) P(X > 53.5)

P(X > 53.5) = P(Z > $\frac{53.5-50}{\sqrt{20}}$) = P(Z > 0.783) =0.2823 + 0.0008 = 0.2831

(v) P(X < 59.8)

P(X < 59.8) = P(Z < $\frac{59.8-50}{\sqrt{20}}$) = P(Z < 2.191) = 0.5 + ϕ(2.191)

$= 0.5 + 0.4826 + 0.0001 = 0.9857$

(vi) P(X < 62.3)

$P(X < 62.3) = P(Z < \frac{62.3-50}{\sqrt{20}}) = P(Z < 2.750) = 0.5 + \phi(2.730$

$= 0.5 + 0.4970 = 0.9970$

Thank You

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