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Subsidiary Mathematics

SENIOR FIVE term 2

TOPIC 3/4: Permutations and Combinations

Permutations and combinations

Permutation

A permutation is an ordered arrangement of a number of objects

Consider digits 1, 2 and 3; find the possible arrangements of the digits

123, 132, 321 }
231, 321, 321 } The total of six

This problem may also be solved as follows:

Given the three digits above, the first position can take up three digits, the second position can take up two digits and the third position can take up 1 digit only

1 st position	2 nd position	3 rd position
3	2	1

The total is thus $3 \times 2 \times 1 = 6$

If the digits were four say 1, 2, 3, 4 the arrangement would be

1 st	2 nd	3 rd	4 th
4	3	2	1

The total is thus $4 \times 3 \times 2 \times 1 = 24$

In summary the number of ways of arranging n different items in a row is given by $n(n-1)(n-2)(n-3) \times \dots \times 2 \times 1$ and can be expressed as $n!$

If the total number of books is 6

The total number of arrangements = $6!$

= $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways

Example 1

Find the values of the following expression

(a) $5!$

Solution

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

(b) $\frac{8!}{5!}$

Solution

$$\frac{8!}{5!} = \frac{8 \times 7 \times 6 \times 5!}{5!} = 336$$

$$(c) \frac{10!}{6! \times 5! \times 2!}$$

Solution

$$\frac{10!}{6! \times 5! \times 2!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{6! \times 5 \times 4 \times 3 \times 2 \times 1 \times 2 \times 1} = 21$$

- (d) Four different pens and 5 different books are to be arranged on a row. Find
 (i) The number of possible arrangements of items

Solution

$$\text{Total number of items} = 4 + 5 = 9$$

$$\text{Total number of arrangements} = 9!$$

$$= 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 362,880 \text{ ways}$$

- (ii) The number of possible arrangements if the pens must be kept together

Solution

The pen is taken to be one since they are to be kept together. So we consider total number of items to six. The number of arrangements of six items = $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$$= 720 \text{ ways}$$

The arrangement of 4 pens = $4!$

$$= 4 \times 3 \times 2 \times 1 = 24$$

Total number arrangements of all the items = $720 \times 24 = 17,280$

Example 2

- (a) How many arrangements can be made using the letters in the word "TROTTING"?

Solution

TROTTING has 8 letters and 3T'2

$$\text{Number of arrangements} = \frac{8!}{3!} = 6720$$

- (b) In how many of these arrangements are the letters N and G next to each other.

Solution

Since N and G are always together, they are counted as one letter

=> TROTTING has 7 letters and 3T'2

$$\text{Number of arrangements} = \frac{7!}{3!} = 840$$

$$\text{Number of arrangements of N and G} = 2! = 2$$

$$\text{Total number of arrangements} = 840 \times 2 = 1680$$

Multiplication principle of permutation

If one operation can be performed independently in **a** different ways and the second in **b** different ways, then either of the two events can be performed in **(a + b)** ways

Example 3

There are 6 roads joining P to Q and 3 roads joining Q to R. Find how many possible routes are from P to R

From P to Q = 6 ways

From Q to R = 3 ways

$$\text{Number of routes from P to R} = 6 \times 3 = 18$$

Example 4

Peter can eat either matooke, rice or posh on any of the seven days of the week. In how many ways can he arrange his meals in a week?

Solution

For each of the 7 days, there are 3 choices

Total number of arrangements

$$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7 = 2187 \text{ ways}$$

Example 5

There are four routes from Nairobi to Mombasa. In how many different ways can a taxi go from Nairobi to Mombasa and returning if for returning?

- (a) any of the route is taken

$$= 4 \times 4 = 16 \text{ ways}$$

- (b) the same route is taken
 $= 4 \times 1 = 4$ ways
 (c) the same route is not taken
 $= 4 \times 3 = 12$ ways

Example 6

David can arrange a set of items in 5 ways and John can arrange the same set of items in 3 ways. In how many ways can either David or John arrange the items?

Solution

Number of ways in which David arranges = 5

Number of ways in which John arranges = 3

Number of ways in which either David or John arrange the items = $5 + 3 = 8$ ways

The number of permutation of r objects taken from n unlike objects

The permutation of n unlike objects taking r at a time is denoted by ${}^n P_r$ which is defined as

$${}^n P_r = \frac{n!}{(n-r)!}, \text{ where } r \leq n.$$

In case $r = n$, we have ${}^n P_n$ which is interpreted as the number of arranging n chosen objects from n objects denoted by n!

$${}^n P_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n! \Rightarrow 0! = 1$$

Example 7

How many three letter words can be formed from the sample space {a, b, c, d, e, f}

Solution

Total number of letters = 6 and $r = 3$

Total number of words = ${}^6 P_3$

$$= \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times 3!}{3!} = 120 \text{ways}$$

Example 8

Three letters are chosen at random from the word CLOTHINGS. Determine the probability that two of the three letters chosen are consonants. (05markks)

Solution

Total number of letters in CLOTHINGS = 9 letters

Selection of 3 teachers from 9 letters ${}^9 C_3 = 84$

Possible selection

7 consonants	2 Vowels
2	1

$$\begin{aligned} \text{Number of selections} &= {}^7 C_2 \cdot {}^2 C_1 \\ &= 21 \times 2 \\ &= 42 \end{aligned}$$

$$\text{Probability} = \frac{42}{84} = \frac{1}{2}$$

Example 9

Find the possible number of ways of arranging 3 letters from the word MANGOES

Solution

Total number of letter in the word = 7

and $r = 3$

$$\text{Number of ways } {}^7 P_3 = \frac{7!}{(7-3)!}$$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3!}{3!} = 840 \text{ways}$$

Example 10

Find number of ways of arranging six boys from a group of 13

Solution

Number of arrangements = ${}^{13} P_6$

$$= \frac{13!}{(13-6)!} = \frac{13!}{7!}$$

$$= \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7!}{7!}$$

$$= 1235520 \text{ ways}$$

The number of permutations of n objects of which r are alike

The number of permutations of n objects of which r are alike is given by $\frac{n!}{r!}$

Example 11

Find the number of arranging in a line the letters B, C, C, C, C, C, C

The number of ways of arranging the seven letters of which of which 6 are alike = $\frac{7!}{6!} = \frac{7 \times 6!}{6!} = 7 \text{ ways}$

The number of ways of permutations on n objects of which p of one type are alike, q of the second type are alike, r of the third type are alike, and so on.

The number of ways of permutations on n objects of which p of one type are alike, q of the second type are alike, r of the third type are alike given by $\frac{n!}{p! \times q! \times r!}$

Example 12

Find the possible number of ways of arranging the letter of the word MATHEMATICS in line

Solution

The word MATHEMATICS has 11 letters and contains 2 M, 2A and 2T repeated

$$\begin{aligned} \text{The number of ways} &= \frac{11!}{2! \times 2! \times 2!} \\ &= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 2 \times 2} \\ &= 4,989,600 \end{aligned}$$

Example 13

Find the possible number of ways of arranging the letter of the word 'MISSISSIPPI' in line

Solution

The word 'MISSISSIPPI' has 10 letters with 4I, 4S, and 2P

$$\text{The number of ways} = \frac{10!}{4! \times 4! \times 2!} = 34650$$

The number of permutations of the like and unlike objects with restrictions

One should be cautious when handling these problems

Example 14

Find the possible number of ways of arranging The letters of the word MINIMUM if the arrangement begins with MMM?

Solution

There is only one way of arranging MMM

The remaining contain four letters with 2I can be arranged in

$$\frac{1 \times 4!}{2!} = \frac{1 \times 4 \times 3 \times 2!}{2!} = 12 \text{ ways}$$

Example 15

- (a) How many 4 digit number greater than 6000 can be formed from 4, 5, 6, 7, 8 and 9 if:
 - (i) Repetitions are allowed

Solution

The first digit can be chosen from 6, 7, 8 and 9, hence 4 possible ways, the 2nd, 3rd and 4th are chosen from any of the six digits since repetitions are allowed

position	1 st	2 nd	3 rd	4 th
selections	4	6	6	6

$$\begin{aligned} \text{Number of ways} \\ &= 4 \times 6 \times 6 \times 6 = 864 \text{ ways} \end{aligned}$$

- (ii) Repetition are not allowed

The first can be chosen from 6, 7, 8 and 9, hence 4 possible ways, the 2nd from

5, 3rd from 4 and 4th from 3 since no repetitions are allowed

position	1 st	2 nd	3 rd	4 th
selections	4	5	4	3

Number of ways
= $4 \times 5 \times 4 \times 3 = 240$ ways

- (b) Find how many four digit numbers can be formed from the six digits 2, 3, 5, 7, 8 and 9 without repeating any digit.

Find also how many of these numbers

- (i) Are less than 7000
(ii) Are odd

Solution

position	1 st	2 nd	3 rd	4 th
selections	6	5	4	3

Total number of ways = $6 \times 5 \times 4 \times 3 = 360$

- (i) The 1st number is selected from three (2, 3, 5), the 2nd number from 5, the 3rd from 4 and the 4th from 3 digits

position	1 st	2 nd	3 rd	4 th
selections	6	5	4	3

Total number of less than 7000

$$= 3 \times 5 \times 4 \times 3 = 180$$

- (ii) The last number is selected from four odd digits (3, 5, 7, and 9), the 1st number selected from five remaining, 2nd from 4 and 3rd from 3

position	1 st	2 nd	3 rd	4 th
selections	5	4	3	4

Total number of odd numbers formed

$$= 5 \times 4 \times 3 \times 4 = 240$$

- (c) How many different 6 digit number greater than 500000 can be formed by using the digits 1, 5, 7, 7, 7, 8

Solution

The 1st digit is selected from five (5, 7, 7, 7, 8), the 2nd from remaining five, 3rd from four, 4th from three, 5th from two and 6th from one

1 st	2 nd	3 rd	4 th	5 th	6 th
5	5	4	3	2	1

$$\text{Total number} = \frac{5 \times 5 \times 4 \times 3 \times 2 \times 1}{3!} = 100$$

NB. The number is divided by 3! Because 7 appears three times

- (d) How many odd numbers greater than 60000 can be formed from 0, 5, 6, 7, 8, 9, if no number contains any digit more than once

Solution

Considering six digits

Taking the first digit to be odd, the first digit is selected from 3 digits (5, 7, 9) and the last is selected from 2 digits

1 st	2 nd	3 rd	4 th	5 th	6 th
3	4	3	2	1	2

$$\text{Number of ways} = 3 \times 4 \times 3 \times 2 \times 1 \times 2 = 144$$

Taking the first digit to be even, the first digit is selected from 2 digits (6, 8) since the number should be greater than 60000 and the last is selected from 2 odd digits

1 st	2 nd	3 rd	4 th	5 th	6 th
2	4	3	2	1	3

$$\text{Number of ways} = 3 \times 4 \times 3 \times 2 \times 1 \times 2 = 144$$

Considering five digits

Taking the first digit to be odd, the digit greater than 6 are 7 and 9 so first digit is

selected from 2 digits and the last is selected from 2 digits

1 st	2 nd	3 rd	4 th	5 th
2	4	3	2	2

$$\begin{aligned} \text{Number of ways} &= 2 \times 4 \times 3 \times 2 \times 2 \\ &= 96 \end{aligned}$$

Taking the first digit to be even, the first digit is selected from 2 digits (6, 8) since the number should be greater than 60000 and the last is selected from 3 odd digits (5, 7, 9)

1 st	2 nd	3 rd	4 th	5 th
2	4	3	2	3

$$\begin{aligned} \text{Number of ways} &= 2 \times 4 \times 3 \times 2 \times 3 \\ &= 144 \end{aligned}$$

$$\begin{aligned} \text{The total number of selections} \\ &= 144 + 144 + 96 + 144 = 528 \end{aligned}$$

Example 16

The six letter of the word LONDON are each written on a card and the six cards are shuffled and placed in a line. Find the number of possible arrangements if

- (a) The middle two cards both have the letter N on them

Solution

If the middle letter are NN, then we need to find the number of different arrangements of the letter LODO.

With the 2O's, the number of arrangements = $\frac{4!}{2!} = 12$

- (b) The two cards with letter O are not adjacent and the two cards with letter N are also not adjacent

Solution

If the two cards are not adjacent, the number of arrangements = Total number of arrangements of the word

$$\begin{aligned} \text{LONDON} &- \text{number of arrangements} \\ &\text{when the two letters are adjacent} \\ &= \frac{6!}{2!2!} - 24 = 156 \end{aligned}$$

Example 17

In how many different ways can letters of the word MISCHIEVERS be arranged if the S's cannot be together

Solution

There are 11 letters in the word MISCHIEVERS with 2S's, 2I's and 2E's

Total number of arrangements

$$= \frac{11!}{2!2!2!} = 4989600$$

If S's are together, we consider them as one, so the number of arrangements

$$= \frac{10!}{2!2!} = 907200$$

$$\begin{aligned} \therefore \text{the number of possible arrangements of the} \\ \text{word MISCHIEVERS when S's are not together} \\ &= 4989600 - 907200 = 4082400 \end{aligned}$$

The number of permutation of n different objects taken r at a time, if repetition are permitted

Example 18

How many four digit numbers can be formed from the sample space {1, 2, 3, 4, 5} if repetitions are permissible

Solution

The 1st position has five possibilities, the 2nd five, the 3rd five, the 4th five

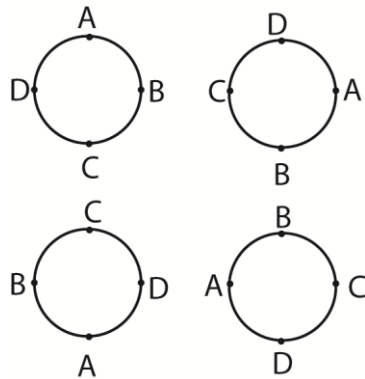
$$\text{Number of permutations} = 5 \times 5 \times 5 \times 5 = 625$$

Circular permutations

Here objects are arranged in a circle

The number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise are different.

Consider four people A, B, C and D seated at a round table. The possible arrangements are as shown below



With circular arrangements of this type, it is the relative positions of the objects being arranged which is important. The arrangements of the people above is the same. However, if the people were seated in a line the arrangements would not be the same, i.e. A, B, C, D is not the same as D, A, B, C. When finding the number of different arrangements, we fix one person say A and find the number of ways of arranging B, C and D.

Therefore, the number of different arrangements of four people around the table is $3!$

Hence the number of different arrangements of n people seated around a table is $(n - 1)!$

Example 19

- (a) Seven people are to be seated around a table, in how many ways can this be done
Solution

$$\begin{aligned} \text{The number of ways} &= (7 - 1)! = 6! \\ &= 720 \end{aligned}$$

- (b) In how many ways can five people A, B, C, D and E be seated at a round table if
(i) A must be seated next to B

Solution

If A and B are seated together, they are taken as bound together. So four people are considered

$$\text{The number of ways} = (4 - 1)! = 3! = 6$$

The number of ways in which A and B can be arranged = 2

The total number of arrangements

$$= 6 \times 2 = 12 \text{ ways}$$

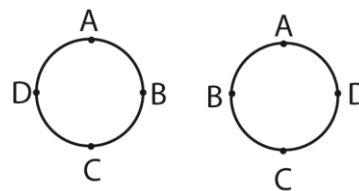
- (ii) A must not seat next to B

If A and B are not seated together, then the number of arrangements = total number of arrangements – number of arrangements when A and B are seated together

$$= (5 - 1)! - 12 = 12 \text{ ways}$$

The number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise arrangements are the same

Consider the four people above, if the arrangement is as shown below



Then the above arrangements are the same since one is the other viewed from the opposite side

$$\text{The number of arrangements} = \frac{3!}{2} = 3 \text{ ways}$$

Hence the number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise arrangements are the same = $\frac{(n-1)!}{2}$

Example 20

A white, a blue, a red and two yellow cards are arranged on a circle. Find the number of arrangements if red and white cards are next to each other.

Solution

If red and white cards are next to each other, they are considered as bound together. So we have four cards. Since anticlockwise and clockwise arrangements are the same and there are two yellow cards, the number of

$$\text{arrangements} = \frac{(4-1)!}{2 \times 2!} = \frac{3!}{2 \times 2!}$$

The number of ways of arranging red and white cards = 2

Total number of ways of arrangements

$$= \frac{3!}{2 \times 2!} \times 2 = 3$$

Revision exercise 1

1. In how many ways can the letters of the words below be arranged
(a) Bbosa [5!]
(b) Precious [8!]
2. How many different arrangements of the letters of the word PARALLELOGRAM can be made with A's separate [83160000]
3. How many different arrangements of the letters of the word CONTACT can be made with vowels separated? [900]
4. How many odd numbers greater than 6000 can be formed using digits 2, 3, 4, 5 and 6 if each digit is used only once in each number [12]
5. Three boys and five girls are to be seated on a bench such that the eldest girl and eldest boy sit next to each other. In how many ways can this be done [2 x 7!]
6. A round table conference is to be held between delegates of 12 countries. In how many ways can they be seated if two

particular delegates wish to sit together [2 x 10!]

7. In how many ways can 4 boys and 4 girls be seated at a circular table such that no two boys are adjacent [144]
8. How many words beginning or ending with a consonant can be formed by using the letters of the word EQUATION? [4320]

Combinations

A combination is a selection of items from a group not basing on the order in which the items are selected

Consider the letters A, B, C, D

The possible arrangements of two letters chosen from the above letters are

AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC.

As seen earlier, the total number of arrangements of the above letters is expressed

$$\text{as } \frac{4!}{(4-2)!} = \frac{4!}{2!} = 12$$

However, when considering combinations, the grouping such as AB and BA are said to be the same groupings such as CA and AC, AD and DA, etc.

So the possible combinations are AB, AC, AD, BC, BD, CD which is six ways.

Thus the number of possible combinations of n items taken r at a time is expressed as ${}^n C_r$ or $\binom{n}{r}$ which is defined as ${}^n C_r = \frac{n!}{(n-r)!r!}$ where $r \leq n$

Hence the number of combinations of the above letters taken two at a time is

$$\binom{4}{2} = \frac{4!}{2!2!} = 6$$

Example 21

Three letters are chosen at random from the word CLOTHINGS. Determine the probability that two of the three letters chosen are consonants.

Solution

Total number of letters in CLOTHINGS = 9 letters

Selection of 3 teachers from 9 letters ${}^9C_3 = 84$

Possible selection

7 consonants	2 Vowels
2	1

Number of selections = ${}^7C_2 \cdot {}^2C_1$

$$= 21 \times 2$$

$$= 42$$

$$\text{Probability} = \frac{42}{84} = \frac{1}{2}$$

Example 22

A committee of four people is chosen at random from a set of seven men and three women

How many different groups can be chosen if there is at least one

(i) Woman on the committee

Solution

Possible combinations

7 men	3 women
3	1
2	2
1	3

The number of ways of choosing at least one woman

$$= \binom{7}{3} \times \binom{3}{1} + \binom{7}{2} \times \binom{3}{2} + \binom{7}{1} \times \binom{3}{3}$$

$$= \frac{7!}{3!4!} \times \frac{3!}{1!2!} + \frac{7!}{2!5!} \times \frac{3!}{2!1!} + \frac{7!}{1!6!} \times \frac{3!}{3!0!} = 175$$

(ii) Man on the committee

7 men	3 women
1	3
2	2
3	1
4	0

The number of ways of choosing at least one man

$$\binom{7}{1} \times \binom{3}{3} + \binom{7}{2} \times \binom{3}{2} + \binom{7}{3} \times \binom{3}{1} + \binom{7}{4} \times \binom{3}{0}$$

$$= \frac{7!}{1!3!} \times \frac{3!}{0!3!} + \frac{7!}{5!2!} \times \frac{3!}{1!2!} + \frac{7!}{4!3!} \times \frac{3!}{2!1!} + \frac{7!}{3!4!} \times \frac{3!}{3!0!}$$

$$= 210$$

Example 23

A group of nine has to be selected from ten men and eight women. It can consist of either five men and four women or four men and five women. How many different groups can be chosen?

Solution

Possible combination

10 men	8 women
5	4
4	5

$$\text{Number of groups} = \binom{10}{5} \times \binom{8}{4} + \binom{10}{4} \times \binom{8}{5}$$

$$= \frac{10!}{5!5!} \times \frac{8!}{4!4!} + \frac{10!}{6!4!} \times \frac{8!}{5!3!} = 29400$$

Example 24

A team of six is to be formed from 13 boys and 7 girls. In how many ways can the team be selected if it must consist of

(a) 4 boy and 2 girls

13 boys	7 girls

4	2
---	---

$$\binom{13}{4} \cdot \binom{7}{2} = \frac{13!}{9!4!} \times \frac{7!}{5!2!} = 15015$$

(b) At least one member of each sex

Possible combinations

13 boys	7 girls
5	1
4	2
3	3
2	4
1	5

$$= \binom{13}{5} \cdot \binom{7}{1} + \binom{13}{4} \cdot \binom{7}{2} + \binom{13}{3} \cdot \binom{7}{3} + \binom{13}{2} \cdot \binom{7}{4} + \binom{13}{1} \cdot \binom{7}{5}$$

$$= \frac{13!}{8!5!} \cdot \frac{7!}{6!1!} + \frac{13!}{9!4!} \cdot \frac{7!}{5!2!} + \frac{13!}{10!3!} \cdot \frac{7!}{4!3!} + \frac{13!}{11!2!} \cdot \frac{7!}{3!4!} + \frac{13!}{12!1!} \cdot \frac{7!}{2!5!}$$

$$= 37037$$

Example 25

A team of 11 players is to be chosen from a group of 15 players. Two of the 11 are to be randomly elected a captain and vice-captain respectively. In how many ways can this be done?

Number of ways of choosing 11 players from 15

$$= \binom{15}{11}$$

A captain will be elected from 11 players and a vice-captain from 10 players

Total number of selection = $\binom{15}{11} \times 11 \times 10$

$$= \frac{15!}{4!11!} \times 11 \times 10 = 150150$$

Example 26

(a) Find the number of different selections of 4 letters that can be made from the word UNDERMATCH.

Solution

There are 10 letters which are all different
Number of selections of 4 letters from 10 is given by $\binom{10}{4} = \frac{10!}{(10-4)!4!} = \frac{10!}{6!4!} = 210$

(b) How many selections do not contain a vowel?

Solution

Number of vowels in the word = 2

Number of letters not vowels = 8

Number of selections of 4 letters from 10 without containing a vowel = selecting 4

letters from 8 consonants =

$$\binom{8}{4} = \frac{8!}{(8-4)!4!} = \frac{8!}{4!4!} = 70$$

Example 27

In how many ways can three letters be selected at random from the word BIOLOGY is selection

(a) Does not contain the letter O

Solution

Number of selections without the letter O = number of ways of choosing three letters from B, I, L, G, y

$$= \binom{5}{3} = \frac{5!}{2!3!} = 10$$

(b) Contain only the letter O

Solution

Number of selections with one letter O = number of ways of choosing two letters from B, I, L, G, y

$$= \binom{5}{2} = \frac{5!}{3!2!} = 10$$

(c) Contains both of the letters O

Solution

Number of selections with two letter O = number of ways of choosing one letter from B, I, L, G, y

$$= \binom{5}{1} = \frac{5!}{4!1!} = 5$$

Example 28

In how many ways can four letters be selected at random from the word BREAKDOWN if the letters contain at least one vowel?

Solution

Vowels: E, A, O (3)

Consonants: B, R, K, D, W, N (6)

Consonants (6)	Vowels (3)
3	1
2	2
1	3

Number of selection of four letters with at least one vowel

$$= \binom{6}{3} \cdot \binom{3}{1} + \binom{6}{2} \cdot \binom{3}{2} + \binom{6}{1} \cdot \binom{3}{3} = 111$$

Example 26

How many different selections can be made from the six digits 1, 2, 3, 4, 5, 6

Solution

Note: this an open questions because selections can consist of only one digit, two digits, three digits, four digits, five digits or six digits

$$\text{Number of selection of 1 digit} = {}^6C_1 = 6$$

$$\text{Number of selection of 2 digits} = {}^6C_2 = 15$$

$$\text{Number of selection of 3 digits} = {}^6C_3 = 20$$

$$\text{Number of selection of 4 digits} = {}^6C_4 = 15$$

$$\text{Number of selection of 5 digits} = {}^6C_5 = 6$$

$$\text{Number of selection of 6 digits} = {}^6C_6 = 1$$

Total number of selections

$$= 6 + 15 + 20 + 15 + 6 + 1 = 63$$

This approach is tedious for a large group of objects.

The general formula for selection from n unlike objects is given by $2^n - 1$.

For the above problems, number of selections = $2^6 - 1 = 63$

Example 29

How many different selections can be made from 26 different letters of the alphabet?

$$\begin{aligned} \text{Number of selection} &= 2^{26} - 1 \\ &= 67,108,863 \end{aligned}$$

Cases involving repetitions

Suppose we need to find the number of possible selections of letters from a word containing repeated letters, we take the selections mutually exclusive

Example 30

How many different selections can be made from the letters of the word CANADIAN?

Solution

There are 3A's, 2N's and 3 other letters

The A's can be dealt with in 4 ways (either no A, 1A's, 2A's or 3A's)

The N's can be dealt in 3 ways (no N, 1N, or 2N's)

The C can be dealt with in 2 ways (no C, 1C)

The D can be dealt with in 2 ways (no D, 1D)

The I can be dealt with in 2 ways (no I, 1I)

The number of selections

$$= 4 \times 3 \times 2 \times 2 \times 2 - 1 = 95$$

Example 31

How many different selections can be made from the letters of the word POSSESS?

Solution

There are 4S's and 3 other letters

The S's can be dealt in 5 ways (no S, 1S, 2N's, 3S's, 4S's, or 5S's)

The P can be dealt with in 2 ways (no P, 1P)

The O can be dealt with in 2 ways (no O, 1O)

The E can be dealt with in 2 ways (no E, 1E)

$$\begin{aligned}\text{Total number of selections} &= 5 \times 2 \times 2 \times 2 - 1 \\ &= 39\end{aligned}$$

Cases involving division into groups

The number of ways of dividing n unlike objects into say two groups of p and q where $p + q = n$ is given by $\frac{n!}{p!q!}$

For three groups of p , q and r provided $p + q + r = n$

$$\text{Number of ways of division} = \frac{n!}{p!q!r!}$$

However, for the two groups above, if $p = q$ then the number of ways of division = $\frac{n!}{p!p!2!}$

For three groups where $p = q = r$

$$\text{then the number of ways of division} = \frac{n!}{p!p!p!3!}$$

Example 32

The following letters a, b, c, d, e, f, g, h, i, j, k, l are to be divided into groups containing

- (a) 3, 4, 5
- (b) 5, 7
- (c) 6, 6
- (d) 4, 4, 4 letters. In how many ways can this be done?

Solution

- (a) Number of ways = $\frac{12!}{3!4!5!} = 27720$
- (b) Number of ways = $\frac{12!}{5!7!} = 792$
- (c) Number of ways = $\frac{12!}{6!6!2!} = 462$

(d) Number of ways = $\frac{12!}{4!4!4!3!} = 5775$

Example 33

Find the number of ways that 18 objects can be arranged into groups if there are to be

- (a) Two groups of 9 objects each
- (b) Three groups of 6 objects each
- (c) 6 groups of 3 objects each
- (d) Three groups of 5, 6 and 7 objects each

Solution

- (a) Number of ways = $\frac{18!}{9!9!2!} = 24310$
- (b) Number of ways = $\frac{18!}{6!6!6!3!} = 2858856$
- (c) Number of ways = $\frac{18!}{3!3!3!3!3!3!6!} = 190590400$
- (d) Number of ways = $\frac{18!}{5!6!7!} = 14702688$

Example 34

- (a) Find how many words can be formed using all letters in the word MINIMUM.

Solution

Number of ways of arranging the letters = $7!$

There are 3M's and 2I's

$$\text{Number of words formed} = \frac{7!}{3!2!} = 420$$

- (b) Compute the sum of four-digit numbers formed with the four digits 2, 5, 3, 8 if each digit is used only once in each arrangement

Solution

Number of ways of arranging a four digit number = $4!$

$$\text{Sum of any four digit number formed} = 2 + 5 + 3 + 8 = 18$$

Total sum of four digit numbers formed

$$= 18 \times 4! = 432$$

- (c) A committee consisting of 2 men and 3 women is to be formed from a group of 5

men and 7 women. Find the number of different committees that can be formed. If two of the women refuse to serve on the same committee, how many committees can be formed?

Solution

$$\begin{aligned} \text{The committees formed} &= {}^5C_2 \cdot {}^7C_3 \\ &= 10 \times 35 = 350 \end{aligned}$$

Suppose two women are to serve together, we take them as glued together, so the number of committees $= {}^5C_2 \cdot {}^6C_3 = 200$

Number of committees in which two women refuse to serve together $= 350 - 200 = 150$

Revision exercise 2

- Find the number of different selection of 3 letters that can be made from the word PHOTOGRAPH. [53]
 - How many of these selections contain no vowel [18]
 - How many of these selections contain at least one vowel? [35]
- find the number of different selections of 3 letters that can be made from the letters of the word SUCCESSFUL. [36]
 - How many of these selections contain only consonants [11]
 - How many of these selections contain at least one vowel [25]
- Find the value of n if ${}^nP_4 = 30 {}^nC_5$ [8]
- How many arrangement can be made from the letters of the name MISSISSIPPI
 - when all the letters are taken at a time [34650]
 - If the two letters PP begin every word [630 ways]
- Find the number of ways in which a one can chose one or more of the four girls to join a discussion group [15 ways]
- Find in how many ways 11 people can be divided into three groups containing 3, 4, 4 people each. [5775]
- A group of 5 boys and 8 girls. In how many ways can a team of four be chosen, if the team contains
 - No girl [5]
 - No more than one girl [85]
 - At least two boys [365]
- Calculate the number of 7 – letter arrangements which can be made with the letters of the word MAXIMUM. In how many of these do all the 4 consonants appear next to each other? [840, 96]
- In how many ways can a club of 5 be selected from 7 boys and 3 girls if it must contain
 - 3 boys and 3 girls [105]
 - 2 men and 3 girls [21]
 - At least one girl [231]
- How many different 6 digit numbers greater than 400,000 can be formed form the following digits 1, 4, 6, 6, 6 7? [100]

Thank You

Dr. Bbosa Science