Subsidiary Mathematics

SENIOR FIVE term 2

TOPIC 4/4: Time Series Analysis

Moving averages

Average = $\frac{sum\ of\ value}{number\ of\ value}$

Moving averages are a series of averages of values

Order of moving averages is number of values required to form an average.

Calculation of moving averages

(a) Moving total is the sum of n values, the first moving average is written against the (n-1)th value.

(b) First moving averages = $\frac{sum\ of\ first\ n\ values}{order\ n}$

(c) Second moving averages = $\frac{sum\ of\ next\ n\ values, excluding\ the\ first\ value}{order\ n}$.

(d) Third moving averages = $\frac{sum\ of\ next\ n\ values, excluding\ the\ first\ \ two\ value}{order\ n}$.

(e) This process continues up to last value of the data

Example 1

The table below shows the enrollment of students in an institution over a period of 5 years.

Year	2003	2004	2005	2006	2007
Number of students	145	182	170	155	213

Calculate

(a) three-year moving averages. (03marks)

Solution

- first three-year moving total = 145 + 185 + 170 = 497
- First three-year moving average = $\frac{145+182+170}{3} = 165.7$
- Second three-year moving total =182+170+155 =507
- Second three-year moving averages = $\frac{507}{3}. = 169.0$
- Third three-year moving total =170+155+213 =538
- Third three-year moving averages = $\frac{538}{3}. = 179.3$

The values are filled in the table as shown below

Year	Tax	Moving totals	
2003	145		
2004	182	497	165.7
2005	170	507	169.0
2006	155	538	179.3
2007	213		
208	x		

(b) number of students enrolled in 2008, given that the fourth moving average is 203. (02 marks)

Let the number enrolled in 2008 be

Fourth three-year moving averages = $\frac{155+213+x}{3}. = 203$

⇨ $368 + x$ = 203 x 3

$x = 609 - 368 = 241$

Example 2

The table below shows quarterly sales of cars for the year 2000, 2001 and 2002 by a company.

YEAR	QUARTER			
	1st	2nd	3rd	4th
2000	390	310	280	355
2001	420	320	305	410
2002	400	350	315	425

(a) Calculate a four point moving average for the data. (06marks)

First four point moving total = 390+310+280+355 =1335

First four point moving average =$\frac{1335}{4}$ =333.75

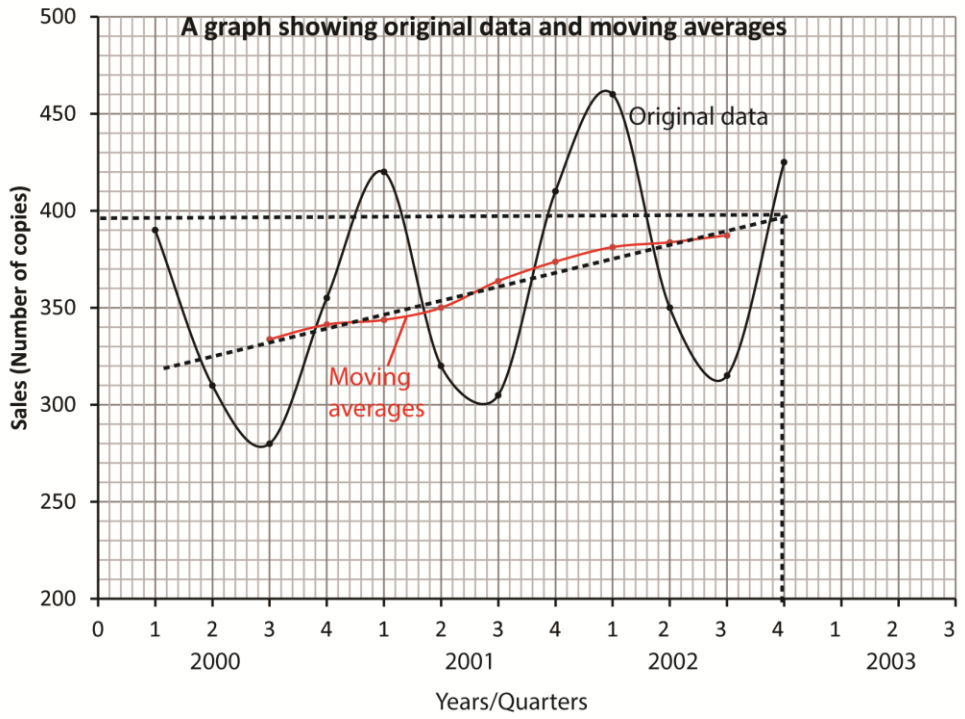
Second four point moving total = 310+280+355+420 =1365

Second four point moving average =$\frac{1365}{4}$ =341.35

The calculations are carried on, the values filled into the table as shown below

Year	Quarter	Sales	Moving totals	Moving averages
2000	1	390		
	2	310		
	3	280	1335	333.75
	4	355	1365	341.35
2001	1	420	1375	343.75
	2	320	1400	350
	3	305	1455	363.75
	4	410	1495	373.75
2002	1	460	1525	381.25
	2	350	1535	383.75
	3	315	1550	387.3
	4	425		
2003	1	x		

(b) (i) Plot a four – point moving average for the data on the same axes. (06marks)



(ii) Comment on the trend of the sales of the cars. (01 mark)

There is a general increase in the sale of cars with the years

(iii) Use your graph to estimate the number of cars sold in the first quarter of 2003. (02marks)

The estimated 10^{th} moving average = 395 cars

Let the number of cars sold in the 1^{st} quarter of 2003 be x

$$\frac{x+425+315+350}{4} = 395$$

x + 1,115 = 395 x 4 =1580

x = 490

Example 3

The table below shows the 3-month moving averages for the quantity of good (in tonnes) manufactured by a certain company from January to August of 2019.

Month	February	March	April	May	June	July
3-month Moving Average (tonnes)	15	17.5	19	20	21.5	22.5

(a) Find the moving totals. (03marks)

Solution

Month	Tax	Moving totals	
Jan			
February		45	15
March		52.5	17.5
April		57	19
May		60	20
June		64.5	21.5
July		67.5	22.5

(b) If 20 tonnes and 10 tonnes of goods were manufactured in February and March respectively, calculate the quantity that was manufactured in January. (02 marks)

Solution

Let the quantity manufactured in January be x

$x + 20 + 10 = 45$

$x + 30 = 45$

$x = 15$

Example 5

The table shows the number of bags of sugar sold by a certain wholesale shop from the year 2009 to 2020

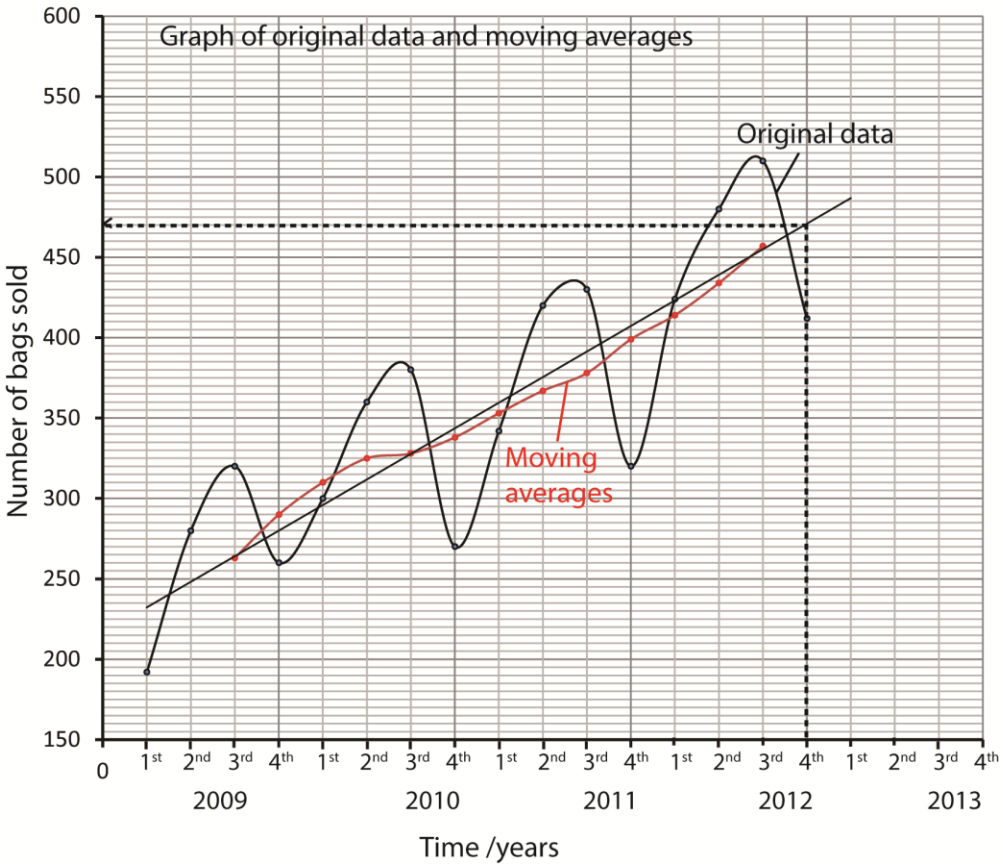
YEAR	QUARTER			
	1ST	2nd	3rd	4th
2009	192	280	320	260
2010	300	360	380	270
2011	342	420	430	320
2012	424	480	510	412

(a) Calculate the four-point averages for the data (06marks)

Year	Quarter	Bags sold	Moving Totals	Moving averages
2009	1st	192		

	2nd	280		
	3rd	320	1052	263
	4th	260	1160	290
2010	1st	300	1240	310
	2nd	360	1300	325
	3rd	380	1310	328
	4th	270	1352	338
2011	1st	342	1412	353
	2nd	420	1462	367
	3rd	430	1512	378
	4th	320	1594	399
2012	1st	424	1654	414
	2nd	480	1734	434
	3rd	510	1826	457
	4th	412		
2013	1st	x		

(b) (i) On the same axes, plot the original data and the four-point moving averages. (05 marks)



(ii) Comment on the trends of the number of bags of sugar sold over the four year period. (01 mark)

There is a general increase in the number of bags sold with increase in time

(iii) Use your graph to estimate the number of bags to be sold in the first quarter of 2013. (03marks)

The estimates 14^{th} moving average = 460

Let the number of bags sold in the first quarter of 2013 be x

$\frac{x+480+510+412}{4} = 470$

x = 1,880 – 1402 = 478

Example 6

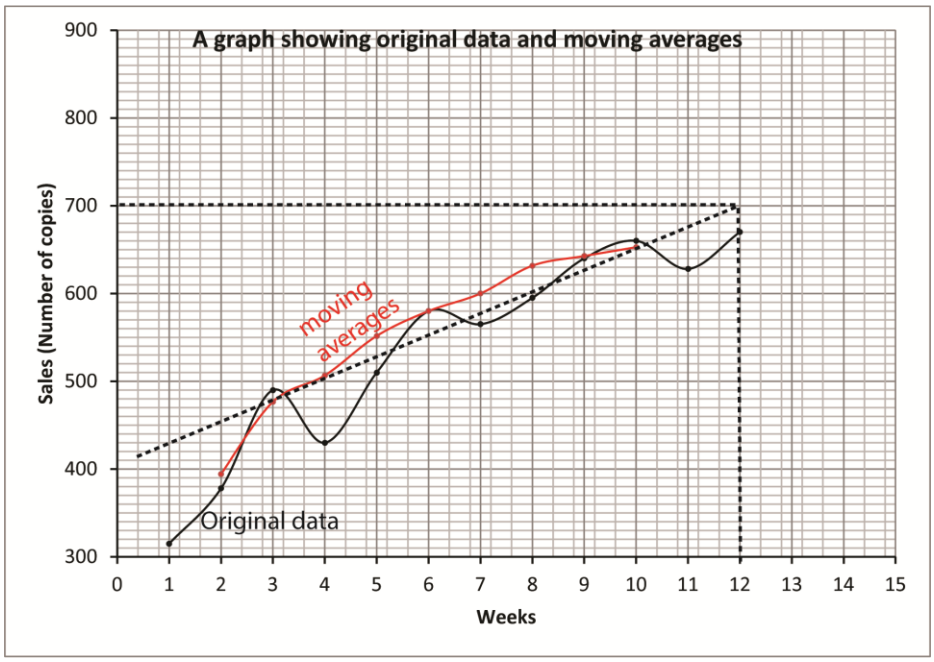
The table shows the sales in thousands of copies by a local Newspaper over a period of 12 weeks.

Week	1	2	3	4	5	6	7	8	9	10	11	12
Number of copies sold	135	378	490	430	510	580	565	595	640	660	628	670

(a) Calculate the 3-weeks moving averages for the copies sold. (06marks)

week	Number of copies	Moving totals	Moving averages
1	315		
2	378	1183	394.3
3	490	1298	476.7
3	430	1430	476.7
5	510	1520	506.7
6	580	1655	551.7
7	565	1740	580
8	595	1800	600
9	640	1895	631.7
10	660	1928	642.7
11	628	1958	652.7
12	670		
13	x		

(b) (i) On the same axes, plot the original data and the 3-weeks moving averages. (06marks)



(ii) Use your graphs to estimate the number of copies sold in the 13th week. (03marks)

From the graph the 11th moving average = 700

Let x be the number of copies sold in the 13th week

$\frac{x+628+670}{3}$ = 720; x = (700x3) – (628 + 670) = 802

Example 7

The table below shows the tax collection of a town council in millions of shillings for six consecutive months.

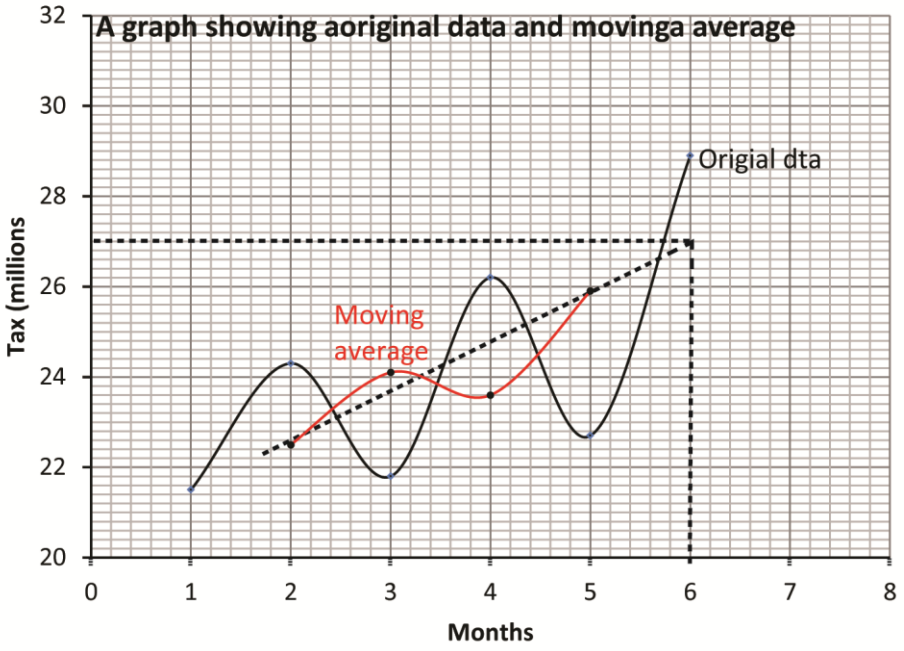
Month	Jan	Feb	Mar	Apr	May	June
Tax (in millions)	21.5	24.3	21.8	26.2	22.7	28.9

(a) Construct the 3-month moving average for the given data. (06marks)

Solution

Month	Jan	Feb	March	April	May	June	**July**
Tax	21.5	24.3	21.8	26.2	22.7	28.9	**x**
Moving totals		67.6	72.3	70.7	77.8		
Moving averages		22.5	24.1	23.6	25.9		

(b) Plot the 3-month moving averages and the original data on the same axes. (06 marks)



(c) Use your graph to estimate the town council's tax collection for the month of July. (03marks)

From the graph the moving average for June = 27 millions

If x are the sales for July

Then $\frac{x+22.7+28.9}{3}$ = 27

x + 51 = 81

x = 30

Defining the trend of the moving averages

The trend of moving average is indicated by the line of the best fit. For the sake of uniformity, the line of the best fit is drawn through the first and last moving average.

(i) A line of the best fit sloping up from left to right shows a general increase in production, sales, performance etc.

(ii) A line of the best fit sloping down from left to right shows a general decrease in production, sales, performance etc.

(iii) A line of best fit that is horizontal (parallel to x-axis) shows constant production, performance, sales.

Revision exercise 1

1. The table below shows the tax collection of a town council in millions of shillings for 8 consecutive months of a given year.

months	1	2	3	4	5	6	7	8
sales	60	45	52	40	45	42	35	48

   (a) Construct a four year moving average
   (b) Plot a graph of the moving averages together with the original data.
   (c) Comment on the trend of sales over 8month period. [general decline of sales over eight months]

2. The table below shows the termly marks scored in mathematics by a certain student in four years of secondary school.

Year	1^{st}	2^{nd}	3^{rd}
1	36	50	54
2	40	45	60
3	39	46	70
4	49	50	x

   (a) Construct the 3-month moving average for the given data
   (b) On the same axes, plot the original data and the 3-month moving averages.
   (c) Use your graphs to estimate the mark x scored in term 3 year 4. [45%]

3. The sales of brand A ina supermarket for a period of five years are given in the table below

Year	1	2	3	4	5
sales	230	241	259	272	288

   (a) Construct the 3-month moving average for the given data
   (b) On the same axes, plot the original data and the 3-year moving averages.
   (c) Use your graphs to estimate the sales of the 6^{th} year [45%]

Price indices

Index numbers are statistical measures that show trends in a variable or group of variable with time. In Economic they are commonly used to calculate changes in prices.

Terms Used

Base year (P_0) is a year or period when prices of items considered being stable or it is a reference year

Current year (P_n) is a year for which the indices are to be computed

Types of price indices

(a) Price relative (P,R) = $\frac{P_n}{P_o} x\ 100$

(b) Simple price index = $\frac{\sum \frac{P_n}{P_o} x\ 100}{n}$

(c) Simple Aggregate Price Index (S.A.PI) = $\frac{\sum P_n}{\sum P_0} x100$

(d) the weighted price index = $\frac{\sum wP_n}{\sum wP_0} x100$

Examples 8

The table below shows the prices of items and their corresponding weights in the years 2000 and 2004

Item	Price (U Shs)		Weight
	2000	**2004**	
Food	55,000	60,000	4
Housing	48,000	52,000	2
Transport	16,000	20,000	1

Using 2000as the base year calculate the weighted price index for the items in 2004. (05 marks)

W.P.I = $\frac{\sum wP_{2004}}{\sum wP_{2000}} x100 = \frac{60{,}000\ x\ 4+52{,}000x2+20000x\ 1}{55{,}000x\ 4+48{,}000\ x\ 2+16{,}000\ x1} x100 = 111.15(2DP)$

Examples 9

The table below shows the expenditure in shillings of University student for the years 2005 and 2006.

ITEM	EXPENDITURE(Shs)		WEIGHT
	2005	2006	
Text books	100,000	120,000	3
Pocket money	50,000	70,000	2
Research	40,000	50,000	1

Using the year 2005 as the base, calculate the weighted aggregate price index. (05marks)

Weighted aggregate index $= \frac{\sum wP_{2006}}{\sum wP_{2005}} \ x \ 100$

$= \frac{3 \ x \ 120{,}000 + 2 \ x \ 70{,}000 + 1 \ x \ 50{,}000}{3 \ x \ 100{,}000 + 2 \ x \ 50{,}000 + 1 \ x \ 40{,}000} x100$

$= \frac{550{,}000}{440{,}000} x100$

= 125

Examples 10

The table below shows the expenditure of a family for the months of January and July in a certain year.

Item	Expenditure (Shs)		Weight
	January	July	
Food	150,000	174,000	8
Rent	50,000	60,000	2
Clothing	100,000	125,000	6
Power	20,000	25,000	1
Water	60,000	90,000	4

(a) Calculate the:

(i) Price relative for each item (03 marks)

Solution

Price relative $= \frac{P_{July}}{P_{June}} \ x100$

Price relative for food $= \frac{174{,}000}{150{,}000} \ x100 = 116$

Price relative for rent $= \frac{60{,}000}{50{,}000} \ x100 = 120$

Price relative for clothing $= \frac{125{,}000}{100{,}000} \ x100 = 125$

Price relative for Power $= \frac{25{,}000}{20{,}000} \ x100 = 125$

Price relative for Water = $\frac{90,000}{60,000}\ x100$ = 150

(ii) Simple aggregate index (04 marks)

Solution

Simple aggregate index = $\frac{\sum P_{July}}{\sum P_{June}}$

$= \frac{174000+60000+125000+25000+900}{150000+50000+100000+20000+60000}$

$= \frac{474,000}{380,000}$

= 1.25

(b) (i) find the weighted aggregate price index. (06 marks)

Solution

Weighed aggregate price index = $\frac{\sum P_{July} x\ w}{\sum P_{June}\ x\ w}$x100

$= \frac{174000\ x\ 8\ +60000x\ 2+125000x6+25000x1+900x4}{150000\ x\ 8\ +50000x2+100000x6+20000x1+60000x4}\ x100$

$= \frac{2,647,000}{1,160,000} x100$

= 123

(ii) comment on your result in (b) (i) (02marks)

Prices increased by 23%

Examples 11

The table below shows the prices in US dollars and weights of five components of an engine, in 1998 and 2005.

COMPONENT	A	B	C	D	E
PRICE ($) 1998	35	70	43	180	480
PRICE ($) 2005	60	135	105	290	800
WEIGHT	6	5	3	2	1

Taking 1998 as the base year

(a) Calculate for 2005the:

(i) simple aggregate price index. (03marks)

S.A.P.I = $\frac{\sum P_{2005}}{\sum P_{1998}} x100 = \frac{60+135+105+290+800}{35+70+43+180+480} x100 = \frac{1390}{808} x100 = 172$

(ii) price relative of each component. (03marks)

Price relative = $\frac{P_{2005}}{P_{1998}}$x100

Price relative for A = $\frac{60}{35}x100 = 171.4$

Price relative for B = $\frac{135}{70}x100 = 192.9$

Price relative for C = $\frac{105}{43}x100 = 244.2$

Price relative for D = $\frac{290}{180}x100 = 161.1$

Price relative for E = $\frac{800}{480}x100 = 166.7$

(iii) weighted aggregate price index. (06marks)

W.A.P.I = $\frac{\sum wP_{2005}}{\sum wP_{1998}}x100 = \frac{60x6+135x5+105x3+290x2+800x1}{35x6+70x5+43x3+180x2+480x1}x100 = \frac{2730}{1529}x100 = 178.5$

(b) Estimate the cost of an engine in 1998 given that its cost in 2005 was 1600 US dollars

Method 1: using S.A.P.I

Let x be the cost

$\frac{1600}{x}x100 = 172$; x = 930.2 US dollars

Method 2 Using W.A.P.I

$\frac{1600}{x}x100 = 178.5$; x= 896.4 US dollars

Examples 12

The table below shows the price (in Ug Shs) of some food Items in January, June and December together with the corresponding weights.

Item	Price (in Ug Shs)			Weight
	January	June	December	
Matooke (1 bunch)	15,000	13,000	18,000	4
Meat (1kg)	6,500	6000	7,150	1
Posho(1kg)	2,000	1,800	1,600	3
Beans (1kg)	2,200	2,000	2,860	2

(a) Simple aggregate price index for June

S.A.P.I = $\frac{\sum P_{June}}{\sum P_{Jan}}x100 = \frac{13,000+6000+1800+2000}{15000+6500+2000+2200}x100 = 88.71$

Comment on your results (05 marks)

There was a general decrease in the prices of items in the month of June by 11.29%

(b) Weighted aggregate price index for December

W.A.P.I = $\frac{\sum wP_{Dec}}{\sum wP_{June}} x100 = \frac{18{,}000x4+7150x1+1600x3+2860x2}{13000x4+6000x1+1800x2+2000x2} x100 = 116.6$

Comment on your results (10 marks)

There was a general increase in the prices of items in the month of December by 11.29%

Examples 13

(a) The table below shows the price (U shs) of flour and eggs in the years of 2000 and 2010

COMMODITY	PRICE (U shs)	
	2000	2010
Flour	3000	5000
Eggs (1tray)	5000	7000

Taking 2000 as the base year, calculate the:

(i) Price relative of each commodity

Price relative = $\frac{P_{2010}}{P_{2000}} x100$

Price relative for flour = $\frac{5000}{3000} x100 = 166.7$

Price relative for eggs = $\frac{7000}{5000} x100 = 140$

(ii) Simple aggregate price index

S.A.P.I = $\frac{\sum P_{2010}}{\sum P_{2000}} x100 = \frac{5000+7000}{3000+5000} x100 = 150$

Comment on your results (08marks)

The prices of items increased by 50% from 2000 to 2010

(b) The data below shows items with their corresponding prices relatives and weights

ITEM	PRICE RELATIVE	WEIGHT
Food	120	172
Clothing	124	160
Housing	125	170
Transport	135	210
Others	104	140

(i) Find the cost of living index

Cost of living index = weighted price index

$= \frac{\sum Pw}{\sum W}$ where p = price relative, w = weights

ITEM	PRICE RELATIVE(P)	WEIGHT (w)	Pw
Food	120	172	20640
Clothing	124	160	19840
Housing	125	170	21250
Transport	135	210	28350
Others	104	140	14560
SUM		852	104640

Cost of living index $= \frac{104640}{852} = 122.8$

(ii) Comment on your result . (07 marks)

The cost of living increased by 22.8%

Revision Exercise 2

1. The cost of making a cake is calculated from the cost of baking flour, sugar, milk and eggs. All these *ingredients are measured in kg. The table below gives the cost of these items in year 1 and year 2*

ITEM (kg)	***Price***		***Weight***
	Year 1	Year 2	
Flour	60	78	20640
Sugar	50	40	19840
Milk	25	30	21250
Eggs	10	15	28350

Using year 1 as base year

(a) (i) Calculate the price relative for each item.

Hence find simple price index for the cost of making a cake.[120]

(ii) Find the weighted aggregate price index for the cost of making a cake. [118]

(b) If the cost of making a cake in year 2 was shs 30. Find the cost in year 1 using the two indices. [25, 25.5]

2. The table below shows the price of commodities in year 1 and year 2 per kg

ITEM (kg)	***Price***	
	Year 1	Year 2
Flour	1200	1800
Sugar	1300	2000
Milk	800	1300
Eggs	700	1000

Find

(i) Price relative of each item in year 2 using year 1 as base year

(ii) Simple aggregate price index in year 2 using year 1 as base year [152.5]

3. Given below are prices of food items and their corresponding weights in Year 1 and year 2

Year	Year1		Year 2	
	Price (shs)	weight	Price (shs)	weight
Chips	800	12	950	10
Bread	350	6	300	7
Cake	250	4	280	5
Fruit	350	8	430	8
Omelette	500	9	600	10
Tea	300	10	300	12
Liver	1500	15	1600	12

Using year 1 as base year, calculate

(i) Price relative of each item

(ii) The simple price index of the items [110]

(iii) Weighted aggregate price index for year 2 [101]

Thank You

Dr. Bbosa Science