Primary 5 Mathematics

Term 1

Theme: Patterns and Sequences

Topic 4/5 - Patterns and Sequences

Learning Outcome: The learner is able to relate and apply simple computation skills involving patterns and sequence in real life situations.

Triangular numbers

- Triangular numbers are special numbers that can be arranged in the shape of a triangle.
- They are formed by adding consecutive natural numbers.
- Example:
    - 1 = first triangular number
    - 1 + 2 = 3 (second triangular number)
    - 1 + 2 + 3 = 6 (third triangular number)

So the sequence is: **1, 3, 6, 10, 15, 21, …**

Formula for finding triangular numbers

$\boldsymbol{T_n = \frac{n(n+1)}{2}}$, where n is the position of the triangular number.

Example 1

(i) For fifth triangular number, n = 5

$\boldsymbol{T_5 = \frac{5(5+1)}{2}}$ **= 15**

(ii) For seventh triangular number, n = 7

$$T_5 = \frac{7(7+1)}{2} = 28$$

Exercise 1

1. Find the 4th triangular number.
2. Find the 8th triangular number.
3. Find the 12th triangular number.
4. Find the 15th triangular number.
5. Find the 20th triangular number.
6. A triangle is formed with dots. The first row has 1 dot, the second row has 2 dots, and the third row has 3 dots, and so on. How many dots are there in the 5th row altogether?
7. A child arranges marbles in a triangular pattern. If he uses 6 rows, how many marbles does he need?
8. A farmer arranges trees in triangular rows. If he plants 10 rows, how many trees are there in total?
9. A teacher arranges chairs in triangular form for a play. If there are 7 rows, how many chairs are used?
10. A pyramid of balls is made with 12 rows. How many balls are used in total?

Square numbers

- A **square number** is the result of multiplying a number by itself.
- Example:
  - 2×2=4 → 4 is a square number.
  - 5×5=25 → 25 is a square number.

So the sequence of square numbers begins: **1, 4, 9, 16, 25, 36, 49, 64, …**

Definition

If n is a natural number, then the square of n is:

$n^2 = n \times n$

Example:

- $7^2=7\times7=49$
- $12^2=12\times12=144$

Exercise 2

1. Find the square of 6, 12, 15, 20, 25, 30, 50
2. A square garden has sides of 15 meters. What is the area of the garden?
3. A square tile has sides of 12 cm. What is the area of the tile?
4. A farmer builds a square pen with sides of 25 meters. What is the area of the pen?
5. A square playground has sides of 40 meters. What is the area of the playground?
6. A square piece of land has sides of 100 meters. What is the area of the land?

Prime Numbers

- A **prime number** is a natural number greater than 1 that has **exactly two factors**: **1** and **itself**.
- Prime numbers cannot be divided evenly by any other number.

Example

- **2** → factors are 1 and 2 → prime.
- **3** → factors are 1 and 3 → prime.
- **5** → factors are 1 and 5 → prime.
- **7** → factors are 1 and 7 → prime.

☞ Note:

- **1 is not prime** (it has only one factor).
- The smallest prime number is **2** (and it is the only even prime number).

Key Points

- Prime numbers have **exactly two factors**.
- All prime numbers except 2 are **odd**.
- Composite numbers have **more than two factors**.

Exercise 3

1. Identify Prime Numbers

   (i) Is 2 prime?
   (ii) Is 9 prime?
   (iii) Is 17 prime?
   (iv) Is 21 prime?
   (v) Is 29 prime?

2. A farmer has 13 cows. Can he divide them equally into groups other than 1 and 13? Is 13 prime?
3. A teacher has 25 books. Can she divide them equally into groups other than 1 and 25? Is 25 prime?
4. A shopkeeper has 19 oranges. Can he divide them equally into groups other than 1 and 19? Is 19 prime?
5. A student has 27 pencils. Can he divide them equally into groups other than 1 and 27? Is 27 prime?
6. A baker makes 23 cakes. Can he divide them equally into groups other than 1 and 23? Is 23 prime?

Composite Numbers

- A **composite number** is a natural number that has **more than two factors**.
- In other words, it can be divided exactly by **1, itself, and at least one other number**.
- Composite numbers are the opposite of **prime numbers**, which have only two factors (1 and itself).

Example

- **4** → factors are 1, 2, 4 → composite.
- **6** → factors are 1, 2, 3, 6 → composite.
- **9** → factors are 1, 3, 9 → composite.
- **12** → factors are 1, 2, 3, 4, 6, 12 → composite.

☞ Note:

- **1** is neither prime nor composite.
- **2** is prime (only divisible by 1 and 2).
- The smallest composite number is **4**.

Key Points

- Composite numbers have **more than two factors**.
- They can be **even or odd**.
- All even numbers greater than 2 are composite.

Exercise 4

1. **Identify Composite Numbers**
   (i) Is 9 composite?
   (ii) Is 15 composite?
   (iii) Is 17 composite?
   (iv) Is 21 composite?
   (v) Is 29 composite?
   (vi) Is 35 composite?
   (vii) Is 51 composite?
2. A farmer has 12 cows. He wants to divide them equally into groups. Show two different ways he can group them. Is 12 composite?
3. A teacher has 18 books. She arranges them equally in rows. How many rows can she make? Is 18 composite?
4. A shopkeeper has 25 oranges. Can he divide them equally into groups other than 1 and 25? Is 25 composite?
5. A student has 30 pencils. List all the numbers that can divide 30 exactly. Is 30 composite?
6. A baker makes 36 cakes. He wants to pack them equally into boxes. Show how many boxes he can use. Is 36 composite?

Factorization of Numbers

- **Factorization** means breaking down a number into its **factors** (numbers that divide it exactly).
- Factors are numbers that multiply together to give the original number.

Example:

- Factors of 12 are 1, 2, 3, 4, 6, 12.
- Factorization of 12 can be written as 12=2×2×3.

Key Points

- Factors must divide the number **without remainder**.
- Factorization often uses **prime numbers** (prime factorization).
- Every number can be expressed as a product of prime numbers.

Example

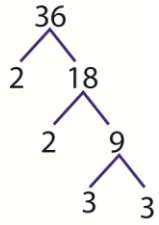
(i) Factorize 36.

- Step 1: Divide by 2 → 36=2×18.
- Step 2: Divide 18 by 2 → 18=2×9.
- Step 3: Divide 9 by 3 → 9=3×3.
- Final Answer: 36=2×2×3×3.

Table

2	36
2	18
3	9
3	3
	1

Factor tree



Hence, 36 = 2 x 2 x 3 x 3

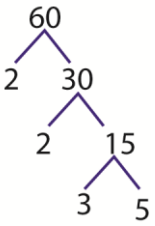
(ii) Factorize 60.

- Step 1: Divide by 2 → 60=2×30.
- Step 2: Divide 30 by 2 → 30=2×15.
- Step 3: Divide 15 by 3 → 15=3×5.
- Final Answer: 60=2×2×3×5.

Table

2	60
2	30
3	15
3	5
5	1

Factor tree



Hence, 60 = 2 x 2 x 3 x 5

Exercise

1. Factorization

   (i) Factorize 24.
   (ii) Factorize 48.
   (iii) Factorize 72.
   (iv) Factorize 100.
   (v) Factorize 84.

2. A farmer has 36 mangoes. He wants to pack them equally into groups. Use factorization to show how many groups he can make.
3. A teacher has 60 pencils. Show how she can divide them into equal groups using factorization.
4. A shopkeeper has 72 oranges. Use factorization to find different ways he can group them.
5. A student has 100 marbles. Show how factorization helps him divide them equally.
6. A baker makes 84 cakes. Use factorization to show how many boxes he can pack them into equally.

Lowest Common Multiple (LCM)

- The **Lowest Common Multiple (LCM)** of two or more numbers is the **smallest number** that is a multiple of all of them.
- Multiples are numbers you get when you multiply a number by 1, 2, 3, 4, …

Example:

- Multiples of 4: 4, 8, 12, 16, 20, …
- Multiples of 6: 6, 12, 18, 24, … ☞ The common multiples are 12, 24, … ☞ The **lowest common multiple (LCM)** is **12**.

Methods of Finding LCM

1. **Listing Multiples**: Write multiples of each number until you find the first common one.

Example:

(i) Find the LCM of 8 and 12.

- Multiples of 8: 8, 16, 24, 32, …
- Multiples of 12: 12, 24, 36, … ☞ LCM = **24**

(ii) Find the LCM of 15 and 20.

- Multiples of 15: 15, 30, 45, 60, …
- Multiples of 20: 20, 40, 60, … ☞ LCM = **60**

2. **Prime Factorization Method**

- Break each number into prime factors.
- Multiply the highest powers of all prime factors.

Example

(i) Find the LCM of 8 and 12.

- Prime factors of 8: $2 \times 2 \times 2 = 2^3$
- Multiples of 12: $2 \times 2 \times 3 = 2^2 \times 3^1$ ☞ LCM = $2^3 \times 3^1$ = **24**

(ii) Find the LCM of 15 and 20.

- Prime factors of 15: 3×5 $3^1 \times 5^1$
- Prime factors of 20: $2 \times 2 \times 3 = 2^2 \times 5^1$ ☞ LCM = $3^1 \times 5^1 \times 2^2$ = **60**

Exercise

1. Find the LCM of
   (i) 6 and 9.
   (ii) 10 and 15.
   (iii) 12 and 18.
   (iv) 8 and 20.
   (v) 16 and 24.

2. Two bells ring every 6 minutes and 8 minutes. After how many minutes will they ring together again?
3. A farmer plants maize every 12 days and beans every 15 days. After how many days will he plant both crops on the same day?
4. A bus arrives at a station every 20 minutes, and another every 30 minutes. After how many minutes will they arrive together?
5. A teacher gives homework every 9 days, and another teacher gives homework every 12 days. After how many days will pupils receive homework from both teachers on the same day?
6. A shop restocks sugar every 18 days and flour every 24 days. After how many days will both be restocked together?

Greatest common factor

- The **Greatest Common Factor (GCF)** of two or more numbers is the **largest number** that divides all of them exactly.
- Factors are numbers that can divide another number without leaving a remainder.

Example:

- Factors of 12: 1, 2, 3, 4, 6, 12
- Factors of 18: 1, 2, 3, 6, 9, 18
- Common factors: 1, 2, 3, 6 ☞ Greatest Common Factor = **6**

Methods of Finding GCF

1. Listing Factors

- Write all factors of each number.
- Identify the largest common factor.

Example

(i) Find the GCF of 20 and 30.

Factors of 20: 1, 2, 4, 5, 10, 20
Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
Common factors: 1, 2, 5, 10 ☞ GCF = **10**

(ii) Find the GCF of 36 and 48.

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
Common factors: 1, 2, 3, 4, 6, 12 ☞ GCF = **12**

2. Prime Factorization Method

- Break each number into prime factors.
- Multiply the common prime factors.

Example

(iii) Find the GCF of 20 and 30.

Prime factors of 20: 2 x 2 x 5
Prime factors of 30: 2 x 3 x 5
Common prime factors: 2, 5 ☞ GCF = 2 x 5 = **10**

(iv) Find the GCF of 36 and 48.

Prime factors of 36: 2 x 2 x 3 x 3
Factors of 48: 2x 2 x 2x 2 x 3
Common prime factors: 2, 2, 2 ☞ GCF 2 x 2 x 3= **12**

Exercise

1. Find the GCF of
   (i) 18 and 24.
   (ii) 28 and 42.
   (iii) 45 and 60.
   (iv) 32 and 48.
   (v) 50 and 75.
2. A farmer has 18 mangoes and 24 oranges. He wants to pack them into boxes with equal numbers of fruits. What is the greatest number of fruits he can put in each box?

3. A teacher has 28 pencils and 42 pens. She wants to share them equally among pupils. What is the greatest number of pupils she can share them among?
4. A shopkeeper has 45 apples and 60 bananas. He wants to arrange them into equal groups. What is the greatest number of groups he can make?
5. A student has 32 marbles and 48 beads. He wants to pack them into equal bags. What is the greatest number of marbles and beads in each bag?
6. A baker has 50 cakes and 75 buns. He wants to pack them equally into boxes. What is the greatest number of cakes and buns in each box?

Sequences

A **sequence** is a list of numbers arranged in a particular order or pattern.

Each number in a sequence is called a **term**.

Sequences can increase, decrease, or follow a special rule.

Types of Sequences

(i) **Increasing sequence** – numbers get bigger (e.g., 2, 4, 6, 8, …).
(ii) **Decreasing sequence** – numbers get smaller (e.g., 100, 90, 80, 70, …).
(iii) **Repeated pattern sequence** – numbers follow a repeating rule (e.g., 1, 3, 1, 3, 1, 3, …).
(iv) **Square numbers sequence** – 1, 4, 9, 16, 25, …
(v) **Triangular numbers sequence** – 1, 3, 6, 10, 15, …

Examples

(i) Find the next two terms in the sequence: 5, 10, 15, 20, …

- Rule: Add 5 each time.
- Next terms: 25, 30.

(ii) Find the next two terms in the sequence: 100, 90, 80, 70, …

- Rule: Subtract 10 each time.
- Next terms: 60, 50.

Exercise

1. Find 3 more terms of each of the following sequences
   (i) 2, 4, 6, 8, ___, ___, ___
   (ii) 10, 20, 30, 40, ___, ___, ___
   (iii) 50, 45, 40, 35, ___, ___, ___
   (iv) 1, 3, 5, 7, ___, ___, ___
   (v) 15, 25, 35, ___, ___, ___
2. A farmer plants trees in rows. The first row has 2 trees, the second row has 4 trees, the third row has 6 trees. How many trees will be in the 5th row?
3. A shopkeeper arranges boxes in stacks. The first stack has 10 boxes, the second has 20, the third has 30. How many boxes will be in the 6th stack?
4. A child saves money in a pattern: 5 shillings on Monday, 10 on Tuesday, 15 on Wednesday. How much will he save on Friday?
5. A teacher gives homework in a sequence: 1 question on day 1, 3 questions on day 2, 5 questions on day 3. How many questions will she give on day 6?
6. A builder arranges bricks: 100 in the first pile, 90 in the second, 80 in the third. How many bricks will be in the 7th pile?

Thank You

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